UDC 532.516
DOI: 10.15587/2312-8372.2018.145298

Alekseyenko S., Yushkevich O.

APPLICATION OF NUMERICAL SIMULATION METHODS FOR REDUCTION OF AIRCRAFTS ICE PROTECTION SYSTEMS ENERGY CONSUMPTION

Об'єктом дослідження є процеси гідроаеродинаміки та тепломасопереносу, що відбуваються при зледенінні літальних апаратів під час польоту в несприятливих метеорологічних умовах, а також системи захисту від зледеніння. Одним з проблемних місць в розробках систем захисту від зледеніння є мінімізація їх енергоспоживання при забезпеченні безпеки польотів.

В ході дослідження було використано розроблене програмно-методичне забезпечення, яке дозволяє моделювати процеси зледеніння літальних апаратів. Для опису зовнішнього повітряно-крапельного потоку, а також випадіння вологи на обтічну поверхню застосовано підхід на основі рівнянь Нав'є-Стокса та моделі взаємопроникних середовищ. Чисельне моделювання процесу наростання криги виконано з використанням методу поверхневих контрольних об'ємів, що базується на рівняннях нерозривності, збереження кількості руху та енергії.

Результати проведених досліджень представлені на прикладі обтікання профілю крила NACA 0012 в'язким стисливим повітряно-крапельним потоком. Отримані більш точні розподіли основних параметрів потоку на межі пограничного шару, конвективного теплообміну вздовж обтічної поверхні, а також основних величин, що входять до рівнянь масових та теплових балансів. Це пов'язано з тим, що запропонований підхід враховує в'язкість та стисливість потоку, а також має ряд особливостей при описі зовнішнього потоку. Зокрема використовується модифікована модель турбулентності Spalart-Allmaras, що враховує шорсткість стінки. Завдяки цьому забезпечується можливість визначення коефіцієнта конвективного теплообміну за отриманим розподілом температурного поля. У порівнянні з відомими традиційними методами, що використовують інтегральні співвідношення, такий підхід дозволяє враховувати передісторію потоку, може бути застосований у випадку досить великих швидкостей і складних крижаних форм, у задачах у тривимірній постановці. Також такий підхід дає можливість визначити аеродинамічні характеристики профілів з наростами криги з урахуванням шорсткості поверхні.

Результати роботи можуть бути використані при оптимізації роботи систем захисту від зледеніння та визначення шляхів зменшення енергетичних витрат при роботі таких систем.

Ключові слова: *зледеніння аеродинамічних поверхонь, системи захисту від зледеніння, математичне моделювання процесу наростання криги.*

1. Introduction

The problem of icing of aircraft in adverse weather conditions is one of the key to ensuring flight safety. Supercooled water droplets contained in the clouds can freeze when they hit the leading edge of the fuselage, wings, blades, the tail elements of airplanes, helicopters, unmanned and other aircraft, air intakes, and parts of aircraft engines. This significantly changes the structure of the flow, aerodynamic characteristics and operational properties of the aircraft.

The development of anti-icing systems and the determination of their effectiveness is a complex problem, in addition to reliability, for such systems there are strict requirements on the volume, weight and power consumption. The anti-icing systems can be divided by the nature of the work into:

– anti-icing – systems for removal of already formed ice, as a rule, of cyclic action;
– de-icing – which, in principle, do not allow the ice appearance;
– by the principle of action on: mechanical – pneumatic, electropulse, ultrasonic, hydrophobic coatings;
– physical and chemical – liquid, thermal (air-thermal, electro-thermal).

At the same time, the thermal de-icing system can operate in full modes, when all the drop-down moisture evaporates, and incomplete evaporation, when the drop-down moisture in the liquid state drains along the streamlined surface.

Reducing energy costs during the operation of anti-icing systems can be achieved by more accurately determining the optimal cycle time for the system to be turned off while ensuring an acceptable level of negative impact on the aircraft of the resulting inter-cyclical ice. When operating a thermal de-icing system, energy costs can be reduced by accurately determining the coefficient of convective heat transfer along a rough, streamlined surface. On the one hand, it has a dominant influence on heat and mass transfer on an icy surface and, in fact, determines the shape of the resulting ice buildup. On the other hand, the convective heat transfer coefficient plays a key role in determining sufficient local heat fluxes that must be supplied to each element of the protected surface [1].

When solving this problem, one of the effective tools for studying the processes of icing are numerical simulation methods that allow one to obtain data on the distribution of the parameters of an airborne flow along a streamlined icy surface. This will allow to determine in which places of the aircraft and what form during the specified time interval ice buildups will form, and how these buildups will affect the aircraft.

Therefore, it is relevant to study the processes of hydrodynamics and heat and mass transfer occurring during icing of aircraft.

2. The object of research and its technological audit

The object of research is the processes of hydrodynamic and heat and mass transfer occurring during icing of aircraft during flight in adverse meteorological conditions, as well as the system of protection against icing.

Frost forming on aerodynamic surfaces of aircraft can affect the handling characteristics in different ways depending on the location, quantity and type of ice. However, the most common phenomena caused by icing include the decrease in lift and the angle of stall flow on the wing, on the tail elements, the loss of longitudinal stability and controllability of the aircraft as a whole. In addition, the resulting ice layer destroys the structure of the flow, causes a loss of thrust, an increase in the turbulent wake, and leads to an increase in weight.

The development of anti-icing systems and determining their effectiveness is a very difficult problem. Flight tests, although being the most reliable means of research, have some significant drawbacks. In addition to their extremely high cost, in this case is not an exhaustive and quite informative tool. On the one hand, there are difficulties of determining the location of icing conditions and ensuring the repeatability of experimental conditions, and on the other hand, taking into account the danger of a situation, the range of acceptable experimental conditions is significantly narrowed. Therefore, to understand the various mechanisms of ice formation, the degree of its influence on aerodynamics and aircraft controllability and de-icing, it is more efficient to use modeling techniques. However, experimental methods also require the use of expensive and complex equipment — cooled high-speed wind tunnels equipped with a reproduction system for icing-causing precipitation, do not provide a complete picture of the distribution of parameters in the area under study. Experiments conducted under ground conditions can't accurately reproduce icing conditions in flight, they require the use of scale models.

As a result, in modern conditions there is a need to apply numerical modeling methods that will allow:

– reduce the time and cost of developing anti-icing systems;
– create a tool for assessing the effectiveness of anti-icing systems;
– improve the understanding of the peculiarities of the influence of changes in the geometry of aerodynamic surfaces, due to the formation of ice buildup, on the flow pattern.

And, thus, to create more advanced anti-icing systems.

At the same time, one of the problem areas in the development of anti-icing systems is minimization of their energy consumption while ensuring flight safety.

3. The aim and objectives of research

The aim of research is analysis, with the help of the developed software and methodological support, the main physical processes that occur during the solidification of supercooled water falling on the streamlined surface, and their influence on the shape of the ice formed.

To achieve this aim it is necessary to perform the following tasks:

1. Develop a methodology to simulate the processes of icing of aircraft when flying in adverse meteorological conditions, as well as to evaluate the effect of ice buildup on aerodynamic characteristics.
2. Create a model that allows, when calculating the external flow, take into account the roughness of the streamlined surface and calculate the heat fluxes in the area of ice formations from the found temperature field.

4. Research of existing solutions of the problem

In order to predict the shape and estimate the impact of ice buildup on the aerodynamic characteristics of aircraft, a number of mathematical models have been developed that describe the ice buildup in different atmospheric conditions. The most famous are:

– LEWICE 2D (USA) [2];
– ONERA (France) [3];
– TRAJICE2D (United Kingdom) [4];
– CANICE (Canada) [5];
– CIRA (Italy) [6];
– 2DFOIL-ICE (Netherlands) [7].

In which the external airborne flow is described using the equations of the potential and the trajectory model, and the description of the ice buildup process is based on the approach proposed in 1953 [8], using semi-empirical dependencies. Such techniques have limited application in the case of sufficiently high velocities and complex forms of ice buildup (due to the presence of local transonic zones and significant pressure gradients). As well as configurations with multi-bodies, neglecting the history of the flow, do not allow to evaluate the influence of the resulting rough ice buildup on the aerodynamic characteristics of the profile. They have limited application in the transition to problems in a three-dimensional formulation, contain some contradictions in describing the physical picture and, accordingly, the thermodynamics of the ice buildup process.

At the same time, such parameters as surface roughness, ice density, residual amount of liquid water are currently insufficiently studied and are taken into account using empirical relationships. In particular, in [9], relations are obtained for estimating the surface roughness. In [10] – to describe the processes of tearing and spraying droplets. And in [11] – to determine the ice density.

It should also be noted that the use of the listed software products may be completely restricted outside the country of origin, or implies the acceptance of license conditions containing restrictions for use in other countries of the latest versions of the product.

 ISSN 2226-3780

Thus, the results of literary analysis allow to conclude that to ensure the safety of the developed aircraft in adverse weather conditions is a promising problem in the development of numerical methods and models that would:

– allow for a given design of the aircraft to assess the danger of flight in a given meteorological conditions;
– could be used in the development of anti-icing systems;
– allow to find ways to reduce energy costs when operating such systems.

5. Methods of research

Ice buildups on the aerodynamic surfaces of aircraft can be formed during flight in adverse weather conditions with an inactive system of protection against icing, or during the cycle of its drawdown. From the point of view of influence on the aircraft, the most sensitive to icing are the front edges of the wings, control surfaces, and tail elements.

Depending on the parameters of the airborne droplet flow, loose, vitreous or mixed ice may form on the streamlined surface. This is determined by the amount of released latent heat of solidification, which can be absorbed by air, liquid and a streamlined surface, or already existing ice.

Loose ice, as a rule, forms smooth growths of a well-streamlined shape, having a not so significant negative impact on the aerodynamic characteristics. Vitreous growths with the possible formation of «feather» formations behind the protrusions downstream are usually inherent in vitreous ice. In this case, the shape of a horn can be characterized by its height, the angle it forms with the line of the chord, and its location along the profile surface. Corn-like growths can form on both the upper and lower parts of the profile, significantly changing the flow structure, up to a critically negative effect on the wing aerodynamics.

Based on the data obtained as a result of the experiments [12, 13], let's suppose that the main mechanisms for moving an unfrozen liquid along a streamlined surface are:

– at a temperature close to the phase transition temperature, and a sufficiently high water content, the water can take the form of a non-continuous film flowing around the «tubercles» of ice, and move under the action of aerodynamic force;
– at a lower temperature and water content, in the area of inhibition, in the far field of ice formations, water moves by spraying followed by «catching up» the rebounded droplets by an external flow and their «secondary» fallout;
– «jumping» and «splashing out» as a result of the collision of supercooled drops with an icing surface.

When calculating the external airborne air flow containing supercooled water droplets, let's use the Reynolds-averaged Navier-Stokes equations in vector-integral form:

$$\frac{\partial q}{\partial t}+\frac{1}{S}\oint_L\left[F_n(q)-F_{vn}(q)\right]dl=H, \qquad (1)$$

where q, $F_n(q)$, $F_{vn}(q)$ – the vectors of independent variables, non-viscous and viscous flows; H – vector describing the interfacial interaction.

In the simulation of turbulence, the *Spalart-Allmaras* one-parameter differential model [14] with the correction for the rough wall [15] is applied.

The discretization of the Reynolds-averaged Navier-Stokes equations is performed using the implicit finite-volume method. To calculate the convective terms, the *Roe* scheme is used [16]. Viscous terms are approximated by a three-point pattern with a second order of accuracy. The block-matrix system of algebraic equations is solved by an iterative Gauss-Seidel algorithm.

Simulation of ice buildup is performed by iterations with increments in time under conditions determined by the parameters of the atmosphere. A mathematical model describing the motion of supercooled droplets makes it possible to calculate the flow of moisture falling on the streamlined surface. Then, at each time step, on the basis of the thermodynamic model of ice buildup, the form of the existence of moisture is determined, the thickness of the accumulated ice layer and the surface roughness are calculated. At the same time, the process of modeling the ice growth is carried out at the macro and micro levels.

The method of surface control volumes [17–19] is used to describe the thermodynamic processes of phase transformations of water precipitated in the form of supercooled droplets on a streamlined surface. The balance of mass and energy for the control volume can be obtained on the basis of the laws of conservation of mass, energy and momentum (Fig. 1). External heat and mass flows, which are included in the equations of mass and energy balance, are determined directly from the solution of the system of equations describing the airborne flow. The following basic assumptions are made:

– physical transformations in the surface control volume occur instantaneously within a time step;
– hydrodynamic processes within a time step are considered quasi-stationary;
– freezing of liquid on the surface occurs from the side of the streamlined body.

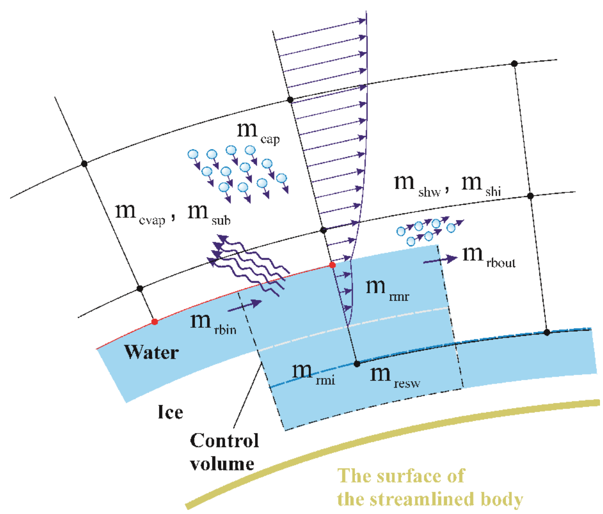

Fig. 1. Scheme of mass flows in the control volume

Mass balance equation. Let's consider a control volume (Fig. 1) located on the surface of the streamlined body [20, 21]. The mass of water entering the reference volume for the period in question consists of the sum of three masses of water in the liquid state:

1) the mass of incoming water from the external flow in the form of supercooled droplets m_{cap};

2) the mass of inflowing water from the previous control volume m_{rbin};

3) the mass of the remaining liquid water from the previous time step m_{resw}:

$$m_{in} = m_{cap} + m_{rbin} + m_{resw}. \quad (2)$$

The mass of water flowing in from the external flow is a part of the supercooled water droplets that are in the external air flow and hit the object. In the case of a two-phase model:

$$m_{cap} = \rho_j \cdot U_{jn} \cdot \Delta b \cdot \Delta s \cdot \Delta t_{acc}, \quad (3)$$

where ρ_j – the concentration of particles near the surface of the body; U_{jn} – normal to the streamlined surface component of the velocity of the second phase; Δb, Δs – width and length of the control volume; Δt_{acc} – step on the rise time of ice.

The mass of flowing water m_{rbin} – the part of the mass of liquid water m_{rbout} from the previous control volume, indicated by the index $i-1$, which is involved in the current control volume, indicated by the index i, external flow:

$$m_{rbin(i)} = m_{rbout(i-1)}. \quad (4)$$

The mass of the remaining water is equal to the mass of residual liquid water m_{rmw}, calculated in the control volume at the previous time step:

$$m_{resw}^{t} = m_{rmw}^{t-\Delta t}. \quad (5)$$

On the other hand, the mass of water m_{in} entering the control volume is spent on the mass of the outgoing water m_{out}, the mass of residual water m_{rmw} and the mass of residual ice m_{rmi}:

$$m_{in} = m_{out} + m_{rmw} + m_{rmi}. \quad (6)$$

The mass of the outgoing water is the mass of water in the solid phase, in the form of a liquid or vapor, which leaves the control volume and consists of five components:

1) the mass of sublimated ice m_{sub};
2) the mass of evaporated water m_{evap};
3) the mass of flowing liquid water m_{rbout};
4) the mass of liquid water torn out by the flow m_{shw};
5) masses of ice torn out by the flow m_{shi}:

$$m_{out} = m_{sub} + m_{evap} + m_{rbout} + m_{shw} + m_{shi}. \quad (7)$$

Heat balance equation. From the energy conservation equation for the control volume, it is possible to obtain the heat balance equation that has the form [19, 20]:

The mass of sublimated ice and the mass of evaporated water are parts of the ice mass and the mass of liquid water that evaporate into the air under the influence of the temperature difference in the boundary layer. The mass of flowing liquid water is the fraction of the mass of liquid water in the control volume, which passes into the next control volume. The mass of torn liquid water is the part of the mass of liquid water inside the control volume, which is emitted into the air under the action of tangential stresses caused by the incoming flow. It can be determined by the Weber number, using the empirical relationship [2]. If there is a stall that is determined from the calculation of the external compressible viscous flow using the *Spalart-Allmaras* turbulence model, let's assume that all unfrozen water flowing from the previous control volume is sprayed and carried away by the external flow.

The mass of the torn-out ice is a part of the ice mass in the control volume, which is thrown into the air due to the detachment of ice crystals under the action of aerodynamic force [21].

Heat balance equation. From the energy conservation equation for the control volume, it is possible to obtain the heat balance equation that has the form [19, 20]:

$$Q_f + Q_{ss} + Q_{sub} + Q_{evap} + Q_{adh} + Q_{kin} + Q_{cd} + Q_{cv} + Q_{rad} = 0, \quad (8)$$

where Q_f – latent heat of solidification; Q_{sub} – latent heat of sublimation; Q_{evap} – latent heat of evaporation; Q_{ss} – internal heat; Q_{cv} – convection heat exchange; Q_{cd} – conductivity heat exchange; Q_{rad} – radiation heat exchange; Q_{adh} – heat of aerodynamic heating; Q_{kin} – heat of kinetic heating.

Based on mass and heat balances, let's calculate the frozen fraction f of water passing through the control volume during Δt_{acc}.

6. Research results

The results of calculations are presented on the example of the test case of the flow around the wing profile of the NACA 0012 with a chord length L=0.3 m with an airborne droplet flow at zero angle of attack, having a velocity of V_∞=129.46 m/s, temperature T_∞=–12.6 °C, pressure p_∞=9.075·10^4 Pa. Using the developed methodology, calculations were performed for a «rough» profile having an average height of equivalent sand grain roughness k_s=0.00020 m. Fig. 2 shows the distribution of the dropout coefficient β along the profile surface, as well as the values of V_e velocity, p_e pressure and T_e temperature at the boundary layer boundary, temperature on the surface of the body T_s and T_{rec} recovery temperature. In the «wet» mode, when a part of the liquid flows over the surface of increasing ice, the surface temperature T_s is equal to the temperature of the phase transition of water. Then, as the distance from the stagnation point increases, a laminar-turbulent transition occurs in the boundary layer. Convective heat transfer increases significantly, all liquid entering the surface control volume freezes, the temperature of the resulting ice is below the phase transition temperature, but above the recovery temperature. In the area where ice was formed in the previous time steps, but where droplets from the external flow no longer fall out, cooling occurs due to the sublimation of ice and the surface temperature is below the recovery temperature.

Fig. 3 shows the distribution of the coefficient of convective heat transfer α and the frozen n fraction along the icing surface. It can be seen that there is a sharp increase in the value of the phase transition coefficient from values of ~0.75·10^3 W/m^2/K in the region of the laminar boundary layer to values of ~1.25·10^3 W/m^2/K in the region of the laminar-turbulent transition. And, then, the subsequent gradual decrease in the value of the phase transition coefficient due to an increase in the thickness of the boundary layer. Accordingly, the frozen fraction in the region of the laminar boundary layer and the stagnation temperature turn out to be small and increase with the turbulization of the boundary layer to 1.

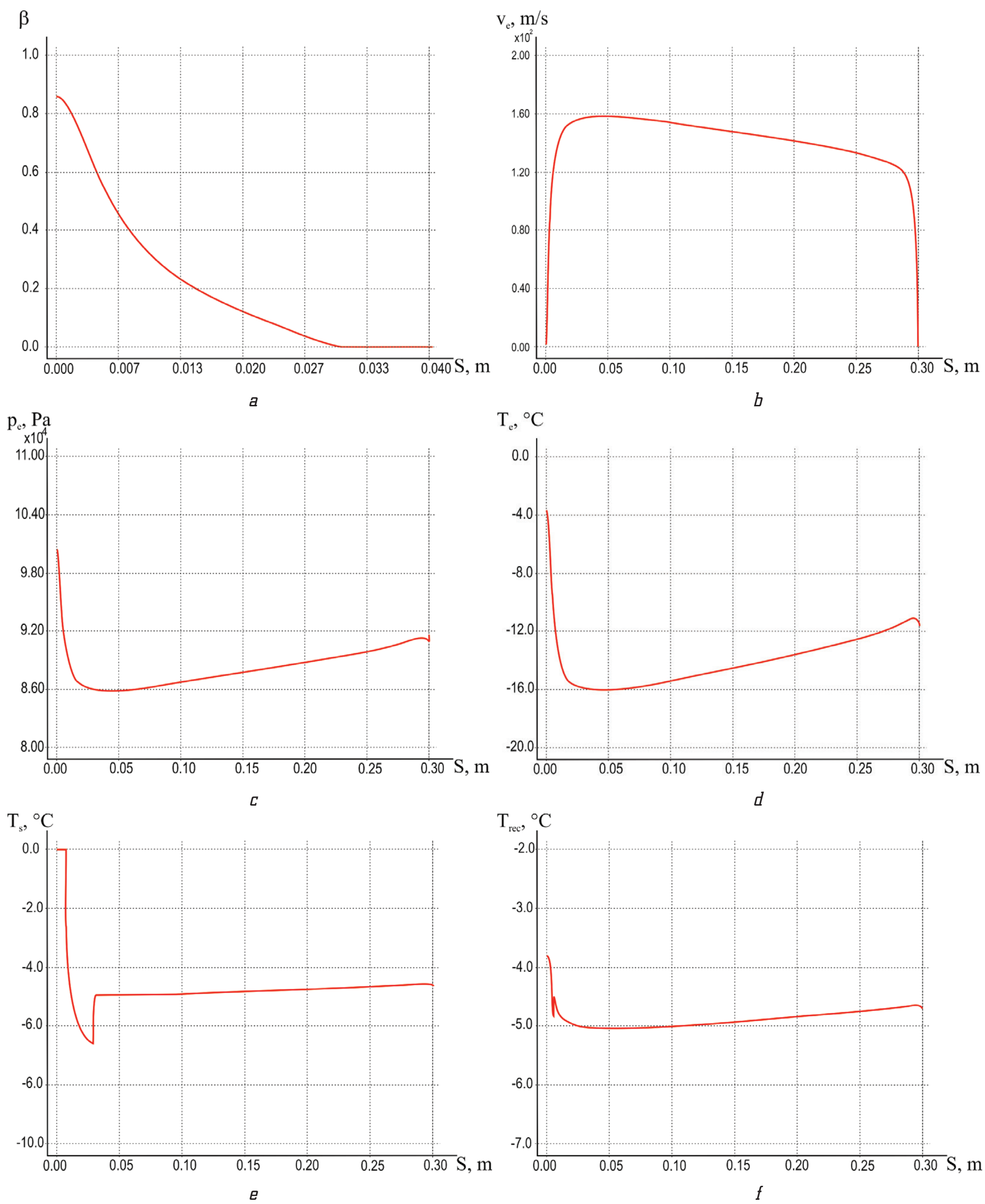

Fig. 2. Parameters of the airborne flow, corresponding to the initial stage of the icing process:
a – dropout coefficient β; *b* – flow velocity at the boundary layer, V_e; *c* – pressure at the boundary layer, p_e; *d* – temperature at the boundary layer, T_e; *e* – temperature on the surface of the profile T_s; *f* – recovery temperature T_{rec}

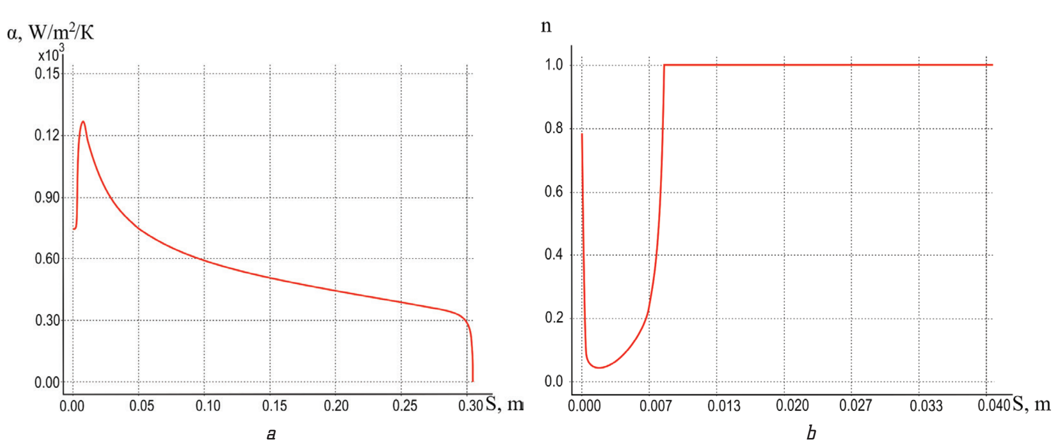

Fig. 3. Distribution of parameters characterizing the shape of an ice buildup along the icing surface:
a – convective heat transfer coefficient, α; *b* – frozen share, n

Fig. 4, 5 summarize the components of heat and mass balances. It has been demonstrated that the main contribution to the heat balance is made by heating due to the released latent heat of the phase transition and cooling due to convection and evaporation. A significant increase in the mass of freezing water occurs during the transition of the boundary layer to the turbulent regime, accompanied by a significant (several times) increase in heat flow due to convection and evaporation. As a result, in the region of the laminar-turbulent transition, growths begin to form, over time, they overlap into large «horn-shaped».

Fig. 6 shows the distribution of isolines of Mach numbers near the streamlined profile, as well as the buildup shape obtained using the developed technique and the experimental form [2].

The considered design case corresponds to a wet icing regime – smooth ice forms in the region of the stagnation point, on the surface and above the surface of which there is a large amount of unfrozen water moving downstream by spraying and «jumping over».

As noted earlier, as the distance from the stagnation area increases, the boundary layer becomes turbulent, the convective heat transfer coefficient and the frozen fraction of the water entering the control volume increase. Protrusions form on the ice surface, on which characteristic ice «horns» are then formed. In the region of these protrusions, the incident flow accelerates, and a flow disruption occurs.

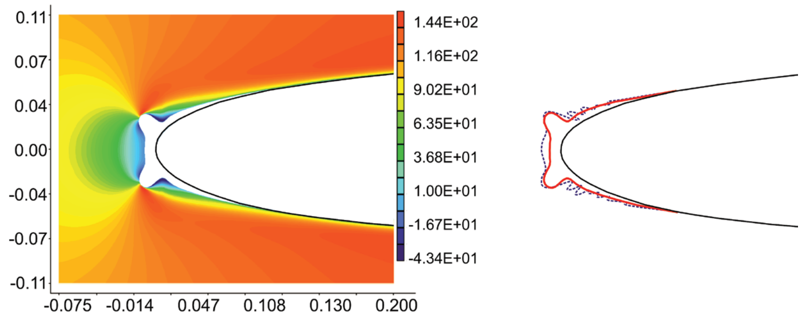

Fig. 6. Builpuo shape, distribution of isolines of Mach numbers near the NACA 0012 profile (α=0°): —— – calculation of this work; – – – – experimental data [2]

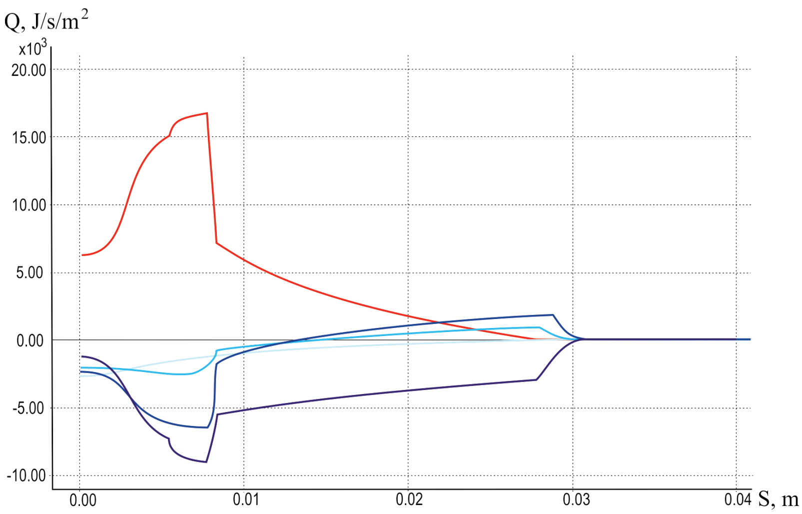

Fig. 4. Components of heat balance: —— – latent heat of evaporation; —— – convective heat transfer minus the heat of kinetic heating and the heat of aerodynamic heating; —— – heat transfer through heat conduction; —— – internal heat; —— – latent heat of solidification

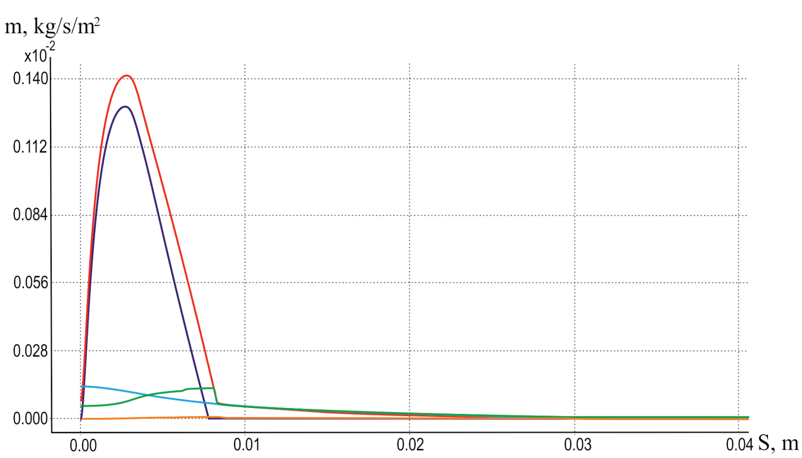

Fig. 5. The components of mass balance: —— – the mass flow rate of supercooled water flowing in from the external flow; —— – the mass flow rate of fluid entering from the previous control volume minus evaporating water; —— – mass flow rate of fluid flowing into the next control volume; —— – mass flow of freezing water; —— – mass flow of evaporating water

There is a good agreement between the experimental and calculated forms of growth in the region of the stagnation point and at some distance along the flow (in the region of large «horns»), but in the far lower ice formation area, where relatively small local protrusions predominate, there is a significant discrepancy in the results.

The proposed method allows to consider icing problems with a heated streamlined surface and, accordingly, to simulate the formation of «barrier» ice during the operation of the anti-icing system [18], as well as to evaluate the effect of ice buildup on the aerodynamic characteristics of the profile.

7. SWOT analysis of research results

Strengths. The developed method for calculating the flow of bodies around an airborne viscous compressible flow describes the process of moisture deposition on a streamlined surface using the model of interpenetrating media. Convective heat transfer is determined by the temperature field obtained from solving the Navier-Stokes equations using a modified turbulence model for a rough wall. This approach allows, in contrast to existing methods:

– more accurately and in a wider range of parameters, describe the change in the geometry of the streamlined bodies and take into account the effect of this change on the aerodynamic characteristics;

– the technique is also valid in the case of sufficiently high velocities and complex forms of ice buildup, in the presence of local transonic zones and significant pressure gradients;

– takes into account the history of the flow, can be applied to configurations with multi-bodies;
– allows to assess the influence of rough ice buildup on the aerodynamic characteristics of the profile;
– allows to go to the tasks in the three-dimensional formulation.

The developed methodology and software and methodological support will allow creating more sophisticated systems for de-icing, and increase the safety of aircraft operations.

Weaknesses. The developed technique requires significant computational resources and has a large time cost. Consequently, an increase in labor resources is required – this leads to an increase in the cost of research.

Opportunities. The approaches used in the methodology will allow to proceed to solving the problem of icing of aircraft in a three-dimensional formulation, to the possibility of a comprehensive analysis of the effects of ice on the aircraft. Provide additional opportunities to create more sophisticated and safer aircraft.

Threats. The emergence of new methods of computational hydrodynamics, the creation of more and more advanced universal commercial software products, an increase in the computing power of computer systems and the development in the field of artificial intelligence will lead to new, more perfect solutions to the problem considered in the work.

8. Conclusions

1. Based on the Navier-Stokes equations using the *Spalart-Allmaras* turbulence model, a method has been developed for calculating airborne viscous compressible flow around a body, which allows simulating the processes of icing of aircraft during flight in adverse meteorological conditions. In contrast to the traditional approach based on the equations of potential, by taking into account the compressibility of the medium, the technique allows to obtain more accurate solutions at flow velocities higher than 0.4 M. The technique is valid in the case of complex forms of ice buildup causing the formation of local transonic zones and significant pressure gradients when it is impossible to neglect the effects of viscosity and compressibility of the medium. Takes into account the history of the flow, can be applied to configurations with multibodies.

2. In the developed methodology, a modified *Spalart-Allmaras* turbulence model is used, taking into account the wall roughness. This allows, when calculating the external flow, to take into account the roughness of the streamlined surface and to calculate the heat fluxes in the region of ice formations from the temperature field found. This solution allows to go to solving problems in a three-dimensional formulation. Also, the technique allows to determine the aerodynamic characteristics of aerodynamic profiles with frost, including the initial stage of icing. The obtained results and the analysis of parameters on the surface of the ice allows to determine ways to optimize energy consumption during the operation of anti-icing systems.

References

1. Fuzaro Rafael C., Mendes Pio D., A. Lima da Silva G. CFD and Boundary Layer Models with Laminar-Turbulent Transition around Airfoils and a Rough Cylinder: Results Validation // SAE Technical Paper Series. 2015. 14 p. doi: http://doi.org/10.4271/2015-01-2163
2. Wright W. B. Users Manual for the Improved NASA Lewis Ice Accretion Code LEWICE 1.6 // National Aeronautical and Space Administration (NASA). Contractor Report. 1995. 95 p.
3. Gent R. W. TRAJICE2 – A Combined Water Droplet and Ice Accretion Prediction Program for Aerofoil // Royal Aerospace Establishment (RAE). Technical Report Number TR90054. Farnborough, 1990. 83 p.
4. Guffond D., Hedde T., Henry R. Overview of Icing Research at ONERA, Advisory Group for Aerospace Research and Development // Fluid Dynamics Panel (AGARD/FDP) Joint International Conference on Aircraft Flight Safety – Actual Problems of Aircraft Development. Zhukovsky, 1993. 7 p.
5. Ice Accretion on Aircraft Wings with Thermodynamic Effects / Tran P. et. al. // American Institute of Aeronautics and Astronautics. 32nd Aerospace Sciences Meeting & Exhibit. AIAA Paper. No. 0605. Reno, 1994. 9 p. doi: http://doi.org/10.2514/6.1994-605
6. Mingione G., Brandi V. Ice Accretion Prediction on Multielement Airfoils // Journal of Aircraft. 1998. Vol. 35, Issue 2. P. 240–246. doi: http://doi.org/10.2514/2.2290
7. Dillingh J. E., Hoeijmakers H. W. M. Accumulation of Ice Accretion on Airfoils during Flight: Conference // Federal Aviation Administration In-flight Icing and Aircraft Ground De-icing. Chicago, 2003. 13 p.
8. Messinger B. L. Equilibrium Temperature of an Unheated Icing Surface as a Function of Air Speed // Journal of the Aeronautical Sciences. 1953. Vol. 20, Issue 1. P. 29–42. doi: http://doi.org/10.2514/8.2520
9. Shin J., Bond T. Experimental and Computational Ice Shapes and Resulting Drag Increase for a NACA 0012 Airfoil // National Aeronautical and Space Administration (NASA). Technical Memorandum. No. 105743. 1992. 10 p.
10. Olsen W., Walker E. Experimental Evidence for Modifying the Current Physical Model for Ice Accretion on Aircraft Surfaces // NASA Technical Memorandum. No. 87184. 1986. 47 p.
11. Jones K. F. The Density of Natural Ice Accretions // Fourth International Conference on Atmospheric Icing of Structure. E. D. F. 1998. Vol. 77, Issue 1. P. 114–118.
12. Interaction of Supercooled Large Droplets with Aerodynamic Profile / Alekseyenko S. et. al. // SAE Technical Paper 2015-01-2118. 2015. 12 p. doi: http://doi.org/10.4271/2015-01-2118
13. An experimental study of freezing of supercooled water droplet on solid surface / Alekseenko S. V. et. al. // Technical Physics Letters. 2016. Vol. 42, Issue 5. P. 524–527. doi: http://doi.org/10.1134/s1063785016050187
14. Spalart P., Allmaras S. A one-equation turbulence model for aerodynamic flow // AIAA Paper. 1992. No. 92 – 0439. 22 p. doi: http://doi.org/10.2514/6.1992-439
15. Aupoix B., Spalart P. R. Extensions of the Spalart–Allmaras turbulence model to account for wall roughness // International Journal of Heat and Fluid Flow. 2003. Vol. 24, Issue 4. P. 454–462. doi: http://doi.org/10.1016/s0142-727x(03)00043-2
16. Roe P. Characteristic-Based Schemes for the Euler Equations. Annual Review of Fluid Mechanics. 1986. Vol. 18, Issue 1. P. 337–365. doi: http://doi.org/10.1146/annurev.fluid.18.1.337
17. Alekseyenko S. V., Prykhodko O. A. Numerical simulation of icing of a cylinder and an airfoil: model review and computational results // TsAGI Science Journal. 2013. Vol. 44, Issue 6. P. 761–805. doi: http://doi.org/10.1615/tsagiscij.2014011016
18. Alekseenko S. V., Prikhod'ko A. A. Mathematical modeling of ice body formation on the wing airfoil surface // Fluid Dynamics. 2014. Vol. 49, Issue 6. P. 715–732. doi: http://doi.org/10.1134/s0015462814060039
19. Prikhod'ko A. A., Alekseenko S. V. Numerical Simulation of the Processes of Icing on Airfoils with Formation of a «Barrier» Ice // Journal of Engineering Physics and Thermophysics. 2014. Vol. 87, Issue 3. P. 598–607. doi: http://doi.org/10.1007/s10891-014-1050-0
20. New Roughness Computation Method and Geometric Accretion Model for Airfoil Icing / Fortin G. et. al. // Journal of Aircraft. 2004. Vol. 41, Issue 1. P. 119–127. doi: http://doi.org/10.2514/1.173
21. Fortin G., Laforte J., Beisswenger A. Prediction of Ice Shapes on NACA0012 2D Airfoil // Anti-Icing Materials International Laboratory, Universite du Quebec a Chicoutimi. 2003. N 01–2154. 7 p. doi: http://doi.org/10.4271/2003-01-2154

Alekseyenko Sergey, *PhD, Associate Professor, Department of Mechatronics, Oles Honchar Dnipro National University, Ukraine, e-mail: alexeyenko_sv@ukr.net, ORCID: http://orcid.org/0000-0003-0320-989X*

Yushkevich Oleg, *PhD, Associate Professor, Department of Mechatronics, Oles Honchar Dnipro National University, Ukraine, e-mail: ReilTO@i.ua, ORCID: http://orcid.org/0000-0002-7199-8424*