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APPLICATION OF THE TECHNIQUE FOR AGGREGATING THE ELEMENTS IN A FORMALIZED GEOMETRIC MODELING OF MULTIFACTOR PROCESSES IN GEOMETRIC ECONOMETRICS

Об'єктом дослідження є моделювання багатофакторних систем в сфері геометричної економетрики. Моделювання економічних, технологічних та будь-яких інших процесів, які відбуваються на реальних суб'єктах господарювання, має свої особливості. Зокрема, його метою є надання підґрунтя для прийняття оптимального управлінського рішення у тій сфері діяльності, яка моделюється. Наразі розроблено широкий спектр методів і моделей.

Одним з найбільш проблемних місць є необхідність врахування великої кількості вихідної інформації різної фізичної природи. Це значно ускладнює моделі. Адекватні моделі є складними, зі значними обмеженнями по кількості факторів, не універсальними. Простіші універсальні моделі є доволі приблизними, з низькою адекватністю. Цих недоліків позбавлений запропонований, у формалізованому геометричному моделюванні багатофакторних процесів, спосіб створення універсальних моделей. Цей спосіб повинен бути здатен враховувати будь-яку скінчену множину факторів, кількість і якість яких можна було б змінювати без перебудови, при цьому, самої моделі.

В ході дослідження використовувався математичний апарат точкового числення Балюби-Найдиша, що дає можливість зручно формалізовувати будь-яку необхідну кількість вихідних факторів різної фізичної природи. На його основі розроблена послідовність побудови формалізованої геометричної моделі з використанням точкових агрегатів, а також визначені її переваги та недоліки. В основу розробленого способу покладено використання властивостей простого відношення трьох точок прямої у точковому численні Балюби-Найдиша.

Завдяки цьому з'явилася можливість розбиття складної багатофакторної задачі на відповідну кількість простих однофакторних задач, що суттєво спрощує обчислення.

Таким чином, запропоновано спосіб створення універсальних геометричних моделей з використанням інструментарію точкового числення Балюби-Найдиша. Цей спосіб відкриває нові можливості моделювання і дослідження багатофакторних систем, в порівнянні з аналогічними відомими методами моделювання. Спосіб універсальний, враховує будь-яку необхідну кількість факторів будь-якої природи. А також надає можливість, в разі зміни факторів, зручного переналаштування без зміни самої моделі.

Ключові слова: *формалізоване геометричне моделювання, агрегування елементів, точкове числення Балюби-Найдиша, параметричний зв'язок, точковий агрегат.*

1. Introduction

The problems of geometric econometrics [1] require the creation of different models for each of them. In practice, a proprietary model is constructed for each problem; in this case, existing methods of mathematical modeling are designed to consider a limited number of factors that can be included in the model. That is why there is always a necessity prior to creating a model to analyze each factor in terms of its impact on the adequacy of the process. In case of inadequacy, it is necessary to qualitatively and quantitatively change the input factors, which practically always entails a change, refinement, or the restructuring of the entire model. In this regard, construction of universal models for any problems related to geometric econometrics that may take into consideration any finite set of factors for multifactor processes is a relevant task and represents a scientifically applied problem. In this case, computer implementation should be executed using a standard personal computer with average capacity while the software realization should require less computing resources and be capable of deployment at computer information systems and automated workplaces. A given problem is considered for the first time in such a statement.

2. The object of research and its technological audit

The object of this research is the modeling of multifactor systems in the field of geometric econometrics. Modeling the economic, technological, and any other processes that occur at actual entities, has its own special features. Specifically, its purpose is to provide a basis for making an optimal management decision in the area of activity that is simulated. A wide range of methods and models have been developed up to now.

One of the most problematic issues is the necessity of taking into consideration a large number of initial information of different physical nature. This greatly complicates the models. The adequate models are complex, with significant constraints for the number of factors; and they are not universal.

3. The aim and objectives of research

The aim of research is to propose, at formalized geometric modelling of multifactor processes, a technique for creating universal models. Such a technique must be able to account for any finite set of factors whose quantity and quality could be changed without adjustment of the model itself.

To accomplish the aim, the following tasks have been set:

1. To analyze existing solutions to the stated problem.
2. To explore patterns in the Balyuba-Naidysh point calculus apparatus in the context of modelling the multifactor systems.
3. To build a sequence of construction of a formalized geometric model using point aggregates, as well as to determine its advantages and disadvantages.

4. Research of existing solutions of the problem

The field of mathematical modeling of energy-saving activities related to public services is intensely investigated by researchers from the scientific schools of Kyiv National University of Construction and Architecture, the National Technical University of Ukraine «Kyiv Polytechnic Institute» and others [2]. Particularly, there is a large body of work associated with geometric modeling in energy saving engineering, including construction, ventilation, lighting and heat and gas supply [3]. An entire range of models were developed in construction, energy generation, and power consumption. Geometric models for energy-efficient structures were proposed in papers [4, 5], while the models of urban networks were covered in study [6]. In economics, particularly finance, while estimating the assets and risks when investing, there is also a significant number of models [7–9].

However, the task on creating an effective system to support management decisions based on the combination of all branches of municipal services to be used in a unified system of models, has remained unsolved. Specifically, the TRACE model [10] fails to comprehensively take into consideration the local peculiarities; the results are very approximate and are not always optimal.

Paper [11] has in detail outlined the ways for solving the problems on multifactor modelling. When making effective management decisions, modeling is practically the only tool to investigate complex economic systems. Analytical methods for studying real-world complex multifactor systems are inefficient because the increased complexity of the system results in a sharp increase in computational operations, which do not always produce an adequate solution anyway.

Paper [12] proposed and noted the possibility of applying point BN-calculus to solve multifactor problems on geometric econometrics. The development of the Balyuba-Naidysh point calculus (BN-calculus) provides many opportunities for the formalized geometric modeling of complex multifactor problems. These works considered some of the universal formalized geometric models, suitable for creating the response surfaces whose construction is needed for management decision making.

5. Methods of research

We have employed the following scientific methods in the course of research:

– an analysis method in the study of existing methods for simulating the multifactor systems;
– a classification method when identifying typical problems in modelling the processes at actual objects as multifactor systems;
– methods of the Balyuba-Naidysh point calculus as tools for the formalization of source data, construction of point forms and building a model;
– a generalization method when determining the universal properties of the model derived.

6. Research results

The B-N point calculus [13] is the geometry of relations between homogeneous geometric shapes or their properties of metric character, which are defined in a single simplex and meet the required conditions for parameters. We shall explain the conditions under consideration using examples. Let points A and C define a straight-line a, points B and D define a straight-line b (Fig. 1).

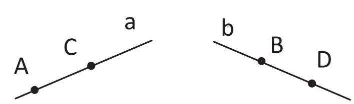

Fig. 1. Examples of straight lines and segments that are not employed in the Balyuba-Naidysh point calculus

The ratios a/b or b/a make no sense for the *B-N* point calculus at all because, from a mathematical point of view, they match the ratio ∞/∞. Therefore, the B-N point calculus considers not the ratio of straight lines, but rather the ratio of certain segments along these straight lines, or along a broken line whose links are straight. In this case, the two segments, the ratio of which is determined, must have a common point. In this regard, ratios AC/BD or BD/AC (Fig. 1) are not used in the *B-N* point calculus, and vice versa (Fig. 2), the ratios of segments AB/BM, AM/MB, CN/ND etc. are employed.

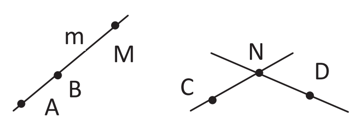

Fig. 2. Examples of geometric shapes for a simple ratio of three points

The existence of a common point for two segments translates their regular ratio into a simple ratio of the three points along a straight line, for example, $AM/MB = ABM$ is a simple ratio of three points (SRTP), which defines parameter t.

In case when SRTP is determined, that is, parameter t, one can choose an endless set of segments along a straight line, the ratios of lengths of which will be equal to the

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value of t. Choosing the two segments from a given set, the ratios of whose lengths is equal to t is called the geometry of number t [14]. It is known that for values $0 \le t \le 1$ the variable point M is located inside the segment AB, if $1 > t > 1$, the variable point M is outside the segment AB and it forms a straight line m (Fig. 2).

It is known that SRTP is an invariant of affine transformations, that is, its value does not change at parallel projection. Underlying the construction of point aggregates [14] is SRTP that determines parameter i. It is always present in these point aggregates in the implicit or explicit forms. Hence, a solution to any problem in the Balyuba-Naidysh point calculus does not require the use of projections. Solving in the Balyuba-Naidysh point calculus always takes place in space; it becomes possible due to the fact that one applies the projection onto the axis of the local or global coordinate systems. However, if it is required to conduct a study along the planes of projection, it is necessary to consider two axes that define the plane of projections and, by applying the already known coordinates of the geometric shape of the solution, to build the required projection. Such capabilities of the *B-N* point calculus are essential in order to build a model of the multifactor process, in which each factor, which is explored in the process, is matched with an axis, with a value of the single coordinate, rather than the plane with the values for two coordinates. The process model that takes into consideration n factors, it is necessary to take a system composed of n axes, in the form of a bundle of straight lines whose angles between the axes are not straight but can accept any form. At such a construction of the formalized geometric model of the process, there is a parametric relationship between a geometric shape of the process and its projections onto the axis, each of which corresponds to a factor of the examined process. Therefore, a multifactor model of the process is not a conventional multidimensional model. Multidimensional models use a projection relationship between a geometric shape of the object and its projections, while the formalized geometric model employs a parametric relation. It is obvious that it is possible to establish a projection relation for the formalized geometric model, but that makes no sense at all, and, for the most part, it requires complicated algebraic transformations.

A geometric model is built using methods and algorithms of traditional mathematics and geometry. The formalized geometric model [1] is constructed applying methods of the *B-N* point calculus. What is the difference between the *B-N* point calculus and traditional mathematics? In the *B-N* point calculus, each step of geometric constructions, the algorithm of solving the problem, is assigned with a point aggregate. The presence of such a sequence of point aggregates formalizes in a step-wise manner a geometric part of the algorithm of solution. Algebraic transformations of the sequence of point aggregates produces a spatial solution to the problem in a point form, which is a formal geometric model of the process. Since in the *B-N* point calculus the parameter is always SRTP, and projecting takes place along an axis, the behavior and influence of each factor in the process can always be found, provided the existence of a spatial solution.

It is known that studying any process requires the construction of a response surface based on the empirical source data (which we shall further call the empirical response surface). There is an example. Let the formalized geometric process model be represented by the empirical response surface (Fig. 3) in the form of a point equation:

$$M = [A_{11}\bar{u}(1-2u) + 4A_{12}u\bar{u} + A_{13}u(2u-1)]\bar{v}(1-2v) + \\ + 4\left[A_{21}\bar{u}(1-2u) + 4A_{22}u\bar{u} + A_{23}u(2u-1)\right]v\bar{v} + \\ + \left[A_{31}\bar{u}(1-2u) + 4A_{32}u\bar{u} + A_{33}u(2u-1)\right]v(2\bar{v}-1), \qquad (1)$$

which is the Balyuba parabolic surface. Here, lines $A_{i1}A_{i2}A_{i3}$ are the parabola that pass through three valid points $A_{i1}A_{i2}A_{i3}$, while the fourth is an endless point defined by parameter $t = 1/2$ along the straight line $A_{i1}A_{i3}$.

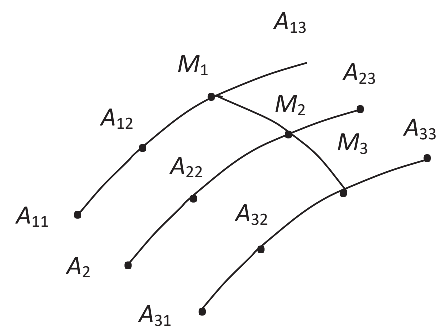

Fig. 3. Geometric scheme of response surface

Given that the edges $A_{i1}A_{i2}A_{i3}$ are parabola and the moving line $M_1M_2M_3$, which helps form the empirical response surface, is also a parabola, hence the surface is denoted as «parabolic»; its construction was proposed in [13].

Point aggregates in the form of a point equation (1) are not suitable for calculations, they only point to the scheme of performing the calculations for each coordinate (factor) separately.

With respect to that the points A_{ij} in a multifactor process can be determined by the finite set of coordinates (factors of the process) and, in this case, the model is not multidimensional, it is unknown then how to represent points A_{ij} in point equation (1). Given that the parameters of model (1) in space are equal to parameters along all coordinate axes, we shall then perform the calculations according to scheme (1). In this scheme, we shall substitute points A_{ij} by the indicators of characteristics for respective factors – X_{ij}, that is, coordinates (2):

$$X_k^m = \left[X_{11}^k\bar{u}(1-2u) + 4X_{12}^k u\bar{u} + X_{13}^k u(2u-1)\right]\bar{v}(1-2v) + \\ + 4\left[X_{21}^k\bar{u}(1-2u) + 4X_{22}^k u\bar{u} + X_{23}^k u(2u-1)\right]v\bar{v} + \\ + \left[X_{31}^k\bar{u}(1-2u) + 4X_{32}^k u\bar{u} + X_{33}^k u(2u-1)\right]v(2\bar{v}-1), \qquad (2)$$

where $k = \overline{1,n}$ is the serial number of the factor; n is the total number of factors in the process; u, v are the parameters of the response surface that vary in the range from 0 to 1; $\bar{u} = 1-u$; $\bar{v} = 1-v$; X_{ij}^k are the numbers that determine characteristics of the factor under different conditions; X_k^m is the empirical response surface for the k-th factor based on a specific characteristic m; $m = \overline{1,m}$ is the number of characteristics for a factor.

It should be noted that because each factor that is taken into consideration when studying a process always has several characteristics *m*, it is possible, for the *k*-th factor, to build *m* response surfaces based on each characteristic. Which response surface, out of *m* to construct, or all together, depends on the purpose of process examination.

The empirical response surface X_k^m from (2) for the formalized geometric model *M* from (1), can be explored as a geometric shape using methods of the *B*-*N* point calculus. The exploration results could help make decisions that match the aims of the investigation. One can see that studying a process model that implies the aggregation of results of geometric study into response surface X_k^m for each factor, that is, the path from an element to the process in general, is a new approach in modeling the processes in order to make substantiated management decisions. It is known that the traditional methods of examining a process, based on its disaggregation, necessitate:

– analysis of the factors;
– determining their significance and, consequently, limiting their number in order to establish, via constraints, a solution domain;
– deriving an objective function; etc., and, in this case, the model is not always adequate.

To improve the adequacy of simulation results to better match the actual ones, it is necessary to qualitatively and quantitatively change the factors included in the model, though this requires the reconstruction of a model, sometimes even the entire model. Contrarywise, the proposed method for aggregating the geometric information about response surfaces, from an element to the formalized geometric process model in general, makes it possible to take into consideration the following:

– all factors, without exception, and their characteristics;
– if the quantity and quality of factors change, the formalized geometric model remains in its original form;
– it is possible to obtain a solution without having to specify the domain of solutions and without deriving an objective function.

In this case, the resulting solution is almost always adequate because it accounts for all factors. It can be refined by changing accents in the model, emphasizing one or another characteristic of all or individual factors. It is also planned to create and investigate the generalizing integrated characteristics of factors and the process in general.

7. SWOT analysis of research results

Strengths. Among the above-mentioned benefits of the proposed technique for aggregating the elements-factors, the most important one should be emphasized. This is the possibility of splitting a process of any complexity into solving the hundreds, thousands, or even tens of thousands, small calculation problems that require split-seconds for calculations.

The application of the technique for aggregating the elements of a process (TAEP) significantly reduces the cost of computational resources and makes it possible to calculate in a real-time mode. That would provide the opportunity to perform a sufficient number of computing experiments involving the model, in order to identify the most acceptable option. The result might be an increase in efficiency and substantiation of management decision making.

Weaknesses. The limits of research into factors and a process in general are the boundaries of a geometric shape in the form of a response surface. The boundaries of a response surface may change only at a change in parameters, characteristics, or conditions for factors.

Opportunities. In the further research, we shall apply the formalized geometric models (FGM) that differ from the one proposed in this paper (1). Thus, by changing FGM, we could find the one that to the largest extent matches the process whose model is constructed.

The prospects of further research include a more detailed development of TAEP, construction of techniques and algorithms for response surfaces, which are the Balyuba parabolic surfaces, devising a procedure for computer modeling involving the formalized geometric models of a multifactor process.

The economic effect of its implementation could be attained based on the results of reducing the cost of computer resources, by optimizing the cost of materials and technologies, by bringing down the expenditures on heating the premises, by the timely made management decisions, etc. In each particular case during application of the technique for aggregating the elements, the economic effect will be determined taking into consideration the patterns of its application.

Threats. When implementing an enterprise information system based on the developed technique for aggregating the elements, it is necessary to build a large database of initial factors. That requires additional expenses. In addition, at the initial stage, there is a significant influence of the human factor, specifically, in case the errors are made when entering the source data, there is a probability to obtain incorrect modeling results.

8. Conclusions

1. We have conducted a comprehensive analysis of modern methods for solving the problem on the geometric modeling of multifactor systems. For almost every type of problems, particularly in the field of geometric econometrics, there are appropriate methods and models. However, despite their large number and diversity, they solve the problem only partially. The adequate precise methods are complex to implement and are developed for a certain specific object. That is, they are not sufficiently flexible for reconfiguration and are not universal. More general methods, such as the models by analogy, are easy to use and versatile, but produce too rough estimates.

2. Given the identified advantages and disadvantages of existing methods of modelling, it is expedient to apply a mathematical apparatus of the Balyuba-Naidysh point calculus. It makes it possible to conveniently formalize any required number of source factors of different physical nature.

3. We have developed a sequence for the construction of a formalized geometric model using the point aggregates; its advantages and disadvantages were defined. Underlying the developed method is the use of the properties of a simple ratio of three points along a straight line in the Balyuba-Naidysh point calculus. This enables the splitting of a complex multifactor problem into appropriate number of simple unifactorial problems, which greatly simplifies computation.

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Thus, we have proposed a technique for constructing universal geometric models using the toolset of the Balyuba-Naidysh point calculus, which opens up new possibilities for modeling and examination of multifactor systems.

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