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MATHEMATICAL MODELING OF SPEED CHANGE OF VEHICLES AT EMERGENCY BRAKING

Об'єктом дослідження є реконструкція розвитку механізму та обставин дорожньо-транспортної пригоди. Ключовим моментом дослідження є встановлення швидкості руху автомобіля у будь-який момент розвитку дорожньо-транспортної пригоди. Саме точність у визначенні швидкості руху автомобіля впливає на характер основних висновків експертизи. При розрахунку швидкості руху автомобіля експерт вирішує зворотну задачу, тобто визначає швидкість за показниками ефективності гальмування автомобіля та довжини сліду гальмування. За останні два десятиріччя змінилася конструкція гальмівної системи автомобіля. Гальма стали обладнувати анти-блокувальною системою. При цьому ефективність гальмування автомобілів значно покращилася. Але виявилось, що експерти не можуть об'єктивно визначити швидкість руху автомобіля, який обладнаний анти-блокувальною системою гальм, тому що такі гальма не залишають слідів гальмування на дорожньому покритті.

При розробці та вирішенні цієї проблеми з визначення швидкості руху автомобіля, який обладнаний анти-блокувальною системою гальм, використані методи диференціювання та інтегрування складної функції. При порівняльному аналізу існуючого та запропонованого методів розрахунку швидкості руху автомобіля в процесі екстреного гальмування застосований графічний метод.

За результатами роботи розроблена математична модель з визначення швидкості руху автомобіля, який обладнаний анти-блокувальною системою гальм. Ця модель дозволяє врахувати вплив дії сил опору повітря, опору руху та опору підйому не тільки в усталеній фазі гальмування, але і під час реакції водія та час спрацьовування гальм. Аналіз математичної моделі показує, що за ці інтервали часу на автомобіль буде діяти певне сповільнення, яке буде залежати від швидкості руху та стану завантаження автомобіля. Причому, дія сили опору підйому здатна значно збільшувати це сповільнення і вливати на зміну швидкості руху автомобіля. Запропонована математична модель більш точно відображує реальний процес екстреного гальмування автомобіля і забезпечує зниження похибки розрахунку швидкості руху автомобіля на 4–8 % у порівнянні з існуючими розрахунковими методами.

Ключові слова: *дорожньо-транспортна пригода, зміна швидкості руху автомобіля, процес екстреного гальмування, анти-блокувальна система гальм.*

1. Introduction

The relevance of the topic and its feasibility are due to the fact that the problem of improving the methods of investigating the circumstances of a road accident through the development and improvement of new calculation methods is constantly in the field of view of expert vehicles.

Analysis of the practice of investigating criminal cases involving acts of criminal violation of road safety and the operation of vehicles is evidenced by the following. In the reports of the inspection of the place of road accidents, the circumstances of the accident, road conditions, the nature of technical damage to vehicles, traces on the road surface, etc. are not always fully reflected. Using information from the DVR during the auto-technical expertise determines the following problems: there are objective complexities of using data from the DVR to determine the speed of movement, deceleration or acceleration, the coordinates of the location of the car on the road. For example, as evidenced by the relevant scientific and technical publications, the existing calculation methods for assessing the braking efficiency and the speed of the car give a deliberately significant error. Therefore, a reasonable question arises about the validity of the application of existing calculation methods for estimating the parameters of the vehicle's movement during the investigation of an accident. The expediency of solving this topical problem is the need to improve the accuracy and objectivity of assessing the circumstances and the mechanism for development of an accident in general.

2. The object of research and its technological audit

The object of research is the reconstruction of the development of the mechanism and circumstances of a road traffic accident. An objective investigation of an accident is carried out by experts and specialists on certain methods. These expert methods allow to calculate the dynamics of traffic in the process of road accident. The key moment to all calculations of the parameters of the vehicle movement is the establishment of the speed of movement at different times. One forward is the vehicle speed before braking. Also, an expert can be tasked with determining the speed of movement at any time during the development of the accident mechanism. Precisely the accuracy in determining the speed of vehicle movement affects the nature of the main conclusions of the examination. When calculating the vehicle speed, the expert solves the inverse problem, that is, in terms of the braking performance of the vehicle, the length of the braking trace determines the speed of the vehicle at the beginning of the braking and at different times. Experts used this method during the 20th century. During this time the design of the vehicle brake system was changed. The brakes were equipped with an anti-blocking system. The effectiveness of braking cars with anti-blocking system (ABS) has improved. And at the beginning of the 21st century, in the developed countries of the world, the relevant regulations were adopted regarding the mandatory use of ABS on new cars. Now the majority of cars, operated in the last decade, are equipped with ABS. But it turned out that experts can't objectively calculate the speed of the vehicle, it is equipped with ABS, because these modern brakes do not leave traces of braking on the road surface. The expert did not understand what method to follow in such case.

3. The aim and objectives of research

The aim of research is improvement of the accuracy of calculating the car speed in the process of investigating an accident.

The objectives of research:

1. To develop a mathematical model for determining the speed of vehicle movement in the process of emergency braking.
2. To analyze the developed mathematical model.

4. Research of existing solutions of the problem

In Ukraine, Belarus, Russia and some other countries in the study of road accidents experts determine the speed of the vehicle according to approved methods. These methods give the best result for the accuracy of determining the speed of movement of the vehicle in the case that the driver managed to apply emergency braking. As a result, braking traces remain on the road surface. In expert practice, these tracks are given special attention [1].

The problem of determining the speed of the vehicle is complicated by the fact that modern vehicle brakes are equipped with ABS, which does not allow wheel locking to improve road safety [2].

In work [3] it is noted that one of the main factors that predetermines the possibility of determining the cause of an accident is the values of the speeds of vehicle movement. This speed at the time of an accident, for example, collisions in existing techniques are determined by the operation of friction tires on the road, taking into account the traces of the braking distance. In many cases, these methods may not be sufficient, since there are no traces when braking modern vehicles equipped with ABS.

In the world automotive science, the formula for determining the coefficient of traction of wheels with the road became dominant, depending on the longitudinal slip of the tire [4]. In the work it is shown that when braking, the tangential reaction R_x for a given normal reaction R_z will reach its maximum value, when the degree of slip in the contact of the wheel with the road surface will reach its optimum value $s \approx 20$ %. That is, this means that vehicles equipped with ABS provide a higher coefficient of traction of the wheels with the road. But with an increase in the initial speed of vehicle movement before deceleration, this coefficient of adhesion decreases. These factors are not taken into account in expert techniques for determining the vehicle speed in the process of emergency braking.

In the opinion of the author [5], the determination of the vehicle speed just before the collision, transfer, pedestrian collision or immovable object is one of the most urgent and difficult task in expert practice. Excess speed is the most common violation of traffic rules. Exceeding the established speed is in causal connection with the fact of the accident and affects the severity of the consequences.

Of course, for the modern auto-technical expertise of interest is the development of new methods for assessing the parameters of vehicle traffic in the investigation of accidents. Thus, in [6], a method is proposed for determining the speed of a vehicle during a collision, when there are no basic parameters for the standard calculation model. The calculation is based on the dynamics of the seat belt load, which is used as a means to limit the movement of people inside the car during a sharp slowdown. This indirectly allows to determine the car speed at the time of the accident. The application of this method is very limited.

At the present time, automated technical facilities have become widespread in the field of traffic management, which are integrated into an intelligent transport system. With a constant increase in motorization, this can reduce emergency, economic, environmental losses in road traffic by 15 % [7]. In case of violation of traffic rules, the creation of an emergency situation, the information obtained in this way can be used to investigate the circumstances of an accident [8]. But such information systems only began to be created and, as a rule, only in large megacities.

In European countries, experts widely use computer simulation of the development of the mechanism of accidents [9, 10]. For example, very close to the experimental data, the results of estimating the braking efficiency of the vehicle are provided by the software CYBID V-SIM-3.0.35 [11]. This program allows to take into account the difference between the braking performance of the vehicle with and without ABS.

In general, it is possible to state that at present for experts in Ukraine there are no clear recommendations for determining the speed of movement of vehicles equipped with ABS. This leads to the possibility of different estimates of the speed of movement of the vehicle in the study of emergency braking of the same vehicle, may have different effects on the findings of the examination as a whole and give a biased picture of the circumstances and mechanism of the accident.

5. Methods of research

The most accurate method for estimating the speed of the vehicle movement and its dynamics of braking is testing and carrying out the experiment. But in the process of an accident a car can get such damage that the experiment will be impossible. Calculation methods used by experts are also impossible to use, since for calculation it is necessary to know the length of the braking track.

In the work, when studying the works of previous researchers and determining the unsolved scientific problem, the method of analysis is used. When developing and solving deterministic mathematical models for determining the speed of vehicle movement during an emergency braking, the methods of differentiating and integrating a complex function are used. In the comparative analysis of computational methods for determining the speed of movement of the vehicle against time and in the process of emergency braking, a graphic method is applied.

6. Research results

For the general account and analysis of the influence of most factors on the dynamics of the car's movement, let's consider the scheme of external forces and moments acting on the car during its braking (Fig. 1).

In accordance with the design scheme, let's write the detailed equation of the vehicle's movement, decelerating:

$$\frac{G_a j}{g}-(G_a\cos\lambda)(\varphi+f)+$$
$$+\sum\frac{J_k\varepsilon}{r_d}-0.5c_x\rho F_x v^2\mp G_a\sin\lambda=0, \quad (1)$$

where P_j – longitudinal inertia force of the vehicle, $P_j = m_a j=(G_a/g)\,j$, H; G_a – vehicle weight, N; P_w – air resistance force, $P_w = 0.5c_x\rho F_x v^2$, N; c_x – streamlining vehicle coefficient; ρ – air density, $\rho\approx 1.225$ kg/m^3; F_x – frontal area of the vehicle, m^2; λ – magnitude of the longitudinal slope of the road, degrees; j – steady deceleration of the vehicle, m/s^2; g – gravity acceleration, m/s^2; φ – traction coefficient of the wheels with the road; f – coefficient of rolling resistance.

The sum of the inertial forces from the rotating masses of the vehicle and its longitudinal inertia force will be the total inertia force of some conventional mass (reduced mass), which can be expressed in terms of the coefficient of influence of the rotating masses, on the basis of which expression (1) takes the form:

$$\frac{G_a j}{g}\delta_j-(G_a\cos\lambda)(\varphi+f)-0.5c_x\rho F_x v^2\mp G_a\sin\lambda=0, \quad (2)$$

where δ_j – coefficient of accounting for rotating masses.

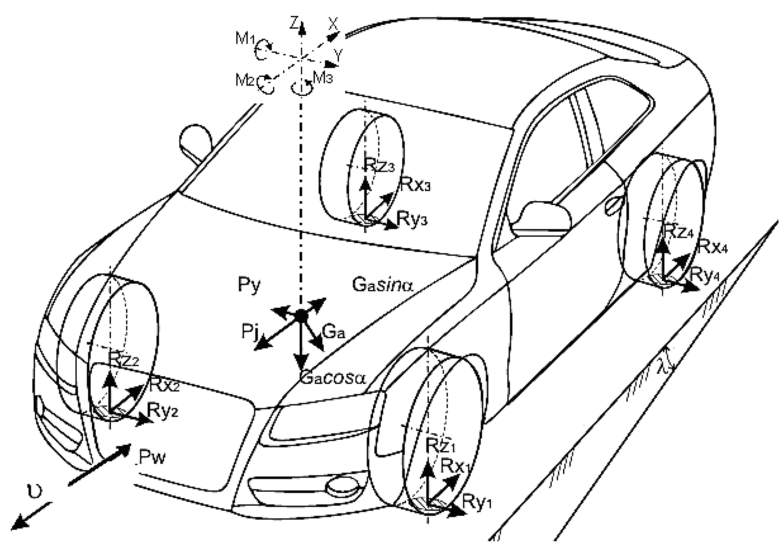

Fig. 1. Diagram of external forces and moments acting on the car during braking: P_j – longitudinal force of car inertia; P_w – air resistance force; G_a – car weight; P_y – lateral force; R_{x1}, R_{x2}, R_{x3}, R_{x4} – longitudinal (tangential) reactions in contact of wheels with the road surface; R_{y1}, R_{y2}, R_{y3}, R_{y4} – lateral reactions in contact of wheels with the road surface; R_{z1}, R_{z2}, R_{z3}, R_{z4} – vertical (normal) reactions in contact of the wheels with the road surface; M_1, M_2, M_3 – inertial moments of the car relative to the axes of the coordinate system; v – direction of vehicle speed; λ – road gradient angle

If there is no wheel slip, δ_j will equal to $\delta_j=1.04+0.04i_i^2$. To simplify the calculation, in view of the slight influence on this calculation, the value δ_j can be assumed as $\delta_j\approx 1.04$ [12].

With the expanded equation of equilibrium of forces (2), let's express the deceleration of the car in case of emergency braking:

$$j=\frac{g}{G_a\delta_j}\Big[(G_a\cos\lambda)(\varphi+f)+0.5c_x\rho F_x v^2\pm G_a\sin\lambda\Big],$$

$$j=\frac{g}{\delta_j}\left[(\cos\lambda)(\varphi+f)+\frac{0.5c_x\rho F_x v^2}{G_a}\pm\sin\lambda\right]. \quad (3)$$

The obtained mathematical model (3) allows to determine the deceleration value for a given speed value. In this case, the speed is an independent variable, abstracted from time. But when the vehicle is decelerated, the speed of movement is constantly decreasing in time. Therefore, in order to take into account such changes, let's write down the mathematical model of the vehicle braking dynamics in a differential form:

$$-\frac{dv}{dt}=\frac{g}{\delta_j}\left[(\cos\lambda)(\varphi+f)+\frac{0.5c_x\rho F_x v^2}{G_a}\pm\sin\lambda\right], \quad (4)$$

or with the initial conditions:

$$\begin{cases}-\dfrac{dv}{dt}=\dfrac{g}{\delta_j}\left[(\varphi+f)\cos\lambda\pm\sin\lambda+\dfrac{0.5c_x\rho F_x v^2}{G_a}\right],\\ v(0)=v_0,\end{cases} \quad (5)$$

where v_0 – speed of vehicle movement at the beginning of braking with maximum intensity, m/s.

To solve this mathematical model, let's write it in a simplified form by making a substitution of complex constants with new variables:

$$-\frac{dv}{dt}=\frac{g}{\delta_j}(b+v^2W), \quad (6)$$

where

$$W=\frac{0.5c_x\rho F_x}{G_a},\ \ \text{s}^2/\text{m}^2;$$

$$b=(\varphi+f)\cos\lambda\pm\sin\lambda.$$

Further, let's express the speed of vehicle movement, as:

$$\frac{dv}{(b+v^2W)}=-\frac{g}{\delta_j}dt$$

and take the indefinite integral:

$$\int\frac{dv}{(b+v^2W)}=-\frac{g}{\delta_j}\int dt+C. \quad (7)$$

Let's reduce the left-hand side of expression (7) to the table integral of the form:

$$\int\frac{dx}{(a^2+x^2)}=\frac{1}{a}\text{arctg}\frac{x}{a}+C.$$

To do this, let's perform the following conversion:

$$\frac{1}{W}\int\frac{\mathrm{d}v}{\left(\frac{b}{W}+v^2\right)}=-\frac{g}{\delta_j}\int\mathrm{d}t+C. \quad (8)$$

After integration:

$$\frac{1}{\sqrt{bW}}\operatorname{arctg}\left(v\sqrt{\frac{W}{b}}\right)=-\frac{g}{\delta_j}t+C,$$

$$\operatorname{arctg}\left(v\sqrt{\frac{W}{b}}\right)=\sqrt{bW}\left(-\frac{g}{\delta_j}t+C\right),$$

$$v\sqrt{\frac{W}{b}}=\operatorname{tg}\left[\sqrt{bW}\left(-\frac{g}{\delta_j}t+C\right)\right]. \quad (9)$$

In order to determine the constant C, let's substitute into the expression (9) the values of the variables t and v, which correspond to the initial conditions of integration, namely, $t=0$ and $v(0)=v_0$. Then the change in the speed of the vehicle movement as a function of time during emergency braking will occur according to the function:

$$v=\sqrt{\frac{b}{W}}\operatorname{tg}\left[\sqrt{bW}\left(-\frac{g}{\delta_j}t+C\right)\right], \quad (10)$$

where C – const, $C=\frac{1}{\sqrt{bW}}\operatorname{arctg}\left(v_0\sqrt{\frac{W}{b}}\right)$, m/s.

With the help of equation (10), it is possible to determine the vehicle speed depending on the current time, taking into account the geometric and weight parameters of the vehicle, the effects of external forces and operating coefficients (Fig. 2).

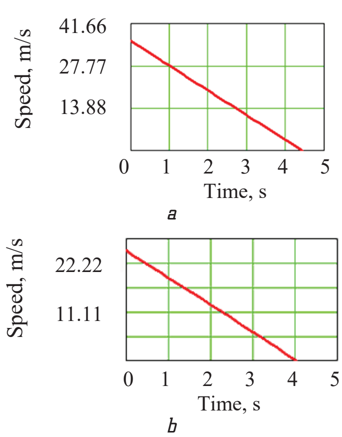

Fig. 2. Results of the calculation of the vehicle speed function as a function of time: *a* – Ford Mondeo, $v_0=36$ m/s, $b=(\cos\lambda(\varphi+f)\pm\sin\lambda)=0.85$, $G=2050$ g H, $F=1.9$ m^2, $\rho=1.2$ kg/m^3, $\delta_j=1.04$, $c_x=0.3$; *b* – Neoplan Tourliner, $v_0=25$ m/s, $b=(\cos\lambda(\varphi+f)\pm\sin\lambda)=0.7$, $G=9000$ g H, $F=8.23$ m^2, $\rho=1.2$ kg/m^3, $\delta_j=1.04$, $c_x=0.79$

Thus, the proposed mathematical model allows to determine the change in the speed of movement from time to time, taking into account many other parameters – $\{\varphi, f, \lambda, \delta_j, c_x, F_x, G_a\}$.

To analyze the effect of the coefficient of traction of the wheels on the road on the speed of vehicle movement, let's write this dependence in the form of a linear equation of the form:

$$\varphi=\varphi_0-Av, \quad (11)$$

where φ_0 – cohesion coefficient of the wheels at the speed of movement close to 0 m/s; A – linear coefficient of the function slope, s/m.

Substituting function (11) in expression (4), let's obtain a complicated mathematical model of the form:

$$-\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{g}{\delta_j}\left(\varphi_0\cos\lambda-Av\cos\lambda+f\cos\lambda\pm\sin\lambda+v^2W\right), \quad (12)$$

after the transformation of which:

$$\frac{\mathrm{d}v}{\left(\varphi_0\cos\lambda-Av\cos\lambda+f\cos\lambda\pm\sin\lambda+v^2W\right)}=-\frac{g}{\delta_j}\mathrm{d}t.$$

The solution of this differential equation is an indefinite integral of the form:

$$\int\frac{\mathrm{d}v}{\left(\varphi_0\cos\lambda-Av\cos\lambda+f\cos\lambda\pm\sin\lambda+v^2W\right)}=$$
$$=-\frac{g}{\delta_j}\int\mathrm{d}t+C_2. \quad (13)$$

This integral is reduced to the tabular form of the integral of the form:

$$\int\frac{\mathrm{d}x}{\left(a^2+x^2\right)}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+C.$$

After integration:

$$\frac{-2\sqrt{2}\operatorname{arctg}\left[\frac{\sqrt{2}\left(-2vW+A\cos\lambda\right)}{\sqrt{-A^2+8W\left(f+\varphi_0\right)\cos\lambda-A^2\cos(2\lambda)+8W\sin\lambda}}\right]}{\sqrt{-A^2+8W\left(f+\varphi_0\right)\cos\lambda-A^2\cos(2\lambda)+8W\sin\lambda}}=$$
$$=-\frac{g}{\delta_j}t+C_2. \quad (14)$$

For a further solution, let's simplify expression (14) by making a substitution of the variables:

$$\frac{-2\sqrt{2}\operatorname{arctg}\left[\frac{\sqrt{2}\left(-2vW+A\cos\lambda\right)}{B}\right]}{B}=-\frac{g}{\delta_j}t+C_2, \quad (15)$$

where

$$C_2=\frac{-2\sqrt{2}\operatorname{arctg}\left[\frac{\sqrt{2}\left(-2v_3W+A\cos\lambda\right)}{B}\right]}{B};$$

$$B=\sqrt{-A^2+8W\left(f+\varphi_0\right)\cos\lambda-A^2\cos(2\lambda)+8W\sin\lambda},\ \text{m/s}.$$

Further, from equation (15) let's express v:

$$\operatorname{arctg}\left[\frac{\sqrt{2}\left(-2vW+A\cos\lambda\right)}{B}\right]=-\frac{B}{2\sqrt{2}}\left(-\frac{g}{\delta_j}t+C_2\right),$$

$$\frac{\sqrt{2}\left(-2vW+A\cos\lambda\right)}{B}=-\operatorname{tg}\left(\frac{B}{2\sqrt{2}}\left(-\frac{g}{\delta_j}t+C_2\right)\right),$$

$$-2vW\frac{\sqrt{2}}{B}=-\operatorname{tg}\left[\frac{B}{2\sqrt{2}}\left(-\frac{g}{\delta_j}t+C_2\right)\right]-\frac{\sqrt{2}}{B}A\cos\lambda,$$

$$v=-\frac{B}{2\sqrt{2}W}\left[-\operatorname{tg}\left(\frac{B}{2\sqrt{2}}\left(-\frac{g}{\delta_j}t+C_2\right)\right)-\frac{\sqrt{2}A\cos\lambda}{B}\right],$$

then at the end:

$$v=\frac{B}{2\sqrt{2}W}\left[\operatorname{tg}\left(\frac{B}{2\sqrt{2}}\left(-\frac{g}{\delta_j}t+C_2\right)\right)+\frac{\sqrt{2}A\cos\lambda}{B}\right]. \quad (16)$$

The suggested mathematical model allows to take into account the influence on the vehicle speed of the dependence $\varphi(v)$ for a certain tire brand.

Thus, on the basis of a more accurate description of the emergency braking of the vehicle, a mathematical model is proposed that allows one to take into account the effect of air resistance on the change in the speed of movement of the vehicle. And also the dependence of the adhesion coefficient of the wheels to the road on the change in the speed of movement, which was not taken into account earlier in the theoretical estimation of the vehicle speed.

In addition, existing mathematical models for determining the vehicle speed determine the effect of external forces – air resistance P_w, rolling resistance $P_f=fG_a\cos\lambda$ and resistance $P_f=fG_a\cos\lambda$ only in the deceleration area with constant deceleration S_j. Calculations show that in case of emergency braking of the vehicle, the component S_j of the stopping path S_0 will vary depending on the initial speed of braking. So when braking at a speed of 60 km/h, the stopping distance of the car will be $S_0=51.8$ m, of which most of the stopping distance the car will move without braking and only $S_j=20.1$ m the car will pass with a constant slowdown.

At a speed of 90 km/h the stopping distance of the car will be 92.8 m, of which half the way the car will move without braking. And only at the initial braking speed of 120 km/h, when the stopping distance of the car is 143.8 m, most of this way the car will pass with a constant deceleration and a fairly large part – without braking. This means that the existing methods of calculating the speed of the vehicle during emergency braking can't account for the effect of the forces of air resistance, rolling resistance of wheels and resistance to lifting by 40–60 % of the length of the stopping road of the vehicle.

Therefore, to more accurately determine the speed of the vehicle in the process of emergency braking, a hypothesis is proposed about the need to take into account the effect of external forces of resistance to movement on the whole section of the stopping road of the vehicle. This hypothesis has a certain basis, corresponds to the physical essence of the movement of the real vehicle.

Let's write the equation of equilibrium of external forces and the moments acting on the vehicle during the reaction time of the driver to the danger and the operation of the brakes:

$$\frac{G_a j_w}{g}\delta_j-G_a f\cos\lambda-0.5c_x\rho F_x v^2\mp G_a\sin\lambda=0, \quad (17)$$

where j_w – vehicle deceleration under the influence of external forces (up to the moment of activation of the brakes), m/s^2.

Let's write a mathematical model of the dynamics of vehicle traffic during the time of the driver's reaction to the danger and the operation of the brakes in a differential form and with the initial conditions:

$$-\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{g}{G_a\delta_j}\left(G_a f\cos\lambda+0.5c_x\rho F_x v^2\pm G_a\sin\lambda\right), \quad (18)$$

$$\begin{cases}-\dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{g}{\delta_j}\left(f\cos\lambda\pm\sin\lambda+\dfrac{0.5c_x\rho F_x v^2}{G_a}\right),\\ v(0)=v_a,\end{cases} \quad (19)$$

where v_a – speed of vehicle movement at the moment of the driver's reaction to danger, m/s.

Let's try to solve the problem of calculating the speed of the vehicle in the process of emergency braking in stages. First, the driver begins to react to the danger after a time t_1 and then depresses the brake pedal, but the start of the brakes is not instantaneous, but with some delay t_2. During these time intervals t_1 and t_2 the car will go the way S_{1+2}, where the braking will occur only under the influence of the forces of air resistance, rolling resistance of the wheels and resistance to lifting, if any. Further on the site S_3 during the deceleration time t_3, the vehicle begins to grow intensively under the action of braking forces, and the speed of movement decreases. And in the fourth section during the time t_4, the vehicle brakes with maximum efficiency (with a constant deceleration).

Known in this task are the technical parameters of the vehicle, the value of the speed of the vehicle before deceleration, the type and condition of the road surface, as well as the value of the time intervals t_1, t_2, t_3, that are taken according to expert data. Accordingly, the algorithm for solving this problem is as follows. It is necessary to compile and solve mathematical models of the change in the vehicle speed for each segment of inhibition.

The speed function on the segments of the stopping path is represented in a differential form. On the site:

$$\begin{cases}-\dfrac{\mathrm{d}v}{\mathrm{d}t}=j_w,\\ v\ (0)=v_a.\end{cases} \quad (20)$$

To solve equation (20), it is necessary to integrate it:

$$\begin{cases}\int\mathrm{d}v=-j_w\int\mathrm{d}t+C_{1v},\\ C_{1v}=v(0)=v_a.\end{cases} \quad (21)$$

After integrating the table integral of the form $\int dt = t + C$, let's obtain an expression for calculating the speed of vehicle movement on the section S_{1+2}:

$$v = v_a - j_w t. \quad (22)$$

Hence the speed of the vehicle at the end of the site S_{1+2} (on the back of the site S_3):

$$v_2 = v_a - j_w (t_1 + t_2). \quad (23)$$

Expression (23) is valid for the assumption that the deceleration j_w appears instantaneously in the case when the ratio of the time of appearance of the deceleration to the time of its action will be relatively large. In the case when the time of appearance of deceleration j_w will be comparable with the time of this deceleration, the function of increasing deceleration can be submitted as a linear function of the form:

$$C = j(t) = \frac{j_w}{t_2} t.$$

If, in the first approximation, let's assume that the deceleration j_w increases in proportion to the given time interval, we obtain one more formula for calculating the velocity of movement of the TS in the section S_{1+2}:

$$v_2 = v_a - \frac{j_w t}{2}. \quad (24)$$

Let's consider the process of braking of the vehicle in the area S_3 of the deceleration growth under the action of braking forces. The function of increasing the deceleration j of the vehicle in this part of the braking can be given as a linear function of the form $y = ax + b$, which is given by the coordinates $(y_1 = 0,\ x_1 = 0)$, $(y_2 = j_s,\ x_2 = t_3)$, where j_s – steady deceleration of the car, m/s^2. Then the coefficients of the linear function are equal to $a = j_s / t_3$, $b = 0$. The function itself will take the form $C = j(t) = (j_s / t_3) t$. The differential equation of the change in the speed of vehicle movement on the section S_3 will have the form:

$$\begin{cases} -\dfrac{dv}{dt} = \dfrac{j_s t}{t_3}, \\ v(0) = v_2, \end{cases} \quad (25)$$

the solution of which is the integral:

$$\begin{cases} \int dv = -\dfrac{j_s}{t_3} \int t dt + C_{3v}, \\ C_{3v} = v(0) = v_2. \end{cases} \quad (26)$$

After integrating, let's obtain an expression for calculating the speed of vehicle movement in the segment S_3 of the stopping track:

$$v = v_2 - \frac{j_s t^2}{2 t_3}. \quad (27)$$

Hence the vehicle speed at the end of the section S_3 will be:

$$v_0 = v_2 - \frac{j_s t_3}{2}. \quad (28)$$

If in S_j the factors affecting the braking process of the vehicle are other factors than the effect of external forces, for example, a change in the coefficient of traction of the wheels with the road from the speed of movement, then it is possible to express the braking efficiency through the steady deceleration:

$$\begin{cases} -\dfrac{dv}{dt} = j_s, \\ v(0) = v_0 = v_2 - \dfrac{j_s t_3}{2}, \end{cases} \quad (29)$$

$$\begin{cases} \int dv = -j_s \int dt + C_{4v}, \\ v(0) = v_0 = v_2 - \dfrac{j_s t_3}{2} = C_{4v}. \end{cases} \quad (30)$$

After integrating, let's an expression for calculating the car speed any time t in the segment S_j of the stopping path:

$$v = v_2 - \frac{j_s t_3}{2} - j_s t = v_a - j_w (t_1 + t_2) - \frac{j_s t_3}{2} - j_s t. \quad (31)$$

On the basis of the calculated formulas, an expression can be written for calculating the initial speed of the vehicle equipped with ABS, in case of emergency braking:

$$v_a = j_w (t_1 + t_2) + \frac{j_s t_3}{2} + j_s t_4 = j_w (t_1 + t_2) + j_s \left(\frac{t_3}{2} + t_4 \right). \quad (32)$$

An analysis of the developed mathematical model shows that when the influence of external forces of resistance to the movement on the entire extension of the stopping track on the vehicle is taken into account, the speed of the vehicle's movement from time to time will vary more intensively. This will affect the shortening of the stopping path of the TS in comparison with the calculations that are performed according to known methods (Fig. 3).

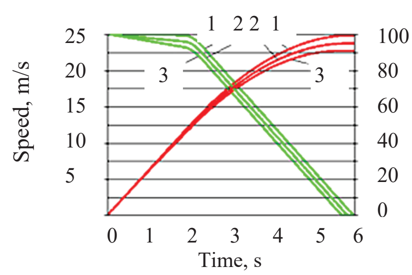

Fig. 3. Influence of calculation of change of speed of vehicle movement on length of a stopping way: 1 – under existing expert method; 2 – according to the developed method; 3 – according to the developed method, on an ascent with a slope of 6 %

The difference in determining the speed of the vehicle during emergency braking will be 4–8 % depending on the chosen calculation method. It should be noted that under real conditions of brake tests, the vehicle speed will change even more intensively (it is below curve 3). This can be explained by the fact that vehicles equipped with ABS have higher braking efficiency.

7. SWOT analysis of research results

Strengths. According to the research results, a mathematical model is developed to determine the car speed, which is equipped with ABS. The proposed mathematical model more accurately reflects the actual process of emergency braking and the change in the car speed, provides a reduction in the calculation error in comparison with existing calculation methods.

Weaknesses. The calculation process according to the proposed mathematical model requires more initial data than the usual expert calculation. To obtain this data, it is necessary to have additional sources of objective information, for example, records from the DVR. This increases the cost of conducting a thorough study.

Opportunities. The next step is the application of the developed mathematical model in the computer simulation of the reconstruction of an accident to improve the objectivity of the conclusions of expert studies.

Threats. At present, for the reconstruction of the road accident there are no clear recommendations for determining the speed of the car equipped with ABS. This leads to the possibility of different estimates of the speed of the same vehicle during the reconstruction of road accidents by various specialists. This may have different effects on the findings of the examination as a whole and give a biased picture of the circumstances and the development of the accident mechanism.

8. Conclusions

1. A mathematical model is developed to determine the car speed in the process of emergency braking. The model allows to take into account the impact of external forces of resistance to movement in the current phase of braking, as well as during the reaction of the driver and the timing of the brakes. An analysis of the mathematical model shows that during these time intervals a certain deceleration will act on the car, which will depend on the speed of movement and loading of the car. The action of the resistance force to lifting can significantly increase the deceleration. All this significantly affects the calculation of the car speed during the investigation of an accident.

2. Analysis of the developed mathematical model shows that when considering the effect on the vehicle of external forces of resistance to movement throughout the extension of the stopping distance, the speed of the vehicle's movement from time to time will vary more intensively. The proposed mathematical model provides a reduction in the error in calculating the speed of the car during an emergency braking by 4–8 % compared with the existing expert method.

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