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THE DEVELOPMENT OF A THREE-DIMENSIONAL MODEL OF THE ICE GROWTH PROCESS ON AERODYNAMIC SURFACES

Об'єктом дослідження є процеси гідроаеродинаміки і тепломасопереносу, що відбуваються при зледенінні аеродинамічних поверхонь літальних апаратів під час польоту в несприятливих метеорологічних умовах. Одними з проблемних місць при розробці програмно-методичного забезпечення, що дозволяє моделювати процеси зледеніння, є складності при переході до вирішення задачі в тривимірній постановці. А також наявність в існуючих методиках протиріч при описі фізичної картини і, відповідно, термодинаміки процесу наростання криги.

В ході дослідження використовувалися експериментальні та аналітичні методи дослідження фізичних процесів наростання криги на обтічних поверхнях, засновані на поетапному аналізі взаємодії переохолоджених крапель з поверхнею і їх подальшим замерзанням на кромці крила. Запропонована модель процесу наростання криги заснована на використанні методу поверхневих контрольних об'ємів, що базується на рівняннях нерозривності, збереження кількості руху і енергії. На основі нових отриманих експериментальних даних про фізику обмерзання, в методиці моделювання наростання криги запропоновано виділити окремо процеси утворення об'ємної структури крига-вода і подальшого повного замерзання цієї структури. На першому етапі процесу кристалізації рідини, в рамках кроку за часом зледеніння, переохолоджена рідина, що міститься в краплях, які потрапляють на обтічну поверхню, переходить в стан термодинамічної рівноваги. Тобто прихована теплота затвердіння, що виділяється при утворенні крижаної долі в структурі крига-вода, буде дорівнювати внутрішній теплоті, необхідній для нагрівання переохолодженої рідини від температури крапельок до температури фазового переходу. На другому етапі буде замерзати вода, що міститься в структурі крига-вода за рахунок теплових втрат конвекцією, випаровуванням, сублімацією, теплопровідністю (за мінусом прихованої теплоти затвердіння, кінетичного і аеродинамічного нагріву). Необхідно зазначити, що вода, яка буде замерзати, також сковуватиме і просторову крижану структуру. При цьому для визначення напрямку переміщення рідини уздовж обтічної поверхні було застосовано метод послідовних наближень.

У порівнянні з відомими традиційними методиками такий підхід дозволяє більшою мірою враховувати надзвичайно складні для математичного опису реальні фізичні процеси обмерзання аеродинамічних поверхонь.

Результати роботи можуть бути використані при оптимізації роботи систем захисту від обмерзання і визначенні шляхів зниження енергетичних витрат при роботі таких систем.

Ключові слова: *зледеніння аеродинамічних поверхонь, системи захисту від зледеніння, математичне моделювання процесу наростання криги.*

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1. Introduction

Icing is a generally recognized serious safety issue in aviation. The most common negative phenomena that can be caused by icing of the aerodynamic surfaces of aircraft are a decrease in lift and stall angle from the wing, as well as tail elements, a loss of longitudinal stability and, accordingly, a sudden loss of controllability. The development of icing protection systems and determining their effectiveness is a very complex problem. According to accepted safety standards, the main research tools that should be included in the certification plan for aircraft for icing conditions are:

– flight tests in vivo icing;
– experiments in wind tunnels, both «dry» and simulating icing conditions;
– numerical methods.

It should be noted that experimental methods, on the one hand, are quite expensive, require sophisticated equipment, and, on the other hand, do not give a complete picture of the distribution of the parameters of the air-drop flow in the studied area. In addition, experiments conducted under ground conditions can't accurately reproduce the icing conditions in flight, and require the use of large-scale models. Therefore, there is a need to apply methods of numerical modeling in order to reduce the time and cost of developing ice protection systems. Moreover, the calculations should be based on evaluating the effectiveness of numerical methods, understanding the features of the influence on the flow pattern of changes in the geometry of aerodynamic surfaces due to the formation of ice growths. All these are prerequisites for the creation of the most advanced systems of protection against icing at present.

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Thus, to ensure the flight safety of aircraft in adverse weather conditions, the urgent problem is the development of software and methodological support that allows to simulate icing conditions, as well as the development of models of the process of ice growth on aerodynamic surfaces.

2. The object of research and its technological audit

The object of research is the processes of hydroaerodynamics and heat and mass transfer that occur when icing the aerodynamic surfaces of aircraft during flight in adverse weather conditions.

To date, a number of well-known techniques and software products have been developed in various countries that allow modeling icing processes, where the external air-drop flow is described using the equations of potential and the trajectory model. It should be noted that the description of the process of ice growth is based on the approach proposed in 1965 [1], using semi-empirical dependencies.

Such techniques are of limited use at sufficiently high speeds and complex forms of ice growths (due to the presence of local transonic zones and significant pressure gradients), configurations with multi-bodies. In addition, they neglect the history of the flow and do not allow to evaluate the effect of rough outgrowths of ice formed on the aerodynamic characteristics of the profile. They also have limited application in the transition to tasks in three-dimensional formulation, contain some contradictions in the description of the physical picture and, accordingly, the thermodynamics of the process of ice growth.

3. The aim and objectives of research

The aim of research is development of a mathematical model of the process of ice growth taking into account the physical processes that occur when supercooled water drops fall on a streamlined surface with their subsequent movement and curing.

To achieve this aim, it is necessary to complete the following objectives:

1. To carry out an analysis of experimental studies of the icing of aerodynamic surfaces: the interaction of supercooled water droplets with a streamlined surface, the mechanism of moisture freezing and further movement along the surface is covered with ice.
2. Based on the conducted experimental studies, formulate, with basic assumptions and simplifications, a method for calculating the ice growth on a streamlined surface.

4. Research of existing solution of the problem

Various groups of researchers all over the world have developed methods that allow simulating the processes of icing of aerodynamic surfaces in a two-dimensional setting, for example:

– LEWICE (USA) [1];
– ONERA (France) [2];
– TRAJICE2D (Great Britain) [3];
– CANICE (Canada) [4];
– CIRA (Italy) [5];
– 2DFOIL-ICE (Netherlands) [6].

It should be noted that most of these methods are being finalized to simulate the growth of ice in a three-dimensional setting [7, 8]. Moreover, the behavior of surface water in these techniques is described, not taking into account its physical condition, using simplifying hypotheses. It is believed that the fluid moves along the surface of the icing in the form of a film, based on the hypothesis that is developed in [9, 10] and others. This assumption does not take into account that the surface fluid may have a different physical state, which is determined by the thermodynamic balance, physical properties supercooled water, the action of aerodynamic, gravitational force, surface tension.

It should also be noted that these relations re obtained for the relatively simple geometry of the streamlined body, and, accordingly, they have limited application in the case of complex geometry and meteorological conditions, significantly different from those considered.

5. Methods of research

In the work, experimental and analytical methods are used to study the physical processes of ice growth on aerodynamic surfaces. These methods are based on a phased analysis [11, 12] of interaction with subsequent icing of supercooled droplets on the wing edge when an air-droplet flow with high water contention occurs.

The initial stage of the icing of the streamlined surface is characterized by deformation of the droplets, fly in the form of «flattening» before the introduction of the form of a kind of disk («pancakes»). In this case, droplets can break up into smaller particles, splatter and combine with neighboring surface moisture particles into clusters, which also have the form of «pancakes». Then, under the influence of surface tension, these «pancakes» are «pulled», take a hemispherical shape, which in turn can be combined into larger surface drops. This process is accompanied by a relatively fast curing of part of the volume of supercooled water that flies in, which causes it to heat up to the solidification temperature and the subsequent (slower) freezing process of the remaining fluid. The outflow of heat occurs as a result of convection, partial disruption of microparticles and evaporation of water, sublimation of ice into the external stream, as well as through heat exchange with an aerodynamic surface having a negative temperature. Surface drops, freezing, remain motionless, or spread, forming irregularities, depressions in which unfrozen water can remain. A qualitative picture of the temperature distribution on the streamlined surface of icing is shown in Fig. 1. At the same time, let's assume that fluid supercooled droplets fly in from an external flow and have a temperature close to the temperature of the incident flow. The fluid located in the hollows between the ice tubercles is a two-phase substance «ice-water», which, during freezing, can have a varying heterogeneous structure in volume, but at the same time has a temperature close to the phase transition temperature. The temperature of the peaks of the ice hillocks will be slightly lower than the temperature of the phase transition, as a result of more intense heat transfer through higher local flow rates in the region above the peaks of the hillocks. The temperature of the ice immediately below the border with the ice-water structure will be equal to the phase transition temperature. However, due to heat exchange with the «cold» surface of the wing, the temperature deep into the ice layer will decrease.

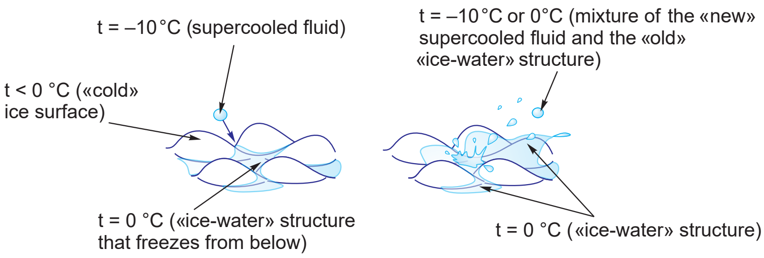

Fig. 1. The temperature distribution on the streamlined surface (in the conditions of the experiment conducted: t_∞=–10 °C, v_∞=33 m/s, LWC=2.4 g/m^3, d_k=150 microns)

When a supercooled droplet enters the icing surface, intense mixing and movement along the surface of the masses of the ice-water structure with a «new» supercooled fluid will occur. At the same time, the time that exceeds the duration of interaction with the droplet that flies in, the «new» fluid will still remain supercooled and will go into the state of thermodynamic equilibrium already in a «calm» substance, located in the hollows between the ice tubercles. Let's assume that the amount and time spent by the fluid on the tops of the icy hillocks is determined by the speed of the incoming flow, capturing this fluid below the flow (spreading or «snatching» with subsequent sedimentation). In this case, the mechanisms for moving the fluid will be:
- relatively slow uniform movement under the action of aerodynamic forces (in the case of accumulation of a sufficient volume of fluid in the element of the streamlined surface);
- local movement due to droplet energy, flies;
- and, with a relatively greater speed, by spraying and «jumping» the fluid with its subsequent loss below the stream (Fig. 2).

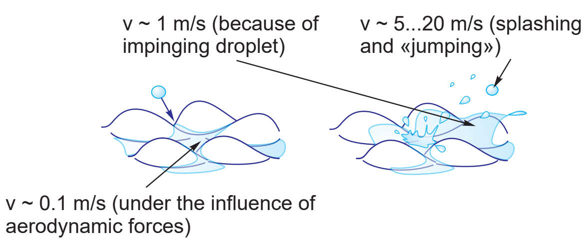

Fig. 2. The mechanism of fluid movement along a streamlined surface (under the conditions of the experiment conducted: t_∞=–10 °C, v_∞=33 m/s, LWC=2.4 g/m^3, d_k=150 microns)

The influence of temperature (ceteris paribus) on the mechanism of fluid movement and the structure of the ice formed is illustrated in Fig. 3. Let's consider the icing mode when moisture exists on the streamlined surface in the fluid state as «wet» (Fig. 3, *a*, *b*). And when the droplets fly in, freeze almost immediately, creating ice «columns» and, in the future, form loose ice – «dry» (Fig. 3, *c*).

According to the well-known, as well as obtained own experimental data and accepted position, the following assumptions and simplifications are made when developing the methodology and mathematical model for calculating the ice growth:
- movement of fluid along the streamlined surface, which occurs with the help of «whirling», in contact with the surface of the icing of droplets, fly in, «jumping» and spraying, followed by the loss of fluid downstream. And also the flow between the ice «hillocks» under the influence of aerodynamic forces is considered as an average steady motion;
- crystallization process of the fluid in the control volume occurs in two stages:
1) the relatively rapid formation of the spatial structure of «ice-water» in the volume of the «new» supercooled fluid flies from the external stream in proportion f_i (which is determined by the temperature of the supercooled drops);
2) and, slower, complete solidification, on the part of already existing ice, of the ice-water structure contained in the control volume, in the proportion n;
- fluid in the form of an ice-water structure that enters from neighboring control volumes and leaves the control volume of the considered one is in a state of thermodynamic equilibrium and has a temperature averaged over the volume equal to the water solidification temperature;
- ice-water structure leaves the control volume in the ratio f_{iav} (average value over the volume of fluid f_i and f_{i-1av};
- surface area of evaporation is taken equal to the area of the outer surface of the control volume;
- evaporation and sublimation of the ice-water structure occurs in proportion f_{iav};
- the lower boundary of the outer computational domain passes along the tops of the irregularities, where the air-drop flow rate is taken equal to 0.

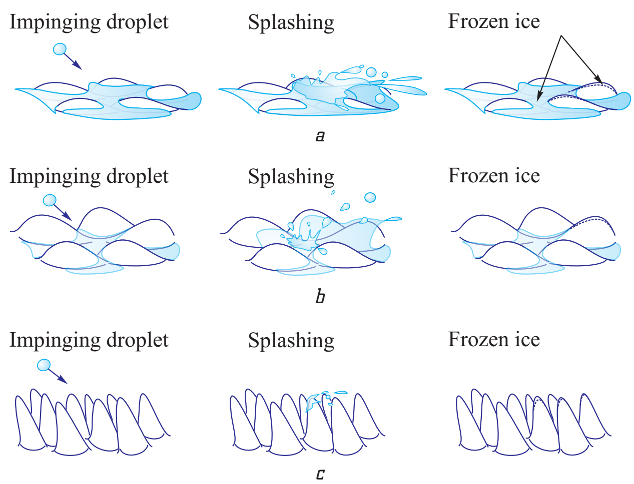

Fig. 3. The effect of temperature on the mechanism of movement and subsequent solidification of surface moisture: *a* – 2.5 °C; *b* – 10.0 °C; *c* – 15.0 °C (under the conditions of the experiment conducted: v_∞=33 m/s, LWC=2.4 g/m^3, d_k=150 microns)

At the same time, the icing process of the aerodynamic surface is divided into time steps during which the process of ice growth is considered quasi-stationary. That is, all the parameters of the oncoming flow, as well as mass and heat fluxes, are assumed constant during this step and corresponding to the moment of its beginning. Physical transformations within a time step occur instantly.

When creating a mathematical model at the macro level, let's consider fair the transition to considering the fluid moving along the streamlined surface in the form of the substance ice-water, freezing from the side of the streamlined body (Fig. 4).

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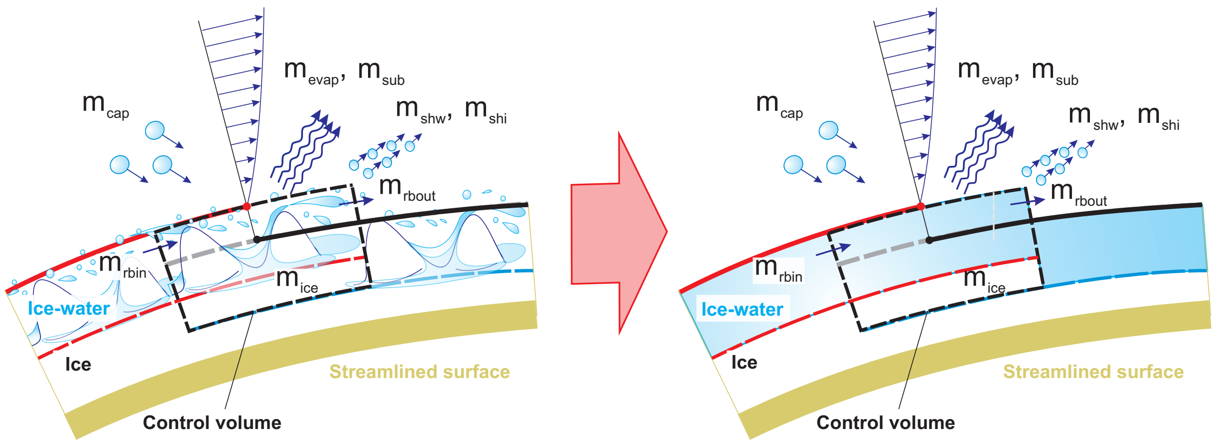

Fig. 4. Scheme of transition to a mathematical model of ice growth

Based on the obtained experimental data, let's assume that at the micro level, at the initial stage of the icing process, surface roughness can be determined using the technique proposed in [13]. In part, the assumption is that the unfrozen fluid is on the streamlined surface in the form of drops that freeze on the surface side, and the maximum possible height of these drops is calculated from the equilibrium condition of the aerodynamic forces and surface tension forces acting on the drop. In this case, the surface roughness will be determined by the maximum possible height of the surface droplets.

Since it is established that in the future, in the icing process of the aerodynamic surface, the physical picture of the process of moving and freezing the surface fluid differs from the generally accepted assumption [2], let's consider it appropriate to use the empirical relations given in [14, 15] to determine the surface roughness.

The developed assumption allows using the method of surface control volumes for a three-dimensional analytical description of the process of ice growth. The equations of mass and thermal balances can be obtained on the basis of the laws of conservation of mass, energy and momentum for a control volume located on the surface of the body according to the methods given in [16, 17].

6. Research results

Let's consider the control volume located on the surface of the streamlined body (Fig. 5). Let's denote the current control volume as P, and four neighboring, respectively, N, E, W, S. r_{sn} is the unit vector in the direction from S to N, and r_{we} is the unit vector in the direction from W to E. In Fig. 5, the grid shown in black corresponds to the previous time step, and the one shown in red corresponds to the current time step.

The mass of fluid that enters the control volume for the considered period of time m_{in} consists of a mass of water, is precipitated from an external stream in the form of supercooled droplets m_{cap} and a mass of fluid in the form of an ice-water structure moves from neighboring control volumes $\sum m_{rbin}$ [13]:

$$m_{in} = m_{cap} + \sum m_{rbin}. \tag{1}$$

A mass of water flies in from an external stream – the mass of a part of supercooled water droplets that are in an external air-drop stream that hit a streamlined surface. In the case of the application of a model of interpenetrating media:

$$m_{cap} = \sum_{k=1}^{n} \left(\rho_{jk} U_{jnk}\right) \Delta s \Delta t_{acc}, \tag{2}$$

where ρ_{jk} – concentration of supercooled droplets in the airborne droplet at the surface of the streamlined body, corresponding to the k-th distribution interval; U_{jnk} – droplet velocity component normal to the streamlined surface, corresponding to the k-th distribution interval; n – the number of distribution intervals; Δs – area of the control volume; Δt_{acc} – step along the time of ice rise.

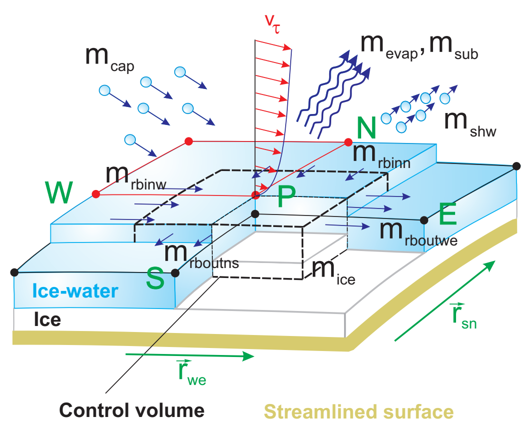

Fig. 5. Scheme of mass flows in the control volume

The mass of the ice-water structure that moves from the previous control volume can be represented as the sum of the fluid and ice components:

$$m_{rbin} = m_{rbinw} + m_{rbinice}, \tag{3}$$

where $m_{rbinice} = m_{rbin} \cdot f_{i-1_{cp}}$, $f_{i-1_{cp}}$ – the rate of ice averaged over the volume in the ice-water structure in the previous control volume.

Let's designate as $\sum m_{rbin}$ – the part of the masses of fluid transferred from the neighboring control volumes defined by the indices N, E, W, S to the present control volume P:

$$\sum m_{rbin} = m_{rbinw} + m_{rbine} + m_{rbins} + m_{rbinn}, \quad (4)$$

where the index *rbinw* denotes the mass of fluid moves to the control volume *P* from the control volume *W*, *rbine* – from the control volume *E*, *rbins* – from the control volume *S*, *rbinn* – from the control volume *N*.

On the other hand, the mass of fluid m_{in} entering the control volume is spent on the mass of fluid m_{out} eliminating the control volume, and the mass of ice m_{ice} formed during the time step of the ice rise is also eliminated [13]:

$$m_{in} = m_{out} + m_{ice}. \quad (5)$$

The mass of eliminated fluid generally consists of five components:

1) the mass of sublimated ice m_{sub};
2) the mass of evaporated water m_{evap};
3) the mass of fluid in the form of an ice-water structure, moves to adjacent control volumes $\sum m_{rbout}$;
4) masses of the removed fluid m_{shw};
5) masses torn out by a stream of ice m_{shi} [13]:

$$m_{out} = m_{sub} + m_{evap} + \sum m_{rbout} + m_{shw} + m_{shi}. \quad (6)$$

The mass of sublimated ice and the mass of evaporated water, m_{sub} and m_{evap} are the parts of the ice mass and the mass of water in the fluid state, which evaporate into the air under the influence of temperature differences in the boundary layer. The mass of the removed fluid m_{shw} is the part of the mass of fluid that is inside the control volume, which is pulled out by the free flow under the influence of shear stresses. It can be determined by the Weber number using empirical relationships [14]. If there is a stall, consider that all the fluid moves from the control volume, is introduced by an external stream. The mass of torn ice is the part of the ice mass located in the control volume that is released into the air as a result of the separation of ice crystals under the influence of aerodynamic force or other factors.The mass of the ice-water structure that leaves the control volume can be represented as the sum of the fluid m_{rbout_w} m_{rbin_w} та and ice $m_{rbout_{ice}}$ $m_{rbin_{ice}}$ components:

$$m_{rbout} = m_{rbout_w} + m_{rbout_{ice}}, \quad (7)$$

where $m_{rbout_{ice}} = m_{out} f_{iav}$, $f_{i_{av}}$ – the volume-average rate of ice in the ice-water structure in the current control volume.

Let's designate as $\sum m_{rbout}$ the mass of fluid that was in the current control volume, and which moves to adjacent control volumes:

$$\sum m_{rbout} = m_{rboutwe} + m_{rboutsn}, \quad (8)$$

where the *rboutwe* index denotes the mass of water that moves from the control volume *P* to neighboring control volumes *W* and *E*, and the *rboutsn* index – to the neighboring control volumes *S* and *N*.

Then the mass balance equation will take the form:

$$m_{cap} + \sum m_{rbin} + m_{resw} = \\ = m_{sub} + m_{evap} + \sum m_{rbout} + m_{shw} + m_{shi} + m_{ice} + m_{rmw}. \quad (9)$$

The positive direction of fluid mass flows into the control volume is shown in Fig. 5 [18]. The direction of fluid flows into the control volume is consistent with the direction of a single vector. That is, let's assume that if it is equal to zero, then there is no movement of fluid in the direction from *S* to *N*; if positive, the fluid moves through the face between the control volumes *P* and *N*, if negative, through the face between the control volumes *P* and *S*.

Then the section of the fluid parts, which is eliminated from outside the control volume, can be determined using the following relations [18]:

$$m_{rboutwe} = \frac{f_{we}}{|f_{we}| + |f_{sn}|} \sum m_{rbout}, \\ m_{rboutsn} = \frac{f_{sn}}{|f_{we}| + |f_{sn}|} \sum m_{rbout}, \quad (10)$$

where $f_{we} = \vec{f}_{\tau\_air} \cdot \vec{r}_{we}$ and $f_{sn} = \vec{f}_{\tau\_air} \cdot \vec{r}_{sn}$ – the components of the rate of the fluid-solid mixture in the direction of shear pressure from *W* to *E* and from *S* to *N*, respectively.

The mass of fluid moves into the considered control volume can be found as follows [18]:

$$m_{rbinw} = \left[\left| \begin{matrix} W \\ m_{outwe}, 0 \end{matrix} \right|\right], \ m_{rbine} = \left[\left| \begin{matrix} E \\ -m_{outwe}, 0 \end{matrix} \right|\right], \\ m_{rbins} = \left[\left| \begin{matrix} S \\ m_{outsn}, 0 \end{matrix} \right|\right], \ m_{rbinn} = \left[\left| \begin{matrix} N \\ -m_{outsn}, 0 \end{matrix} \right|\right], \quad (11)$$

where the sign $\left[\left| \quad \right|\right]$ means the choice of the maximum of the given values; $\begin{matrix} W \\ m_{outwe} \end{matrix}$ and $\begin{matrix} E \\ m_{outwe} \end{matrix}$ – the mass of fluid moving in the direction of *WE* in the control volumes *W* and *E*, respectively; $\begin{matrix} S \\ m_{outsn} \end{matrix}$ and $\begin{matrix} N \\ m_{outsn} \end{matrix}$ – the mass of fluid moving in the direction of *SN* in the control volumes *S* and *N*, respectively.

The mass of the fluid m_w – as part of the mass of the ice-water structure included in the control volume m_{in} can be defined as [13]:

$$m_w = (1 - n) m_{in}. \quad (12)$$

The mass of fluid in the form of an ice-water structure moves from the current control volume $\sum m_{rbout}$ to the mass of the fluid m_w minus the mass of the evaporized m_{evap} and torn out external fluid stream m_{shw} [17]:

$$\sum m_{rbout} = m_w - m_{evap} - m_{shw}. \quad (13)$$

The residual ice mass m_{rmi} is the part of the fluid mass in the form of an ice-water structure that enters, which freezes, and it corresponds to the amount of ice m_{ice} accumulated during the time step minus the mass of sublimated ice and torn out by the ice mass flow [13]:

$$m_{rmi} = m_{ice} - m_{sub} - m_{shi}. \quad (14)$$

Mass of ice m_{ice} is the part of the mass of fluid water in the form of an ice-water structure that enters m_{in}, which freezes:

$$m_{ice} = n m_{in}. \tag{15}$$

For the control volume, from the energy conservation equation, it is possible to obtain the heat balance equation, which has the form (Fig. 6):

$$Q_{kin} + Q_{aerod} + Q_{lat_{i-w}} + Q_{lat} - Q_{evap} -$$
$$- Q_{sub} - Q_{conv} - Q_{cond} - Q_{sens} = 0, \tag{16}$$

where Q_{kin} – heat of kinetic heating; Q_{aerod} – heat of aerodynamic heating; $Q_{lat_{i-w}}$ – latent heat of solidification released during the formation of the ice-water structure; Q_{lat} – latent heat of solidification released during freezing of the ice-water structure; Q_{evap} – latent heat of evaporation; Q_{sub} – latent heat of sublimation; Q_{conv} – convection heat exchange; Q_{cond} – heat transfer conductivity; Q_{sens} – internal heat.

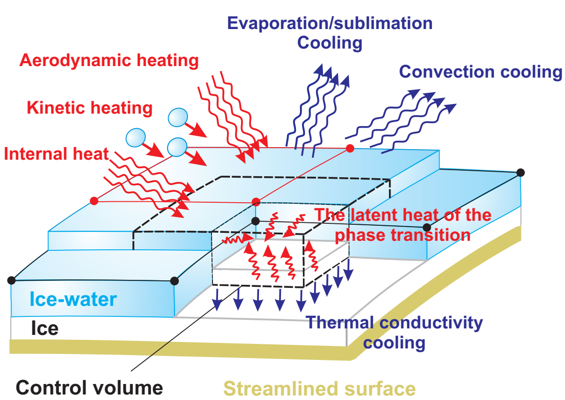

Fig. 6. The scheme of heat flows in the control volume

At the first stage of the fluid crystallization process, as part of the step on the icing time, the supercooled fluid contained in the droplets that fall on the streamlined surface passes into a state of thermodynamic equilibrium. That is, the latent heat of solidification released during the formation of an ice fraction in the ice-water structure $Q_{lat_{i-w}}$ will be equal to the internal heat Q_{sens} required to heat the supercooled fluid from the temperature of the droplets T_d to the temperature of the phase transition T_f:

$$Q_{lat_{i-w}} = Q_{sens}, \tag{17}$$

$$Q_{lat_{i-w}} = m_{cap_{ice}} L_f, \tag{18}$$

$$Q_{sens} = m_{cap} c_{p_w} \left(T_f - T_d\right), \tag{19}$$

where c_{p_w} – the specific heat of water.

In this case, the mass of water flies in from the external airborne droplet stream m_{cap} in the form of supercooled droplets will be divided into the mass of water m_{cap_w} and the mass of ice $m_{cap_{ice}}$ in the ice-water structure that has formed:

$$m_{cap} = m_{cap_w} + m_{cap_{ice}}. \tag{20}$$

That is, $m_{cap_{ice}} = m_{cap} f_i$, whence the rate of ice in the spatial ice-water spatial structure is defined as:

$$f_i = \frac{c_{p_w}\left(T_f - T_d\right)}{L_f}, \tag{21}$$

after which there is an average value of the ice fraction in the spatial ice-water structure over the fluid volume $f_{i_{av}}$.

At the second stage, the water contained in the ice-water structure will freeze due to heat loss by convection, evaporation, sublimation, thermal conductivity (minus the latent heat of solidification, kinetic and aerodynamic heating). In this case, the water will also freeze the ice structure, it is contained in this water in the proportion $f_{i_{av}}$.

When part of the mass of water contained in the ice-water structure in the control volume freezes, the latent heat of solidification is released:

$$Q_{lat} = m_{freeze} L_f, \tag{22}$$

where m_{freeze} – the mass of ice formed in the ice-water structure; L_f – specific heat of solidification of water.

In this case, the mass of ice formed during complete curing of the ice-water structure will be equal to:

$$m_{ice} = \frac{1}{1 - f_{i_{cp}}} m_{freeze}. \tag{23}$$

The kinetic heat Q_{kin} is the energy generated by the change in the speed of supercooled water droplets that hit a streamlined surface [13]:

$$Q_{kin} = \frac{m_{cap} U_d^2}{2}, \tag{24}$$

where U_d is the speed of water droplets, determined from the calculation of the external air-drop flow.

The heat of aerodynamic heating generated by the friction of air on the surface of the streamlined body, Q_{aerod}, is formed inside the boundary layer when the temperature changes from the value in the incident flow T_∞ to the average temperature in the boundary layer, which is called the recovery temperature T_{rec} [13]:

$$Q_{aerod} = h_{cv}\left(T_{rec} - T_\infty\right) \Delta s \Delta t_{acc}, \tag{25}$$

where h_{cv} – convective heat transfer coefficient; Δs – area of the control volume; Δt_{acc} – step along the time of ice rise.

Part of the ice mass present in the control volume sublimates, while the heat is absorbed:

$$Q_{sub} = -m_{sub} L_{sub}, \tag{26}$$

where m_{sub} – mass of sublimated ice, L_{sub} – specific heat of sublimation.

Part of the mass of water in the fluid phase, which is present in the control volume, evaporates, absorbing heat:

$$Q_{evap} = -m_{evap} L_{vap}, \tag{27}$$

where m_{evap} – mass of evaporated water, L_{vap} – specific heat of evaporation.

The convective heat transfer Q_{cv} between the air flow and the streamlined surface, when the flow and the surface have a different temperature, can be determined by the temperature field, or be described using the relation:

$$Q_{cv} = h_{cv}\left[f_w r_A + (1 - f_w)\right](T_\infty - T_s)\Delta s \Delta t_{acc}, \quad (28)$$

where h_{cv} – convective heat transfer coefficient; T_∞ – temperature of the unperturbed flow; T_s – surface temperature.

The relative contribution of various quantities to the heat balance is shown in Fig. 6 for typical flight conditions in icing conditions [19]. Fig. 6 shows that heating mainly occurs due to the release of latent heat of the phase transition, and cooling, mainly due to convection, evaporation and/or sublimation and the internal heat of supercooled water drops. It should be noted that internal heat makes a significant contribution if the temperature of the incident flow (i. e., the temperature of supercooled drops) is sufficiently low (~ –30 °C). It is also seen that the convective heat transfer coefficient plays the most significant role in the area of the stagnation point, since convective heat flux and flux due to evaporation and sublimation depend on it (60 % of negative values in the heat balance depend on this coefficient).

Based on the mass and thermal balances, the fraction of the ice-water structure n passing through the control volume over the time Δt_{acc} is calculated, the mass of ice is completely frozen m_{ice} and the thickness of the ice layer that formed h_{ice}:

$$n = \frac{1}{1 - f_{icp}} \frac{Q_{evap} + Q_{conv} + Q_{cond} - Q_{kin} - Q_{aerod}}{L_f\left(m_{in_w} + m_{in_{ice}} + m_{cap}\right)}, \quad (29)$$

$$h_{ice} = \frac{m_{ice}\Delta t}{\rho_{ice}}, \quad (30)$$

where the ice density is determined depending on the surface temperature T_s.

In wet mode, let's assume that the surface temperature is 0 °C and assume that the density of smooth ice is 917 kg/m^3. In dry mode, the surface temperature is lower than the solidification temperature, the density is calculated by the empirical formula proposed in [13]:

$$\rho_{ice} = 917 \cdot \left(\frac{X}{X + 1.3}\right)^2, \quad (31)$$

where X – Macklin coefficient:

$$X = \frac{d_d}{2} \cdot \frac{U_\infty}{T_f - T_s}, \quad T_f > T_s, \quad (32)$$

which is a dimensional number and depends on the arithmetic mean diameter of supercooled water droplets d_d, expressed in microns, undisturbed flow velocity U_∞, expressed in m/s, icing surface temperature T_s and temperature of the phase transition of water T_f in K.

7. SWOT analysis of research results

Strengths. A model of the process of ice growth on aerodynamic surfaces has been developed. It can be used in software and methodological software that allows describing the icing of aircraft during flight in adverse weather conditions.

The proposed model, unlike the existing ones, will allows to evaluate the effect of rough outgrowths of ice formed on the aerodynamic characteristics of the profile. And also go to the solution of the problem in a three-dimensional formulation and to a greater extent take into account the real physical processes of icing of aerodynamic surfaces extremely difficult for the mathematical description

Weaknesses. The developed technique, unlike the existing ones, requires significant computing resources and has large time expenditure. So, the required increase in labor resources will increase the cost of research.

Opportunities. The approaches used in the technique will allow to move on to solving the problem of icing aircraft in a three-dimensional formulation, to the possibility of a comprehensive analysis of the effect of ice on an aircraft. They will provide additional opportunities for creating more advanced and safer aircraft.

Threats. The emergence of new methods of computational fluid dynamics, the creation of increasingly sophisticated universal commercial software products, an increase in the computing power of computer systems and developments in the field of artificial intelligence will lead to new, more advanced solutions to the problems considered in this work.

8. Conclusions

1. On the basis of new experimental data obtained on the physics of icing, it was proposed to separate the formation of the ice-water volumetric structure and subsequent complete freezing of this structure separately in the methodology for modeling the formation of ice outgrowths. This makes it possible to take into account, to a greater degree, the real physical processes of icing of aerodynamic surfaces that are extremely complex for mathematical description.

2. To describe the process of ice growth in a three-dimensional setting, the method of surface control volumes was developed and the main simplifications and assumptions were formulated. In this case, the equation of mass and thermal balances is obtained on the basis of the laws of conservation of mass, energy and momentum for a control volume located on the surface of a streamlined body. To determine the direction of fluid movement along the streamlined surface, the method of successive approximations is applied.

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