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PRACTICAL APPLICATION OF MATHEMATICAL MODELS OF ELECTRO-THERMO-MECHANICAL PROCESSES IN INDUSTRIAL INDUCTION FURNACES WITH THE AIM OF INCREASING THEIR ENERGY EFFICIENCY

Об'єктом дослідження є промислова індукційна сталеплавильна піч. Одним з найбільш проблемних місць індукційних сталеплавильних печей є низька енергоефективність через їх конструктивну недосконалість та існуючий технологічний процес теплової роботи, що призводить до перевитрат електричної енергії.

В ході дослідження використовувалося математичне моделювання впливу електромагнітного поля на елементи індукційної печі. Також було проведено досліди по перевірці дії електромагнітного поля на траєкторію та швидкість руху розплаву металу як в лабораторних умовах, так і в індукційній печі. Для дослідження руху розплаву металу було виготовлено спеціальний термопоплавок. Поплавок складався з керамічної термостійкої втулки, всередину якої був вставлений стрижень з вольфраму. Для більшої достовірності експерименту вага поплавка дорівнювала вазі розплаву такого ж об'єму. Завдяки проведеним дослідам в лабораторних умовах та на печі було розроблено алгоритм режимів роботи електричного індуктора. Проведено аналіз режимів роботи індуктора на різних частотах. Виявлено вплив частоти струму, який живить індуктор на глибину проникнення електромагнітного поля. Зі зменшенням частоти струму (f<50 Гц) глибина проникнення збільшується і навпаки, при збільшенні (f>50 Гц) – зменшується. Також підтверджено, що максимальна дія електромагнітного поля на розплав зосереджена всередині (по висоті) розплаву.

Розглянуто основні питання застосування на практиці математичної моделі електротермомеханічних процесів, що виникають у промислових індукційних печах під час нагрівання й розплавлення різних металів та їхніх сплавів, які мають широке застосування в машинобудуванні. Використана система рівнянь у формі крайових задач електродинаміки для квазістаціонарного магнітного поля, нестаціонарної теплопровідності та неізотермічної термопластичності. Застосування на практиці запропонованих методів використання можливостей математичного моделювання електрометалургійних процесів є підґрунтям для створення сучасних комп'ютерних програм з метою підвищення енергоефективності за рахунок суттєвого зменшення зайвих, недоцільних втрат електроенергії.

Ключові слова: *індукційна сталеплавильна піч, математичне моделювання електротермомеханічних процесів, крайові задачі електродинаміки, індукційний нагрів.*

1. Introduction

At many engineering and motor-building enterprises, melting is used in induction furnaces of various metals and their alloys in order to obtain blanks which shape is as close as possible to the geometric dimensions of the finished part. This, in turn, makes it possible to reduce the cost of further machining and increase the competitiveness of the entire production [1].

Induction heating followed by melting is the most non-contact method of transmitting electromagnetic energy to the body; it is heated by exciting electric currents (Foucault currents) in it under the action of an alternating electromagnetic field created by an inductor. Also, the positive characteristics of induction furnaces should include the effect of continuous mixing of the melt under the action of an electromagnetic field, which makes it possible to obtain a high homogeneity of the alloy structure, which is extremely necessary in modern engineering.

But today, in the operation of induction furnaces [2], there are significant overspending of electricity through constructive imperfection and the inconsistency of the existing technological (metallurgical) process with modern European requirements for energy saving. Therefore, it is relevant to study ways to improve the energy efficiency of induction steel-melting furnaces.

2. The object of research and its technological audit

The object of research is an industrial induction steel-melting furnace.

The primary winding serves as an inductor, which is streamlined by alternating current. The secondary winding and at the same time the load – the metal itself, which is loaded into the crucible and placed inside the inductor. An alternating current flows through the inductor and creates an electromagnetic field. The electromagnetic field penetrates the lead metal inside the inductor and causes eddy currents according to the law of induction. This leads to the heating of the metal. If a sufficient amount of electricity is supplied, the process of melting the metal begins. The formation of electromagnetic fields leads to the appearance of electromagnetic forces. These electromagnetic forces in an induction furnace cause the effect of the movement of the melt and the formation of a dome-like molten metal dome.

Fig. 1 shows how the electromagnetic field inside the inductor acts on the melt.

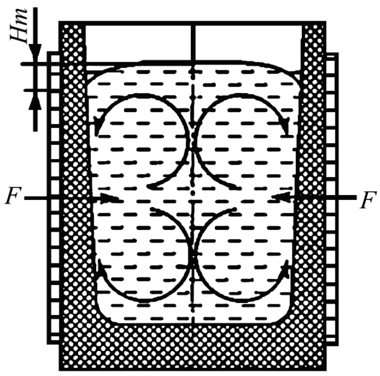

Fig. 1. The action of the electromagnetic field inside the inductor on the melt: F – effort; H_m – meniscus height

One of the most problematic places in induction steel-melting furnaces is low energy efficiency due to their constructive imperfections and the existing technological process of thermal work, which leads to excessive energy consumption.

3. The aim and objectives of research

The aim of research is identification of the conditions for the thermal operation of an induction steel-melting furnace.

To achieve this aim it is necessary to solve the following objectives:

1. To develop an algorithm of operating modes of the steel-melting complex on the basis of induction conversion of electrical energy.
2. To identify the distribution of the electromagnetic field inside the inductor.
3. To make a special thermal float to study the movement of the molten metal.

4. Research of existing solutions of the problem

The main objects of research of electro-thermo-mechanical processes taking place in induction furnaces during their work is an inductor and a metal (alloy), which is to be melted. Various physical phenomena occurring in the cavity of an induction furnace, sufficiently described in the framework of continuum mechanics by the integral and field characteristics of the electromagnetic, thermal and stress-strain state, as well as the parameters of phase transformations in [3, 4].

It is known that the most effective method for development and design of induction furnaces, as well as melting technology, is mathematical modeling. There are a large number of relevant mathematical models [5, 6]. However, in the majority they are mainly theoretical, and the proposed analytical solutions and techniques are rather complicated. This significantly limits their use. Therefore, in practice, apply simplified options. But they do not provide enough information to create modern computer programs, which would provide the necessary increase in the efficiency of the technological (electrometallurgical) process.

In 1992 ABB (Sweden) has created a two-station melting complex, which operates from a single dual channel power source. It redistributes the flow of energy between the furnaces in the melting cycle (the Twin-Power melting system) [7]. The company Inductortherm (England) has developed a system of two-post mid-frequency melting «DUAL-TRAK». The creation of such systems made it possible to increase the efficiency of using the installed capacity of the power supply system of furnaces, the melting devices performance and reduce its cost by 30–40 % [8].

Two-station melting installations of the DMI type (double-station melting installation) have a double-channel distribution power supply system, which provides the mode of simultaneous melting of metal in two furnaces. It allows to perform independent power control on each furnace at a constant power at the input of the power supply system [9]. In this case, the DMI performance is growing by 20–25 % compared to the single-station.

The method of focusing power in different zones of the crucible at different stages of melting allows to intensify the metallurgical processes for the preparation of alloys [10].

With the displacement of the low-frequency components of the current in the sections of the inductor by 120 deg. el., additional traction forces are created in the deep layers of the molten bath during the crucible walls. Their direction (up or down) depends on the phase sequence of the low-frequency components of the current in the inductor sections [11].

An example of the design of a continuous induction furnace is the horizontal furnace developed by Ajax Magnethermic (USA) [12]. It is a ceramic tunnel located inside a horizontal cylindrical inductor and has a loading and unloading bath at the ends.

The design of turbo-inducting crucible furnaces (TICF) of continuous and semi-continuous actions is based on the designs of traditional ICFs (induction crucible furnace), which are complemented by magnetodynamic devices that rotate the metal around the crucible axis [13].

Thus, the results of the analysis allow to conclude that the designs of induction furnaces have their differences, each of which is aimed at improving the melting process, but not at energy saving. Therefore, the study of this issue is promising.

5. Methods of research

The existing simplified computational methods are classified as follows: analytical solutions of the corresponding boundary value problems, numerical methods, engineering and practical approaches. At the moment, physical modeling is used in the laboratory, where the role of the melt is played by mercury in a glass flask and torsion balance. In experiments on an induction furnace, a heat-resistant ceramic float is used, which shows the movement of the melt.

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6. Research results

In order to conduct energy audit of the whole process of heating and subsequent melting of the metal in induction furnaces, it is necessary to determine its main stages, which have their own characteristics and, accordingly, require different costs of electricity. This significantly affects the total energy consumption per unit of production. Fig. 2 shows the algorithms of operating modes of the steel-making complex on the basis of induction conversion of electric energy.

Despite the fact that most of the metals and their alloys used in mechanical engineering, have a slight magnetic and electrical anisotropy, they can be attributed to isotropic. In nonmagnetic conductive materials, magnetic anisotropy is absent, but their electrical properties are considered isotropic. Based on this, when carrying out calculations of the electromagnetic state of the conductive elements of an induction furnace (conductor, magnetic conductor, material, heats up (melts)), the magnetic and electrical properties can be considered isotropic. And in environments that do not conduct electric current (air, heat insulation, etc.). Conductance currents are insignificant compared to displacement currents. The presence of the delay effect is due to the presence of bias currents [14].

The source of the alternating electromagnetic field in the conductor (inductor) emits into the environment electromagnetic waves, the propagation of which in the non-current-conducting elements of an induction furnace can be characterized using the retention function:

$$Q \sim e^{i\omega\left(t-\frac{r}{u}\right)} = e^{i\omega t}\left(1-\frac{i\omega r}{u}+\dots\right), \tag{1}$$

where r/u – the delay time during which the electromagnetic wave travels the distance r; u – the propagation velocity of the electromagnetic field in a non-conducting medium; ω – the frequency of the disturbing field.

Existing induction furnaces are classified by design, scope and energy parameters. The main energy parameters that must be considered in the design include: power, current frequency, efficiency, permissible maximum temperature, time required to complete a complete metallurgical cycle.

But despite the constructive diversity, they have common characteristics regarding the process of converting the energy of an electromagnetic field into thermal energy.

As is known, the following indicators are used to determine the energy characteristics of an induction furnace: electric, thermal, and overall efficiency, which can be calculated using the following formulas:

$$\eta_e = \frac{P_2}{P_e}, \tag{2}$$

$$\eta_t = \frac{P_t}{P_2} = \frac{P_t}{P_t + \Delta P_t}, \tag{3}$$

$$\eta_0 = \eta_e \cdot \eta_m, \tag{4}$$

where P_2 – the total power that is transmitted directly to the heated metal; P_e – electric power supplied to the inductor; P_t – net power; ΔP_t – heat loss power.

The value η_0 of industrial induction furnaces, as a rule, is in the range of 0.42–0.72 and largely depends on:

– structural design of the basic elements of induction furnaces;

– properties of the heated material in the case under study η_e=0.6...0.8, η_t=0.7...0.9;

– technological parameters of the metallurgical process modes (heating rate, maximum melting temperature, etc.).

In order to ensure stable operating conditions limited by temperature and the level of equivalent voltages, a study is conducted of the structural and technological parameters of an industrial induction furnace. This makes it possible to determine the magnitude of the mechanical forces and heat generation in its main elements. The analysis of temperature fields and the stress-strain state in the material is heated.

To solve this problem on the basis of a combination of electro-thermo-mechanical effects, it is necessary to apply a mathematical model [15, 16], which is represented by a related system of the following equations:

$$\operatorname{rot}\mu^{-1}\operatorname{rot}\dot{\bar{E}} = -i\omega\gamma\left(\dot{\bar{E}} - \dot{\bar{E}}_0\right), \tag{5}$$

$$\operatorname{rot}\gamma^{-1}\operatorname{rot}\dot{\bar{H}} = -i\omega\mu\dot{\bar{H}}, \tag{6}$$

$$\operatorname{div}\dot{\bar{E}} = 0, \tag{7}$$

$$\operatorname{div}\mu\dot{\bar{H}} = 0, \tag{8}$$

where μ – magnetic permeability; γ – specific conductivity.

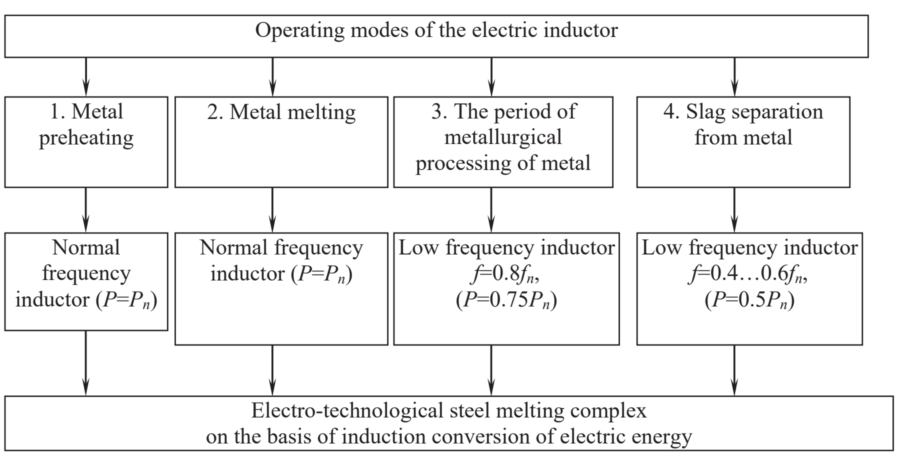

Fig. 2. Algorithms of operating modes of a steel-melting complex on the basis of induction conversion of electric energy

When forming the boundary conditions in a specific case, it is necessary to take into account the specifics of electromagnetic fields in induction furnaces, more precisely: for close electromagnetic fields at a sufficient distance from the inductor ($r > r_0$), the full shielding condition can be applied:

$$\frac{\partial \dot{\bar{E}}(\bar{r}_0)}{\partial n} = 0, \quad (9)$$

$$\frac{\partial \dot{\bar{H}}(\bar{r}_0)}{\partial n} = 0, \quad (10)$$

or full attenuation:

$$\dot{\bar{E}}(\bar{r}_0) = 0, \quad (11)$$

$$\dot{\bar{H}}(\bar{r}_0) = 0. \quad (12)$$

The conjugation conditions are reflected relative to the complex amplitude vectors. $\dot{\bar{E}}$ and $\dot{\bar{H}}$ [13]:

$$\bar{n} \cdot \left(\dot{\bar{E}}_l - \dot{\bar{E}}_t\right) = 0, \quad (13)$$

$$n \cdot \left(\mu_l^{-1} \operatorname{rot} E_l \operatorname{rot} \dot{\bar{E}}_t\right) = 0, \quad (14)$$

$$\bar{n} \cdot \left(\dot{\bar{H}}_l - \dot{\bar{H}}_t\right) = 0, \quad (15)$$

$$\bar{n} \cdot \left(\gamma_l^{-1} \operatorname{rot} \dot{\bar{H}}_l - \gamma_t^{-1} \operatorname{rot} \dot{\bar{H}}_t + \dot{\bar{E}}_{0t}\right) = 0. \quad (16)$$

Thus, boundary value problems are formulated for determining the vectors of complex amplitudes of strengths $\dot{\bar{E}}$ and magnetic $\dot{\bar{H}}$ fields in the basic elements of an induction furnace.

Thermal calculations should be carried out in the case of non-stationary thermal conductivity, and non-isothermal thermoelastic-plasticity within the framework of the flow theory with isotropic reinforcement associated [14, 15] by the von Mises conditions:

$$\operatorname{div}\left[\lambda(T) \operatorname{grad} T\right] + Q_1 + Q' = pc\left(T \frac{\partial T}{\partial t}\right), \quad (17)$$

$$\operatorname{div} \dot{\sigma} + \bar{F}_1 + \bar{F}' = 0, \quad (18)$$

$$\dot{\varepsilon} = \frac{1}{2}\left(\bar{\nabla}\bar{u} + \bar{u}\bar{\nabla}\right), \quad (19)$$

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^T, \quad (20)$$

$$\dot{\varepsilon}^e = \frac{1}{E}\left[(1+V)\dot{\sigma} - 3V\sigma_0 \dot{I}\right], \quad (21)$$

$$d\dot{\varepsilon}^p = \frac{3}{2}\frac{d\dot{\varepsilon}_i^{p^*}}{\sigma_i}\dot{s}, \quad (22)$$

$$\dot{\varepsilon}^T = a\Delta T\dot{I}, \quad (23)$$

where $\bar{u}$ – displacement vector; $\dot{\sigma}$, $\dot{\varepsilon}$ – stress and strain tensors; $\dot{s}$ – stress tensor deviator; $\dot{\varepsilon}^e$, $\dot{\varepsilon}^p$, $\dot{\varepsilon}^T$ – tensors of elastic, plastic and temperature components of deformation; $d\varepsilon_i^{p^*}$ – intensity of plastic deformation; σ_i – stress intensity; σ_0 – average stress value; $\bar{F}_1$, Q_1 – period-mediated τ components of ponderomotive forces and Joule heat sources; $\bar{F}'$, Q' – volumetric power and thermal effects of non-electromagnetic origin; p – density of the heated material; c – constant heat capacity; λ – thermal conductivity coefficient; E – Young's modulus; V – Poisson's ratio; a – thermal expansion coefficient.

The influence of the electromagnetic field on the elements of an induction furnace is characterized by ponderomotive forces and sources of Joule heat of constant power Q [16]. For conductive ferromagnetic materials of elements of an induction furnace:

$$\vec{F} = \gamma\mu\vec{E}' \cdot \vec{H} + \frac{\mu - \mu_0}{\mu}\operatorname{grad}\left(\mu^2\vec{H} \cdot \vec{H}\right), \quad (24)$$

$$Q = y\vec{E}' \cdot \vec{E}', \quad (25)$$

where $\vec{E}' = \vec{E} + \vec{E}_0$.

In the case of quasi-electromagnetic fields with the effective value of the complex amplitudes $\dot{E}'$ and $\dot{H}$, the reflection of the ponderomotive forces and constant Joule heat power is possible [13] as the sum of the indirect for the period $\tau = 2\pi/\omega$. As well as periodic components in time:

$$F' = F_1'' + F_2', \quad (26)$$

$$Q = Q_1 + Q_2, \quad (27)$$

where

$$\vec{F}_1 = \frac{\gamma\mu}{2}\left(\dot{\vec{E}}' \cdot \dot{\vec{H}}^* + \dot{\vec{E}}^* \cdot \dot{\vec{H}}\right) + \frac{\mu - \mu_0}{\mu}\operatorname{grad}\left(\mu^2\dot{\vec{H}} \cdot \dot{\vec{H}}^*\right), \quad (28)$$

$$\vec{F}_2 = \frac{\gamma\mu}{2}\left(\dot{\vec{E}}' \cdot \dot{\vec{H}}e^{2i\omega t} + \dot{\vec{E}}'^* \cdot \dot{\vec{H}}'^* e^{-2i\omega t}\right) + \\ + \frac{\mu - \mu_0}{2\mu}\operatorname{grad}\left(\mu^2\left(\dot{\vec{H}} \cdot \dot{\vec{H}}e^{2i\omega t} + \dot{\vec{H}}^* \cdot He^{-2i\omega t}\right)\right), \quad (29)$$

$$Q_1 = \gamma\dot{\vec{E}}' \cdot \dot{\vec{E}}'^*, \quad (30)$$

$$Q_2 = \frac{\gamma}{2}\left(\dot{\vec{E}}' \cdot e^{2i\omega t} + \dot{\vec{E}}'^* \cdot \dot{\vec{E}}'^* e^{-2i\omega t}\right). \quad (31)$$

It is necessary to note the fact that periodic in time $\vec{F}_2$, Q_2 cause mechanical oscillations (vibrations) of the structural elements of an induction furnace, which explains the «buzz» during its operation. To reduce the harmful effects of vibrations and prevent the effect of self-destruction of an induction furnace, measures are recommended to firmly secure its main elements [17].

In order to verify the correctness of the presented theoretical calculations, relevant experiments are repeatedly carried out both in laboratory conditions and on industrial equipment under real production conditions.

Experiment No. 1 – check the action of the electromagnetic field on the molten metal in the laboratory.

Mercury was used as a metal melt, which has all the similar properties. A glass vessel containing mercury was placed inside a cylindrical inductor. Torsion scales was used as a measuring device, «impeller» which is made of fluoroplastic and immersed in mercury. This made it possible to investigate the influence of the electromagnetic field on the «molten metal».

During the experiment, the dependence of the penetration depth of the electromagnetic field as a function of current frequency was revealed: as the frequency of the current feeding the inductor decreases ($f<50$ Hz), the penetration depth increases and vice versa ($f>50$ Hz) the penetration depth decreases. It was also confirmed that the maximum effect of the electromagnetic field on the melt is concentrated inside (along the height) of the melt (Fig. 3).

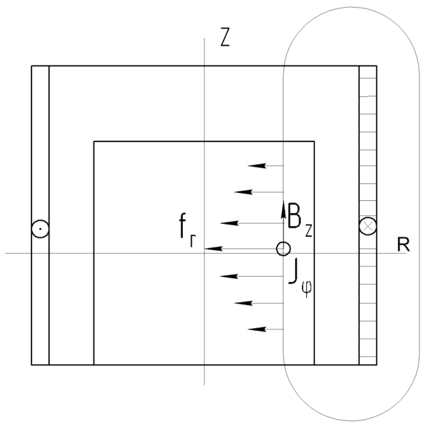

Fig. 3. The distribution of electromagnetic values in the middle of the inductor

The appearance of a spherical «cap» on the surface of the melt, which has a «bald spot» in the center, which does not contain slag, and therefore can't protect the central part of the surface from unwanted intense oxidation, should be referred to as negative. As a result, the quality of finished products is deteriorating. At present, this defect can be eliminated only in low-power vacuum induction furnaces.

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Experiment No. 2 – Study of the influence of the electromagnetic field on the trajectory and speed of movement of the molten metal (alloy) in an induction furnace.

Despite the fact that the melting point of the metal (alloy) is quite high 1000...1500 °C (depending on the grade), and the currents and movements that occur in the middle of the melt under the influence of the electromagnetic field and temperature are mostly chaotic, investigate their trajectories and speed of movement with high reliability is impossible. Therefore, a method of approximate visual approbation was used.

For this purpose, a special thermal float was made with a ceramic heat-resistant sleeve, into which a tungsten rod was inserted (Fig. 4).

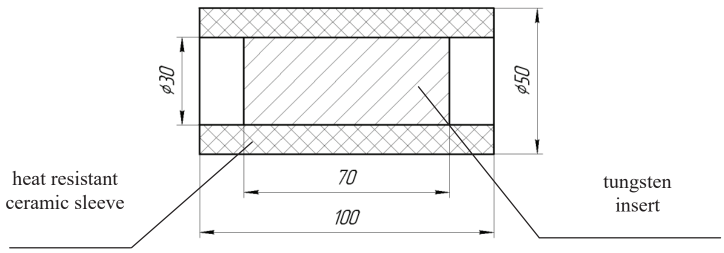

Fig. 4. «Float» structure

For greater accuracy of the experiment, the weight of the float is equal to the weight of the melt of the same volume. That is, in fact, the float is mainly affected by the action of electromagnetic fields, which penetrated the tungsten rod and forced it to move along its trajectory.

The influence of temperature fields can't be physically completely eliminated, but the errors from their action, especially at the final stage of melting, are not significant because of the actual uniformity of the alloy temperature. Therefore, the main attention is paid to the influence of the electromagnetic field of the melting process and obtaining opportunities to control the metallurgical process according to a given program in order to minimize electricity consumption.

During the experiment on an industrial induction furnace, the following were found:

– at the initial stage of the metallurgical process, the surface parts of the metal pieces are first melted. «Liquid» gradually «flows» to the bottom of the furnace, and solid particles under the action of the electromagnetic field begin to «float» in the formed liquid. This accelerates the further melting of the entire volume, which is loaded into the furnace;

– after complete melting at the stage of aging at a given temperature (deoxidation), the thermal melt was immersed in the melt, which periodically appeared on the surface and disappeared in depth. Fig. 5 shows the change in the thermal conditions of an induction steel-melting furnace at different periods and stages of melting.

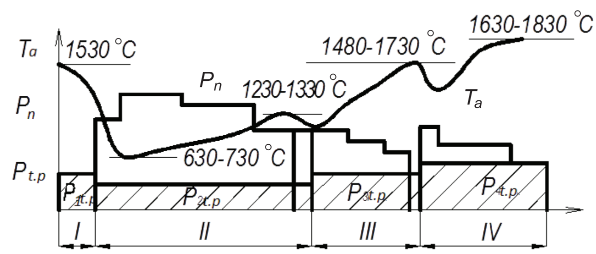

Fig. 5. Changes in the thermal conditions of an induction steel-melting furnace at different periods and stages of melting: I – furnace preparation; II – melting; III – oxidation; IV – recovery; T_a – the actual temperature, P_n – nominal power, $P_{t.p.}$ – power of the technological process for the corresponding period

Despite the randomness of the appearance on the surface and the sinking of the thermal float, it was possible to detect such a pattern: with a decrease in the frequency of the current $f<50$ Hz, it appeared on the surface less frequently and approached the geometric center of the induction furnace. On the contrary, with an increase in the frequency of the current $f>50$ Hz, it appeared more often, but at a considerable distance from the geometric center of the induction furnace.

According to the research results, it is possible to draw the following conclusion: the action of the electromagnetic field over the entire period of the metallurgical cycle is positive. In order to achieve maximum energy saving indicators, it should be controlled by the magnitude and frequency of the current; it creates it depending on the actual state of individual cycles of the metallurgical process.

7. SWOT analysis of research results

Strengths. Theoretical and practical laboratory studies of electro-thermo-mechanical processes in industrial induction furnaces have shown that there are reserves for improving energy efficiency that need to be used, combining mathematical modeling with practical operating experience.

The positive characteristics of induction furnaces should include the effect of continuous mixing of the melt under the action of an electromagnetic field, which makes it possible to obtain a high homogeneity of the structure of the alloy and reduce the time they work.

Weaknesses. To obtain significant results in the development of measures to save electricity in the process of melting metals (alloys) by induction it is necessary to focus on world achievements using computer technology.

Opportunities. The introduction of fundamentally new approaches to the design of induction furnaces and the development of modern energy-efficient technological processes using software packages similar to, for example, are recommended.

– VEM Antriebtechnik (Germany) for the selection of electrical components;
– OPAL (Switzerland) with complex modeling and identification procedures;
– CTSizing (Control Techniques, USA) for process control.

This will allow specialists, designers, technologists, metallurgists at the design stage to determine the technical and economic indicators of their work and provide for measures to improve energy efficiency.

Threats. Threats include:
– complexity of the design of the induction furnace;
– increase in the cost of designing and developing software.

8. Conclusions

1. Thanks to the experiments conducted in the laboratory and on the furnace, an algorithm is developed for the operation modes of the electric inductor. The analysis of the inductor operation modes at different frequencies is carried out. The influence of the frequency of the current supplying the inductor to the penetration depth of the electromagnetic field is revealed. With decreasing current frequency ($f<50$ Hz), the penetration depth increases and vice versa, with increasing ($f>50$ Hz) it decreases. It is also confirmed that the maximum effect of the electromagnetic field on the melt is concentrated inside (along the height) of the melt.

2. The distribution of the electromagnetic field inside the inductor and its effect on the elements of an induction furnace are shown. It forms electrodynamic forces directed inward. The operating conditions of the furnace with the indication of the actual temperature at various stages of melting (from 630 °C to 1830 °C) are revealed.

3. Special thermal float are made to study the movement of the molten metal. The float consists of a ceramic heat-resistant sleeve, into which a tungsten rod is inserted. For greater accuracy of the experiment, the float weight is equal to the weight of the melt of the same volume.

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