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АНАЛИЗ СТЕПЕНИ ЭФФЕКТИВНОСТИ МАСКИРОВКИ ПАЛУБНОЙ АВИАЦИИ ОТ РАЗВЕД-ДИВЕРСИОННЫХ ГЛАЙДЕРОВ ПУТЕМ ИСКУССТВЕННО ФОРМИРУЕМОЙ БУФЕРНОЙ ЗОНЫ

Проведен анализ одного из технических решений средств эффективной маскировки самолетов тактической палубной авиации корабельного базирования. Рассмотрен механизм достижения невидимости самолета искусственным формированием в жидкости ограждающей поверхности межоболочечного пространства в виде зоны каустики, конфокальной внутренней поверхности внешней оболочки. Сформулированы условия эффективного достижения требуемой степени скрытности самолета на палубе авианосца.

Ключевые слова: угол аберрации, ультразвуковое излучение, резонанс волнового совпадения, зоны каустики, ограждающая поверхность.

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INVESTIGATION OF THE INFLUENCE OF GRAVITATIONAL FORCES ON THE PROCESS OF DISPLACEMENT OF VISCOPLASTIC FLUIDS

Досліджено чисельне моделювання процесу двовимірної двохфазної фільтрації в'язкопластичної нафти та води з урахуванням гравітаційних сил, деяких властивостей рідин, а також відносних фазових проникностей і капілярних сил на основі різностно-ітераційного методу в рухомих сітках. Для дослідження впливу цих факторів на процес фільтрації розроблено обчислювальний алгоритм, що володіє властивістю адаптованості до особливостей завдань і відрізняється високою точністю.

Ключові слова: *гравітаційні сили, метод змінних напрямків, локально-одномірні схеми, адаптивна сітка, в'язкопластична рідина.*

1. Introduction

Mathematical modeling of oil production processes, as a rule, reduces to solving boundary value problems for systems of nonlinear partial differential equations. Their research can be carried out by analytical or approximate methods. Analytical solutions can be obtained by substantially simplifying the models of real processes, when most of the main parameters are not taken into account, for example, the inhomogeneity of the seams, the non-stationarity of the operating modes, the compressibility of phases, the complexity of the geometry of the filtration region, etc. Such solutions are undoubtedly theoretical and methodical, but their practical significance is significantly limited. Accounting for factors that determine the specific conditions of oil production, significantly complicates mathematical modeling and generates the need for numerical modeling. For numerical modeling of filtration processes within the framework of the adopted model, first of all, it is necessary to develop economical numerical methods with high accuracy.

The tasks of multiphase filtration have specific features, which often do not allow the use of traditional finite-difference methods in a numerical solution. Therefore, there is a need to develop difference schemes in adaptive grids [1, 2], which allow to take into account the singularities of the solution.

Adaptive grids reduce the artificial viscosity and oscillation of the numerical solution. And also it is possible to obtain qualitatively and quantitatively acceptable results in the entire region with a small number of nodes in the computational grid, excluding zones where there are singularities of the solution, for example, zones of large gradients.

2. The object of research and its technological audit

The object of research is numerical simulation of the process of two-dimensional two-phase filtration of viscoplastic oil and water, taking into account the gravitational forces, some properties of liquids, and relative phase permeabilities and capillary forces based on the difference-iteration method in moving grids.

Let's consider the spatially-axisymmetric problem of displacement of viscoplastic oil by water in a layer inhomogeneous in reservoir properties, taking into account capillary and gravitational forces. It is assumed that the liquids are compressible, the roof and the base of the formation are impermeable, a perfect production well with a radius $r=r_c$ located in the center, and injection wells on the outline of the formation.

Assuming that the phase potentials $\varphi_i(r,z,\phi)$ do not depend on ϕ, the equations describing the isothermal process of displacement of viscoplastic oil by water in a cylindrical coordinate system can be represented as:

$$\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_1\Psi_1\frac{\partial\varphi_1}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda_1\frac{\partial\varphi_1}{\partial z}\right)=\frac{\partial}{\partial t}(m\rho_1 s_1),\\
&\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_2\frac{\partial\varphi_2}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda_2\frac{\partial\varphi_2}{\partial z}\right)=\frac{\partial}{\partial t}(m\rho_2 s_2),\\
&P_1-P_2=P_k(s_2),\ s_1+s_2=1,\\
&(r,z,t)\in G_T=\{r_c<r<R,\,0<z<H,\,0<t\le T\},
\end{aligned}\tag{1}$$

where index 1 refers to oil, and 2 to water, and H – thickness of the formation.

Let's note that for compressible fluids it is determined by the formulas:

$$\varphi_i=gz+\int_{p_0}^{p}\frac{\mathrm{d}p}{\rho_i(P_i)},\ i=1,2,\tag{2}$$

where g – the acceleration due to gravity; P_0 – some value taken for the start of the pressure report. The function Ψ_i is the same as in [3–5].

Let at the initial time $t=0$ in the reservoir there is residual water. Then the system (1) can be written as the desired functions $P_2(r,z,t)\equiv P(r,z,t)$ and $P_k(r,z,t)$ in the following dimensionless form:

$$\begin{cases}
\dfrac{1}{r}\dfrac{\partial}{\partial r}\left[\left(r\lambda_1\Psi_1\left(\dfrac{\partial p}{\partial r}+\dfrac{\partial p_k}{\partial r}\right)\right)\right]+\\
+\dfrac{\partial}{\partial z}\left[\lambda_1\left(\dfrac{\partial p}{\partial z}+\dfrac{\partial p_k}{\partial z}+\rho_1 g_1\right)\right]=\dfrac{\partial}{\partial t}(m\rho_1(1-s),\\
\dfrac{1}{r}\dfrac{\partial}{\partial r}\left[(r\lambda_2\dfrac{\partial p}{\partial r}\right]+\dfrac{\partial}{\partial z}\left[\lambda_2\left(\dfrac{\partial p}{\partial z}+\rho_2 g_2\right)\right]=\dfrac{\partial}{\partial t}(m\rho_2 s),\\
p_1=p+p_k(s),\\
s_1=1-s,s_2\equiv s,
\end{cases}$$
$$(r,z,t)\in G_T=\{r_c<r<R,\ 0<z<H,\ 0<t\le T\},\tag{3}$$

the initial and boundary conditions have the form:

$$\begin{aligned}
&p(r,z,0)=p^0(r,z),\\
&p_k(r,z,0)=p^0{}_k(r,z),\ t=0;
\end{aligned}\tag{4}$$

$$\begin{cases}
\dfrac{\partial p}{\partial z}+\dfrac{\partial p_k}{\partial z}=-\rho_1 g_1,\\
\dfrac{\partial p}{\partial z}=-\rho_2 g_2,\ z=0,z=H,\ r_c<r<R,\ 0<t\le T;
\end{cases}\tag{5}$$

$$\begin{cases}
p(r,z,t)+p_k(r,z,t)=f_1(z,t),\\
\dfrac{\partial p_k}{\partial r}=0,\ r=r_c,\ 0<z<H,\ 0<t\le T;
\end{cases}\tag{6}$$

$$\begin{cases}
p(r,z,t)=f_2(z,t),\\
\dfrac{\partial p}{\partial r}+\dfrac{\partial p_k}{\partial r}=0,\ r=R,\ 0<z<H,\ 0<t\le T.
\end{cases}\tag{7}$$

If the phase potentials are given at the initial time, then the initial values for the unknown functions are determined from the expression:

$$\begin{aligned}
&P^0(r,z)=\frac{1}{A_2}\left(e^{A_2\varphi_2^0 Z-A_2 g_2 z}-B_2\right),\\
&P_k^0(r,z)=\frac{1}{A_1}e^{A_1\varphi_1^0-A_1 g_1 z}-P^0(r,z).
\end{aligned}$$

As can be seen, the equations of system (3), describing the axisymmetric process of displacement of viscoplastic oil by water, are nonlinear. Therefore, the only effective device for solving such problems is numerical simulation.

3. The aim and objectives of research

The aim of research is development of efficient-economical numerical methods for solving plane-radial two-dimensional (axisymmetric) problems of nonlinear filtration of a multiphase compressible fluid. These methods will take into account the features of the solution and will be suitable for a wide range of tasks. They can also be used to create a software package for performing numerical calculations and studies on the basis of numerical modeling of various nonlinear filtering processes.

To achieve this aim, it is necessary to solve such problems:

1. To construct cost-effective difference schemes and an iterative process for finding the distribution of water saturation.

2. Based on numerical experiments, to discuss the computational implementation of the proposed methodology and give practical recommendations on its application.

4. Research of existing solutions of the problem

Among the works devoted to this topic, it is possible to single out the following. In [6], the problem of displacement from a porous medium of oil is solved with the help of a polymer solution, which obeys the power law of filtration. A numerical approach is proposed in [7], which differs ideologically from the finite element method and finite differences. The authors of [8] consider the determination of formation parameters for flows that do not obey Darcy's linear law, and the possibility of distinguishing nonlinear effects.

An important stage in the development of methods for solving non-stationary two-dimensional problems is the method of variable directions (MVD), based on homogeneous approximation and widely used in many problems of mathematical physics [9–15].

However, this method does not admit a direct generalization to the case of a larger number of dimensions and for parabolic equations of a more general form [9]. For example, three-dimensional problems can be solved using MVD only if the filtration in the vertical direction is insignificant.

A more general method for obtaining economical implicit difference schemes, suitable for equations with variables, and even discontinuous coefficients, for quasilinear two-dimensional non-stationary problems is the method of total approximation or locally one-dimensional schemes (LOS) [9].

5. Methods of research

For the numerical solution of problem (3)–(7) let's apply the difference-iteration method in adaptive grids [1, 16].

To this end, let's introduce a spatiotemporal grid in the region $\bar{G}_T=\{r_c\le r\le R,\ 0\le z\le H,\ 0\le t\le T\}$:

$$\bar{\hat{G}}_{h_{i,n}h_j\tau_n}=\begin{Bmatrix} r_{i,n}=r_{i-1,n}+h_{i,n},\ i=\overline{1,M-1},\ r_{0,n}=r_c,\ r_{M,n}=R;\\ z_j=z_{j-1}+h_j,\ j=\overline{1,J-1},\ z_0=0,\ z_J=H;\\ t_n=t_{n-1}+\tau_n,\ n=\overline{1,N-1},\ t_0=0,\ t_N=T. \end{Bmatrix}$$

To construct cost-effective difference schemes, let's use the LOS method. To this end, let's represent the system of equations (3) in the following operator form:

$$\begin{cases} L_{11}p+L_{11}p_k+L_{12}p+L_{12}p_k+Lw_1=\dfrac{\partial}{\partial t}(m\rho_1(1-s)),\\ L_{21}p+L_{22}p+Lw_2=\dfrac{\partial}{\partial t}(m\rho_2 s), \end{cases} \tag{8}$$

where there are the following designations:

$$L_{11}=r^{-1}\frac{\partial}{\partial r}\left[r\lambda_1(r,p_1,s)\Psi_1\frac{\partial}{\partial r}\right],\ L_{12}=\frac{\partial}{\partial z}\left(\lambda_1\frac{\partial}{\partial z}\right),$$

$$L_{21}=r^{-1}\frac{\partial}{\partial r}\left[r\lambda_2(r,p_2,s)\Psi_1\frac{\partial}{\partial r}\right],\ L_{22}=\frac{\partial}{\partial z}\left(\lambda_2\frac{\partial}{\partial z}\right),$$

where $L=\dfrac{\partial}{\partial z}$, $W_\alpha=\lambda_\alpha\rho_\alpha g_\alpha$, $g_\alpha=\rho_\alpha\dfrac{R_0 g}{\rho_0}$ – dimensionless quantity; R_0,P_0 – characteristic dimensional quantities.

The systems (8) are split into two one-dimensional systems:

– by z:

$$\begin{cases} L_{12}P+L_{12}P_k+Lw_1=\dfrac{1}{2}\dfrac{\partial}{\partial t}(m\rho_1(1-s)),\\ L_{22}p+Lw_2=\dfrac{1}{2}\dfrac{\partial}{\partial t}(m\rho_2 s); \end{cases} \tag{9}$$

– by r:

$$\begin{cases} L_{11}P+L_{11}P_k=\dfrac{1}{2}\dfrac{\partial}{\partial t}(m\rho_1(1-s)),\\ L_{21}p=\dfrac{1}{2}\dfrac{\partial}{\partial t}(m\rho_2 s). \end{cases} \tag{10}$$

On the basis of the grid ϖ_τ introduced over t, let's introduce the points $t_{n+\frac{1}{2}}=t_n+\dfrac{\tau}{2}$ and divide each interval of the form $\left(t_n,t_{n+1}\right]$ into two half-intervals:

$$\left(t_n,t_{n+\frac{1}{2}}\right] \text{ and } \left(t_{n+\frac{1}{2}},t_{n+1}\right].$$

The system of equations (9) and (10) can be approximated, respectively, on the half-intervals $\left(t_n,t_{n+\frac{1}{2}}\right]$ and $\left(t_{n+\frac{1}{2}},t_{n+1}\right]$ by a two-layer implicit difference scheme. Let's obtain a chain of one-dimensional schemes, which let's call LOS:

$$\begin{cases} \Lambda_{12}(\bar{t})\bar{Y}_1+\Lambda_{12}(\bar{t})\bar{Y}_2+\Lambda\bar{W}_1=\dfrac{1}{2}\left(\bar{V}_{11}\bar{Y}_{1,\bar{t}}+\bar{V}_{12}\bar{Y}_{2,\bar{t}}\right),\\ \Lambda_{22}(\bar{t})\bar{Y}_1+\Lambda\bar{W}_2=\dfrac{1}{2}\left(\bar{V}_{21}\bar{Y}_{1,\bar{t}}+\bar{V}_{22}\bar{Y}_{2,\bar{t}}\right) \end{cases} \tag{11}$$

and

$$\begin{cases} \Lambda_{11}(\hat{t})\hat{Y}_1+\Lambda_{12}(\hat{t})\hat{Y}_2=\dfrac{1}{2}\left(\hat{V}_{11}\hat{Y}_{1,\bar{t}}+\hat{V}_{12}\hat{Y}_{2,\bar{t}}\right),\\ \Lambda_{21}(\hat{t})\hat{Y}_1=\dfrac{1}{2}\left(\hat{V}_{21}\hat{Y}_{1,\bar{t}}+\hat{V}_{22}\hat{Y}_{2,\bar{t}}\right), \end{cases} \tag{12}$$

where there are the next designations:

$$\Lambda_{11}(t)Y=r_i^{-1}h_i^{-1}\begin{bmatrix} h_{i+1}^{-1}\left(r\lambda_1\Psi_1\right)_{i+\frac{1}{2},j}\left(Y_{i+1,j}-Y_{i,j}\right)-\\ -h_i^{-1}\left(r\lambda_1\Psi_1\right)_{i-\frac{1}{2},j}\left(Y_{i,j}-Y_{i-1,j}\right) \end{bmatrix},$$

$$\Lambda_{21}(t)Y=r_i^{-1}h_i^{-1}\begin{bmatrix} h_{i+1}^{-1}(r\lambda_2)_{i+\frac{1}{2},j}(Y_{i+1,j}-Y_{i,j})-\\ -h_i^{-1}(r\lambda_2)_{i-\frac{1}{2},j}(Y_{i,j}-Y_{i-1,j}) \end{bmatrix},$$

$$\Lambda_{\alpha 2}(t)Y=h_j^{-1}\begin{bmatrix} h_{j+1}^{-1}(\lambda_\alpha)_{i,j+\frac{1}{2}}(Y_{\alpha,i,j+1}-Y_{\alpha,i,j})-\\ -h_j^{-1}(\lambda_\alpha)_{i,j-\frac{1}{2}}(Y_{\alpha,i,j}-Y_{\alpha,i,j-1}) \end{bmatrix},$$

$$\Lambda W_\alpha=h_j^{-1}\left[\left(W_\alpha\right)_{i,j+\frac{1}{2}}-\left(W_\alpha\right)_{i,j-\frac{1}{2}}\right],\ \alpha=1,2,$$

$$V_{11}=m\rho_1'(1-s),\ V_{12}=m\rho_1'(1-s)-m\rho_1 s',$$

$$V_{21}=m\rho_2' s,\ V_{22}=m\rho_2 s',\ \bar{t}=t^{n+\frac{1}{2}},$$

$$\hat{t}=t^{n+1},\ Y_1=P(r_i,z_j),\ Y_2=P_k(r_i,z_j),$$

$$1\le i\le M-1,\ 1\le j\le J-1,\ 1\le n\le N-1.$$

Approximation of the initial and boundary conditions leads to the systems:

$$Y^0_{1,i,j}=P(r_i,z_j,0),\ Y^0_{2,i,j}=P_k(r_i,z_j,0),\quad 1\le i\le M-1,\ 1\le j\le J-1, \tag{13}$$

$$\begin{cases}\widehat{Y}_{1,i,1}-\widehat{Y}_{1,i,0}=-h_1 g_2\widehat{\rho}_{2,i,0},\\ \widehat{Y}_{2,i,1}-\widehat{Y}_{2,i,0}=h_1(\widehat{\rho}_2 g_2-\widehat{\rho}_1 g_1)_{i,0},\end{cases} \tag{14}$$

$$\begin{cases}\widehat{Y}_{1,i,1}-\widehat{Y}_{1,i,0}=-h_1 g_2\widehat{\rho}_{2,i,0},\\ \widehat{Y}_{2,i,1}-\widehat{Y}_{2,i,0}=h_1(\widehat{\rho}_2 g_2-\widehat{\rho}_1 g_1)_{i,0},\\ \widehat{Y}_{1,i,J}-\widehat{Y}_{1,i,J-1}=-h_J g_2\widehat{\rho}_{2,i,J},\\ \widehat{Y}_{2,i,J}-\widehat{Y}_{2,i,J-1}=h_J(\widehat{\rho}_2 g_2-\widehat{\rho}_1 g_1)_{i,J},\end{cases} \tag{15}$$

$$\begin{cases}\widehat{Y}_{1,0,j}+\widehat{Y}_{2,0,j}=\widehat{\phi}_{1,j}\\ h_1^{-1}\left(\widehat{Y}_{2,1,j}-\widehat{Y}_{2,0,j}\right)=0,\ i=0,\ 1\le j\le J-1,\ 1\le N\le N-1,\end{cases} \tag{16}$$

$$\begin{cases}\widehat{Y}_{1,M,j}-\widehat{Y}_{1,M-1,j}+\widehat{Y}_{2,M,j}-\widehat{Y}_{2,M-1,j}=0,\\ \widehat{Y}_{1,M,j}=\widehat{\phi}_{2,j},\ i=M,\ 1\le j\le J-1,\ 1\le n\le N-1.\end{cases} \tag{17}$$

The solution of the systems under consideration is difficult because of the nonlinearity, since the coefficients of the equations entering into it depend on the unknown functions. Therefore, when solving such systems, iterative methods are used that make it possible to fully exploit the advantages of implicit schemes and to conduct calculations with a larger time step.

Linearization of the nonlinear equations of the system (11)–(17) can be carried out in two ways – by the simple iteration method and the Newton method [17]. In the future let's use the simple iteration method, which essence lies in the fact that on a new time layer the values of nonlinear terms coincide with the values on the previous time layer.

Thus, by opening all the terms in the systems (11) and (12), linearizing the nonlinear terms for P, P_k and s, and introducing the following designation:

$$Az=\begin{bmatrix}az_{11} & az_{12}\\ az_{21} & az_{22}\end{bmatrix},\ cz=\begin{bmatrix}cz_{11} & cz_{12}\\ cz_{21} & cz_{22}\end{bmatrix},$$

$$bz=\begin{bmatrix}bz_{11} & bz_{12}\\ bz_{21} & bz_{22}\end{bmatrix},\ \varphi z=\begin{bmatrix}fz_1\\ fz_2\end{bmatrix},$$

$$AR=\begin{bmatrix}ar_{11} & ar_{12}\\ ar_{21} & ar_{22}\end{bmatrix},\ CR=\begin{bmatrix}cr_{11} & cr_{12}\\ cr_{21} & cr_{22}\end{bmatrix},$$

$$BR=\begin{bmatrix}br_{11} & br_{12}\\ br_{21} & br_{22}\end{bmatrix},\ \varphi R=\begin{bmatrix}fr_1\\ fr_2\end{bmatrix},$$

let's obtain as a result the equations for each direction:

– in z direction:

$$\begin{cases}-CZ^{(e)}_{i,0}Y^{(e+\frac{1}{2})}_{i,0}+BZ_{i,0}Y^{(e+\frac{1}{2})}_{i,1}=-\varphi Z^{(e)}_{i,0},\ j=0,\\ AZ^{(e)}_{i,j}Y^{(e+\frac{1}{2})}_{i,j-1}-CZ^{(e)}_{i,j}Y^{(e+\frac{1}{2})}_{i,j}+BZ^{(e)}_{i,j}Y^{\left(e+\frac{1}{2}\right)}_{i,j+1}=-\varphi Z^{(e)}_{i,j},\\ 1\le j\le J-1,\\ AZ^{(e)}_{i,J}Y^{\left(e+\frac{1}{2}\right)}_{i,J-1}-CZ^{(e)}_{i,J}Y^{\left(e+\frac{1}{2}\right)}_{i,J}=-\varphi Z^{(e)}_{i,J},\ j=J,\end{cases} \tag{18}$$

$$0\le i\le M,\ 0<n<N;$$

– in r direction:

$$\begin{cases}-CR^{(e)}_{0,j}\cdot\widehat{Y}^{(e+1)}_{0,j}+BR^{(e)}_{0,j}\widehat{Y}^{(e+1)}_{1,j}=-FR^{(e)}_{0,j},\ i=0,\\ AR^{(e)}_{i,j}\cdot\widehat{Y}^{(e+1)}_{i-1,j}-CR^{(e)}_{i,j}\cdot\widehat{Y}^{(e+1)}_{i,j}+BR^{(e)}_{i,j}\cdot\widehat{Y}^{(e+1)}_{i+1,j}=-FR^{(e)}_{i,j},\\ 1\le i\le M-1,\\ AR^{(e)}_{M,j}\cdot\widehat{Y}^{(e+1)}_{M-1,j}-CR^{(e)}_{M,j}\cdot\widehat{Y}^{(e+1)}_{M,j}=-FR^{(e)}_{M,j},\quad i=M,\end{cases} \tag{19}$$

$$0\le j\le J,\ 0<n\le N.$$

Thus, the solution of problem (11)–(17) reduces to solving two independent one-dimensional problems:

Problem 1: (11), (13)–(15).

Problem 2: (12), (16), (17).

Problem 1 is solved first, and the initial approximations are chosen from (11). The implementation algorithm is as follows. If denote the index number in parentheses by the iteration number, let's set $Y^{(0)(n+1)}_{1,i,j}=Y^n_{1,i,j},\ Y^{(0)(n+1)}_{2,i,j}=Y^n_{2,i,j}$.

Solving problem 1 by the matrix sweep method [9], let's determine the intermediate values of the grid functions:

$$Y^{(1)\left(n+\frac{1}{2}\right)}_{1,i,j}=Y_1\left(r_i,z_j,t^{n+\frac{1}{2}}\right),\ Y^{(1)\left(n+\frac{1}{2}\right)}_{2,i,j}=Y_2\left(r_i,z_j,t^{n+\frac{1}{2}}\right).$$

Next, taking the values of the grid functions found for the initial approximations for problem 2 and solving it by the matrix sweep method, let's determine the nodal values of the grid functions:

$$Y^{(1)\left(n+\frac{1}{2}\right)}_{1,i,j}=Y_1(r_i,z_j,t^{n+1}),\ Y^{(1)(n+1)}_{2,i,j}=Y_2(r_i,z_j,t^{n+1}).$$

Thus, on the grid $\bar{\bar{\Omega}}_{h_i h_j \tau_{n+1}}$ let's obtain the values of the grid functions in the $(l+1)$-th approximation. Similarly let's define all other approximations.

Iterations continue until such l_m, as the following conditions are fulfilled:

$$\max_{i,j}\left|Y^{(l_m+1)}_{1,i,j}-Y^{(l_m)}_{1,i,j}\right|\le\varepsilon_1,$$

$$\max_{i,j}\left|Y^{(l_m+1)}_{2,i,j}-Y^{(l_m)}_{2,i,j}\right|\le\varepsilon_2,$$

where $\varepsilon_1,\varepsilon_2$ – convergence accuracy.

6. Research results

The carried out methodological calculations show that in the numerical simulation of two-dimensional problems it is also expedient to apply the difference-iteration method in moving grids. During the calculations, an adaptive grid by r was used, which was constructed on the basis of two criteria. By the first criterion for $z=0$, the nodal point was defined:

$$r_1^*(t)=\max\left|\frac{s(r_{i+1},z_1,t)-s(r_i,z_1,t)}{h_{i+1}}\right|,$$

in which the water saturation gradient reaches its maximum value.

According to the second criterion, a similar point was determined on the roof of the formation ($z=H$):

$$r^*_{J\text{-}1}(t)=\max_{0\le i\le M}\left|\frac{s(r_{i+1},z_{J-1},t)-s(r_i,z_{J-1},t)}{h_{i+1}}\right|.$$

The «crushing» of the grid was carried out in a region containing a segment $\left[r_1^*(t), r_{J-1}^*(t)\right]$, and the condition of α-quasi-uniformity of the grid with respect to the variable r was fulfilled.

Let's note that the variable z uses a non-uniform fixed grid, condensing only in the vicinity of $z=0$ and $z=H$.

Numerical calculations were carried out for the following initial data:

$$R=R_0=100\ m;\ H=10\ m;\ r_c=0.1\ m;\ m=0.2;$$

$$k=k_0=10^{-12}\ m^2;\ \mu_1=0.3\text{ poise};\ \mu_2=\mu_0=0.01\text{ poise};$$

$$S^0=0.15;\ \rho_0=1\,\frac{\text{g}}{\text{cm}^3};\ G_1=0.1\,\frac{\text{atm}}{\text{m}}=100\,\frac{\text{dyn}}{\text{cm}^3};$$

$$f_1(t)=90\,\frac{\text{kgf}}{\text{cm}^2};\ f_2(t)=100\,\frac{\text{kgf}}{\text{cm}^2}.$$

If let's neglect the forces of gravity in problem (3)–(7), then the criteria for selecting the points in the vicinity of which the computational grid should thicken are identical to those of the one-dimensional problem [16].

The influence of the initial gradient on the displacement process is illustrated in Table 1, in which for the case $G_1=0$ the columns with the number I correspond to the case, and to the case $G_1=0.001$ – with the number III. It should be noted that the densities of oil and water are determined by the following formulas:

$$\rho_1(p_1)=0.00853p_1+0.82592,$$

$$\rho_2(p_2)=0.01033p_2+0.99989.$$

The value of the water saturation is assumed to be 0.15 at $t=0$ and $z=0$, water pressure – $100\,\frac{\text{kgf}}{\text{cm}^2}$. These functions determine the functions $p^0(r,z)$ and $p_k^0(r,z)$ – the initial conditions for the unknown functions.

At the injection well at $z=0$, the water pressure is set equal $100\,\frac{\text{kgf}}{\text{cm}^2}$, and at $z=0.2$; 0.4; 0.6; 0.8; 1.0 is determined, respectively, 0.9980; 0.9960; 0.9941; 0.9921; 0.9901. The pressure difference (pressure difference at the injection and production wells) was equal $10\,\frac{\text{kgf}}{\text{cm}^2}$ for each z.

The results of the calculations of the water saturation obtained for the distribution of the initial gradient $G_1=5\cdot10^{-4}$ are given in the second column of Table 1. Comparison of columns II and III of Table 1 shows that the filtration process is largely characterized by values G_1 (as in the flat-radial case). Moreover, a decrease in the parameter G_1 leads to the case of the problem for $G_1=0$, which is plausible.

The results of calculations carried out to determine the effect of gravity on the displacement process at $z=0$ are given in Table 2, according to which, even at low thicknesses of productive layers, gravitational forces influence the displacement process, and this influence increases with time. In fact, if at the moment $t=0.08$ (let's note that in the dimensional form 0.02592 corresponds to one month), on the contour the difference in water saturation was 0.0077; at $t=0.24$–0.0122; at $t=1.04$ it becomes equal to 0.0292.

Table 1

The influence of the initial gradient on the displacement process

T	0.1608			0.8808			1.3608		
Columns / r, m	I	II	III	I	II	III	I	II	III
0.001	0.1500	0.1500	0.1500	0.1514	0.1520	0.1528	0.1555	0.1566	0.1672
0.1	0.1500	0.1500	0.1500	0.1516	0.1526	0.1536	0.1564	0.1574	0.1590
0.2	0.1500	0.1500	0.1500	0.1523	0.1530	0.1537	0.1603	0.1614	0.1631
0.3	0.1500	0.1500	0.1500	0.1537	0.1542	0.1547	0.1708	0.1727	0.1752
0.4	0.1500	0.1500	0.1500	0.1574	0.1580	0.1586	0.1963	0.2013	0.2086
0.5	0.1500	0.1500	0.1500	0.1684	0.1694	0.1705	0.2725	0.2822	0.2904
0.6	0.1503	0.1504	0.1504	0.1958	0.1985	0.2022	0.3365	0.3420	0.3495
0.7	0.1515	0.1506	0.1516	0.2771	0.2835	0.2889	0.3833	0.3882	0.3941
0.8	0.1572	0.1574	0.1576	0.3443	0.3483	0.3530	0.4205	0.4257	0.4316
0.9	0.1593	0.1940	0.1951	0.3975	0.4014	0.4061	0.4572	0.4620	0.4680
1.0	0.3189	0.3225	0.3259	0.4466	0.4504	0.4550	0.4953	0.4992	0.5057

Table 2

The influence of gravity on the displacement process at $z=0$

t	0.0808		0.2408		0.6408		1.0408	
g / r, m	0	981	0	981	0	981	0	981
0.001	0.1500	0.1500	0.1500	0.1500	0.1500	0.1500	0.1503	0.1527
0.1	0.1500	0.1500	0.1500	0.1500	0.1500	0.1502	0.1508	0.1529
0.2	0.1500	0.1500	0.1500	0.1500	0.1500	0.1507	0.1518	0.1538
0.3	0.1500	0.1500	0.1500	0.1500	0.1503	0.1514	0.1547	0.1564
0.4	0.1500	0.1500	0.1500	0.1503	0.1513	0.1528	0.1630	0.1641
0.5	0.1500	0.1500	0.1501	0.1511	0.1548	0.1562	0.1839	0.1859
0.6	0.1500	0.1500	0.1503	0.1539	0.1662	0.1675	0.2431	0.2445
0.7	0.1500	0.1502	0.1527	0.1681	0.1976	0.1976	0.3106	0.3188
0.8	0.1507	0.1515	0.1670	0.2384	0.2820	0.2880	0.3594	0.4250
0.9	0.1640	0.1650	0.2358	0.3076	0.3438	0.3576	0.3970	0.4205
1.0	0.2759	0.2816	0.3299	0.3421	0.3945	0.4168	0.4355	0.4647

Table 3 shows the distribution of water saturation by r at time $t=0.32$, obtained on the basis of the use of two-dimensional (without gravity) and plane-radial models with and without initial gradient taken into account.

Table 3

Distribution of water saturation by r

r, m \ G_1	Two-dimensional problem without gravity		Radial filtration	
	$G_1=0$	$G_1=10^{-3}$	$G_1=0$	$G_1=10^{-3}$
0.001	0.1500	0.1500	0.1500	0.1500
50	0.1500	0.1503	0.1500	0.1502
60	0.1511	0.1514	0.1509	0.1511
65	0.1530	0.1533	0.1525	0.1530
70	0.1570	0.1575	0.1567	0.1572
75	0.1654	0.1665	0.1652	0.1658
80	0.1814	0.1835	0.1810	0.1832
85	0.2154	0.2202	0.2146	0.2193
90	0.2672	0.2747	0.2656	0.2742
95	0.2942	0.3004	0.2938	0.2996
100	0.3469	0.3530	0.3463	0.3528

Calculations show that in the two-dimensional problem the water saturation value on the injection well taking into account G_1 is 0.3520, and at the radial filtration it is 0.3528. That is, they differ insignificantly and at the same time, in both cases, at this moment the water advances to the same distance – 50 m. It follows that in modeling the process without taking gravity into account, it is expedient to simplify the geometry of the filtration region, i. e. consider the plane-radial flow, in view of the considerable simplicity of the calculations.

7. SWOT analysis of research results

Strengths. The proposed algorithm can be used for hydro-gas dynamic calculations related to the development and operation of oil fields containing abnormal oil.

This algorithm contributes to:
– saving time;
– reducing the amount of computing process;
– increase the productivity of oil wells;
– increase the speed of calculation.

Weaknesses. The disadvantages of this method include the calculation complexity.

Opportunities. Thanks to the introduction of this method in the oil industry, the oil recovery is expected to increase.

Threats. To implement this method, additional equipment is needed, and, accordingly, it is money costs. Highly qualified personnel are also required to work with this equipment.

8. Conclusions

1. Economical difference schemes are constructed that combine the advantages of explicit and implicit schemes and make it possible to reduce the two-dimensional problem to a chain of one-dimensional problems. A difference-iterative method is also proposed in moving grids for solving two-dimensional (axisymmetric) non-stationary filtration problems of anomalous liquids, by means of which an iterative process is constructed to find the distribution of water saturation.

2. The carried out calculations to determine the influence of gravity on the displacement process have shown that at $z=0$, even at low productive-bed thicknesses, gravitational forces influence the displacement process. And over time this influence increases: if at the time $t=0.08$ on the circuit the difference of water saturation was 0.0077; at $t=0.24$–0.0122, then at $t=1.04$ it becomes equal to 0.0292.

It is shown that when modeling the process without taking gravity into account it is expedient to simplify the geometry of the filtration region, i. e., to consider a plane-radial flow in view of the considerable simplicity of the calculations.

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ИССЛЕДОВАНИЕ ВЛИЯНИЯ ГРАВИТАЦИОННЫХ СИЛ НА ПРОЦЕСС ВЫТЕСНЕНИЯ ВЯЗКОПЛАСТИЧНЫХ ЖИДКОСТЕЙ

Исследовано численное моделирование процесса двумерной двухфазной фильтрации вязкопластичной нефти и воды с учетом гравитационных сил, некоторых свойств жидкостей, а также относительных фазовых проницаемостей и капиллярных сил на основе разностно-итерационного метода в подвижных сетках. Для исследования влияния этих факторов на процесс фильтрации разработан вычислительный алгоритм, обладающий свойством адаптируемости к особенностям задач и отличающийся высокой точностью.

Ключевые слова: гравитационные силы, метод переменных направлений, локально-одномерные схемы, адаптивная сетка, вязкопластичная жидкость.

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