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NUMERICAL INVESTIGATION OF THE PROBLEM OF NONLINEAR THREE-PHASE FILTRATION

Об'єктом дослідження є чисельне моделювання процесу фільтрації нафти, газу та води на адаптивних сітках з урахуванням деяких властивостей рідин під час їх спільної течії. Для отримання адекватного опису процесів необхідно одночасно враховувати вплив більшості з зазначених факторів на фільтрацію. Це в математичному плані призводить до вирішення систем нелінійних диференціальних рівнянь в приватних похідних, складність яких не дозволяє досить глибоко дослідити їх аналітичними методами. Експериментальне ж вивчення цих процесів пов'язано з проведенням тривалих і дорогих лабораторно-промислових експериментів.

Одним з найбільш проблемних місць в теорії багатофазної фільтрації є те, що кроки по просторовій змінній повинні подрібнюватися в областях різкої зміни не тільки градієнта водонасиченості, але і градієнта газонасиченості. Це пояснюється тим, що через дуже малу в'язкість вільний газ під дією градієнта тиску обганяє інші компоненти суміші – воду та нафту.

В ході дослідження використовувалися алгоритм побудови адаптивних сіток, який може бути адаптований до властивостей рішення, а також методи обчислювальної математики, у тому числі разностно-ітераційний метод в рухомих сітках.

Були проведені чисельні експерименти для оцінювання впливу запропонованого методу на переміщення і розміри «нафтового валу» і дано порівняльний аналіз отриманих результатів на основі чисельних розрахунків.

Завдяки цьому показано, що при наближенні нафтового валу до експлуатаційної свердловини з пласта виходить тільки газ, і в міру зменшення в'язкості нафти зменшується час наближення нафтового валу до експлуатаційної свердловини. А також показано, що при збільшенні в'язкості нафти довжина l_v і «приріст» h_v нафтового валу зменшуються, причому зменшення l_v в порівнянні з h_v відбувається з більшою швидкістю. А при збільшенні швидкості фільтрації і різниці тисків геометричні розміри і «приріст» нафтового валу різко збільшуються.

Ключові слова: *капілярні сили, трифазна фільтрація, адаптивна сітка, закон Дарсі, метод прогонки, в'язкопластична рідина.*

1. Introduction

As is known [1], important practical problems of oil and gas production, hydraulic engineering, chemical technology, etc. are associated with the problem of multiphase filtration. This problem is the most urgent for the theory and practice of oil and gas production.

The tasks of multiphase filtering have certain specific features that do not make it possible to always use the finite difference method that has become a classic finite-difference method for their numerical solution. Therefore, there is a need to develop difference schemes on adaptive grids, taking into account the features of the solution [2, 3].

Adaptive grids reduce artificial viscosity and oscillation of a numerical solution. Even if the computational grid has minimal points, they again give satisfactory results in terms of quantity and quality for the entire region (even with the inclusion of zones with solution features).

Therefore, it is relevant to study the processes of joint filtration of oil, gas and water in a porous medium, taking into account:

– real geological field data (elastic properties and formation heterogeneity);
– properties of filtered liquids and gas (compressibility, dependence of viscosities on pressure);
– relative phase permeability and capillary forces.

2. The object of research and its technological audit

The object of research is the numerical simulation of the filtration process of oil, gas and water on adaptive grids, taking into account some properties of liquids during their joint flow.

The tasks of multiphase filtration have specific features that do not make it possible to always use the classical finite difference method in their numerical solution.

One of the most problematic places in the theory of multiphase filtration is that the steps of the spatial variable must be crushed in areas of a sharp change not only in the gradient of water saturation, but also in the gradient of gas saturation. This is explained by the fact that due to the very low viscosity, free gas under the action of the pressure gradient overtakes the other components of the mixture – water and oil. Therefore, there is a need to develop difference schemes on adaptive grids, which makes it possible to take into account the features of the solution.

3. The aim and objectives of research

The aim of research is development of efficient and sufficiently universal numerical methods for solving the problem of one-dimensional three-phase filtration.

To achieve this aim it is necessary to perform the following objectives:

1. Build a mathematical model of the process of one-dimensional three-phase filtration in a porous medium.
2. Build an adaptive grid, taking into account the grinding steps in areas of abrupt change not only the gradient of water saturation, but also the gradient of gas saturation.
3. Develop an algorithm for the difference-iterative method in moving grids.
4. On the basis of numerical experiments, discuss the issues of computational implementation of the proposed method and give practical recommendations for its application.

4. Research of existing solutions of the problem

When conducting hydrodynamic calculations associated with the conduct and operation of oil and gas fields in the water-pressure mode, the Rapoport-Lis two-phase model is mainly used. However, due to the fact that in many fields there is a three-phase flow, it is necessary to consider a model of three-phase flow and evaluate the effect of the gas phase on the displacement process. And this, in turn, leads to the need to develop a numerical method for solving three-phase filtration problems described by a system of nonlinear partial differential equations. To study this process, numerical methods are mainly used [4, 5].

In the works [6, 7], the Buckley – Leverett model is used, and in [4] the effects of capillary forces are also calculated. And for the numerical solution of the problems obtained, the idea of the method is used [8]. However, this method does not take into account the characteristics of multiphase filtration, and in some cases gives qualitatively and quantitatively incorrect results [4].

Among the main directions of solving this problem, identified in the resources of the world scientific periodicals, the works [9–11] can be highlighted. However, in [9], the extrusion is assumed to be piston, and approximate analytical solutions are obtained. A numerical method, ideologically different from the finite difference method and finite element method, is proposed in [10]. It should be noted that modifications of these works are widely used by many authors [11].

In this paper, an effective difference-iterative method in moving grids is proposed, which has the property of adaptability to the features of problems and is distinguished by high accuracy [2].

5. Methods of research

5.1. Mathematical formulation of the problem. Let's suppose a circular horizontal reservoir of radius R and thickness H is considered. It is assumed that the reservoir does not let in fluid from below and above, in the center of the reservoir there is a perfect hydrodynamic production well of radius $r=r_c$, on the contour $r=R$ there is a battery of wells pumping compressible fluid into the reservoir.

Let's suppose that in the reservoir in question there is a three-phase compressible fluid, the phases are immiscible and at the initial moment of time $t=0$ are in a state of capillary equilibrium. Based on the Rapoport-Lias model for liquids with reduced conditions, the isothermal plane-radial nonlinear filtering problem is described by a system of partial differential equations in a dimensionless form:

$$\begin{cases} r^{-1}\dfrac{\partial}{\partial r}\left[r\rho_1 n_1\lambda_1(r,p_1,s_1,s_2,s_3)\dfrac{\partial p_1}{\partial r}\right]=m\dfrac{\partial}{\partial t}(\rho_1,s_1),\\ r^{-1}\dfrac{\partial}{\partial r}\left[r\rho_2\lambda_2(r,p_2,s_1,s_2,s_3)\dfrac{\partial p_2}{\partial r}\right]=m\dfrac{\partial}{\partial t}(\rho_2,s_2),\\ r^{-1}\dfrac{\partial}{\partial r}\left[r\rho_3\lambda_3(r,p_3,s_1,s_2,s_3)\dfrac{\partial p_3}{\partial r}\right]=m\dfrac{\partial}{\partial t}(\rho_3,s_3),\end{cases}$$

$$(r,t)\in\Omega_T=\{r<r_c<R,\ 0<t\le T\}. \quad (1)$$

The following notation is used here: λ_α – phase mobility coefficient:

$$\lambda_\alpha(r,p_\alpha,s_1,s_2,s_3)=k(r)\frac{f_\alpha(s_1,s_2,s_3)}{\mu_\alpha(p_\alpha)};$$

$k(r)$ – permeability; r – polar coordinate; t – time.

Function:

$$\psi_1=\begin{cases}1-G_1/|gradP_1|, & |gradP_1|\ge G_1,\\ 0, & |gradP_1|<G_1,\end{cases}$$

takes into account the viscoplastic property of oil [1, 12]. Here G_1 – the initial pressure gradient.

To the system of equations (1) it is necessary to add the equations of state of liquids and gas:

$$\rho_\alpha=F_\alpha(p_\alpha),\ \alpha=1,2,3, \quad (2)$$

and ratios:

$$s_1+s_2+s_3=1. \quad (3)$$

Let's note that, in general, in problems of multiphase filtration, the pressure $p_{\alpha(\alpha=1,2,3)}$ is not equal to each other and differs by the value of the corresponding capillary pressure, i. e.:

$$p_1-p_2=p_{k1}(s_1,s_2,s_3),$$
$$p_3-p_1=p_{k2}(s_1,s_2,s_3), \quad (4)$$

where p_{k1} – the capillary pressure between the oil and water, and p_{k2} – the capillary pressure between the gas and the oil.

Knowing p_{k1} and p_{k2}, it is possible to use them to express the capillary pressure between gas and water:

$$p_3-p_2=p_{k1}+p_{k2}.$$

It should be noted that functions $f_\alpha(s_1,s_2,s_3)$, $(\alpha=1,2,3)$ have the following properties in the tasks of three-phase filtration [6, 7]:

$$f_1(s_1,s_2,s_3)\equiv 0,\ s_1\le s_1^*,$$
$$f_2(s_1,s_2,s_3)\equiv 0,\ s_2\le s_2^*,$$
$$f_3(s_1,s_2,s_3)\equiv 0,\ s_3\le s_3^*,$$

where values:

$$s_\alpha^*\ \left(\alpha=\overset{n}{1,3}\right)$$

are called residual saturations.

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Let's suppose that the functions unknown in system (1):

$$p_\alpha(r,t),\ \left(\alpha=\overset{n}{1,3}\right).$$

Let's define them from the initial and boundary conditions.

Let's suppose that at time $t=0$, i. e. before the start of operation, the values of the functions being defined are known, i. e.:

$$p_\alpha(r,0)=p_\alpha^0(r),\ r_c\,\mathrm{J}\,r\,\mathrm{J}\,R,\ \alpha=1,2,3. \tag{5}$$

For the problem under consideration, the boundary conditions can be in the following form.

For a production well (i. e., at $r=r_c$), it is assumed that only oil is extracted from the formation, and at this point the capillary pressure jump is not taken into account:

$$\begin{cases} p_1(r,t)=\varphi_1(t), \\ \dfrac{\partial p_1}{\partial r}=\dfrac{\partial p_2}{\partial r}, \\ \dfrac{\partial p_3}{\partial r}=\dfrac{\partial p_2}{\partial r},\ r=r_c,\ 0<t\le T. \end{cases} \tag{6}$$

On the external contour (i. e., at $r=R$), it is assumed that only compressible fluid (water) is pumped into the formation and, accordingly, the first and third phases do not flow:

$$\begin{cases} \lambda_{1,}n_1\dfrac{\partial p_1}{\partial r}=0, \\ p_2(r,t)=\varphi_2(t), \\ \lambda_3\dfrac{\partial p_3}{\partial r}=0,\ r=R,\ 0<t\le T. \end{cases} \tag{7}$$

Thus, according to conditions (5)–(7), the task can be set as follows. It is necessary to find such functions:

$$p_\alpha(r,t)\ (\alpha=1,2,3),$$

so that they would satisfy the system of equations (1) and the initial boundary conditions (5)–(7).

Assuming the existence and uniqueness of the solution to problem (1)–(7), let's give an approximate method for solving it.

5.2. Numerical method for solving the problem. First of all, let's note one specific feature of multiphase filtration tasks. It has been established that during the whole study, usually a sharp change in phase pressures occurs around the wells. Therefore, when modeling multiphase nonlinear filtering processes, the boundary conditions must be approximated with high accuracy, i. e., the grid step around wells must be reduced [2, 3].

Thus, let's cover the area:

$$\Omega=\{(r,t):\ r_c\le r\le R,\ 0\le t\le T\}$$

uneven grid $\overset{n}{\omega}_{hr}=\overset{n}{\omega}_h\, x\, \overset{n}{\omega}_\tau$ as follows.

The nodes of the irregular grid $\overset{n}{\omega}_{hr}=\overset{n}{\omega}_h\, x\, \overset{n}{\omega}_\tau$ by r are defined as follows:

$$\bar r_i=\bar r_{i-1}+h,\ i=\overline{1,M-1};\quad \bar r_0=r_c,\quad r_M=R,$$

$$\bar h_{i+1}=n_i,h_i,\quad i=1,2,\ldots,i_0-2,\ldots,i_0+\bar n;\quad \bar h_1=\bar r_0,$$

$$\bar h_{i_0+1}=\begin{cases} h_{i_0},\ \varepsilon_r\le 2\bar h_{i_0-1}, \\ L_1-\bar r_0-\sum_{k=0}^{i_0}h_k,\ \varepsilon_r>2\bar h_{i_0-1}; \end{cases}$$

$$\bar h_{i_0+2}=\frac{R-L_1}{n^*},\quad h_{i_0+\bar n+i+1}=h_{i_0+\bar n-i+2},\quad i=1,2,\ldots,i_0+n+1.$$

Here

$$n_i=\begin{cases} 1,\ i=1,2,\ldots,\bar n,i_0+2,\ldots,i_0+\bar n, \\ 2,\ i=\bar n+1,\bar n+2,\ldots,i_0-2; \end{cases}$$

$$i_0=\max\left\{i:\ \bar r_0+\sum_{k=1}^{i-1}h_k+6h_{i-2}\le L_1\right\},$$

where N – the number of partitions of the segment $[r_0,L_1]$; $\overset{n}{n}-\overset{n}{h}_1=\overset{n}{r}_0$ – the number of constant steps; n^* – the number of splits of the segment $[L_1,R]$.

For better convergence and accuracy of the results when solving the problem in question using an iterative-difference method, it is advisable to carry out calculations with an increment in time. Thus, the calculations begin with a very small step in t and gradually increase to a given T [2]. In accordance with this, the grid nodes $\overset{n}{\omega}_\tau$ are defined as follows:

$$t_n=t_{n-1}+\tau_n,\ n=1,\overset{n}{N}-1,\ t_0=0,\ t_N=T,$$

$$\tau_n=\begin{cases} l_t\cdot\tau_{n-1}, & 1\le n\le n_{k-1}, \\ \tau_{\max}, & n_k\le n\le N. \end{cases}$$

Here

$$l_t=\begin{cases} 1\ ,k_n\ne 0, \\ \varepsilon_\tau\ ,k_n=0, \end{cases}\quad k_n=\frac{n}{k_c}-\left[\frac{n}{k_c}\right],\ n=1,2,..,n_{k-1},$$

$$n_k=\max\{n:\ \varepsilon_r n\tau_n\le\tau_{max}\},$$

where $[n]$ – integer part; k_c – positive integer.

Thus, increasing the step in t occurs every k_c time layer. In the calculations, $k_c=100$ and $\tau_{in.}=4\cdot 10^{-8}$.

Using an implicit conservative scheme, let's approximate the differential problem (1)–(7) on a non-uniform grid $\overset{n}{\omega}_{h\tau}$ with the following discrete problem:

$$\begin{aligned} &\frac{1}{\hbar}\left[(r\rho_1\lambda_1 n_1)_{i+\frac12}\frac{p_{1,i+1}-p_{1,i}}{h_{i+1}}-(r\rho_1\lambda_1 n_1)_{i-\frac12}\frac{p_{1,i}-p_{1,i-1}}{h_i}\right]^{l+1}= \\ &=r_i m\left[f_{11}\frac{\partial p_1}{\partial t}+f_{12}\frac{\partial p_2}{\partial t}+f_{13}\frac{\partial p_3}{\partial t}\right]_i, \\ &\frac{1}{\hbar}\left[(r\rho_2\lambda_2)_{i+\frac12}\frac{p_{2,i+1}-p_{2,i}}{h_{i+1}}-(r\rho_2\lambda_2)_{i-\frac12}\frac{p_{2,i}-p_{2,i-1}}{h_i}\right]^{l+1}= \\ &=r_i m\left[f_{21}\frac{\partial p_1}{\partial t}+f_{22}\frac{\partial p_2}{\partial t}+f_{23}\frac{\partial p_3}{\partial t}\right]_i, \\ &\frac{1}{\hbar}\left[(r\rho_3\lambda_3)_{i+\frac12}\frac{p_{3,i+1}-p_{3,i}}{h_{i+1}}-(r\rho_3\lambda_3)_{i-\frac12}\frac{p_{3,i}-p_{3,i-1}}{h_i}\right]^{l+1}= \\ &=r_i m\left[f_{31}\frac{\partial p_1}{\partial t}+f_{32}\frac{\partial p_2}{\partial t}+f_{33}\frac{\partial p_3}{\partial t}\right]_i, \end{aligned} \tag{8}$$

$$p_\alpha(r_i,0)=p^0_{\alpha,i},\ n=0,\ 0\le i\le M,\ \alpha=1,2,3, \qquad (9)$$

$$\begin{cases} p^{n+1}_{1,i}=\varphi^{n+1}_1, \\ h_1^{-1}\left(p^{n+1}_{1,1}-p^{n+1}_{1,0}\right)=h_1^{-1}\left(p^{n+1}_{2,1}-p^{n+1}_{2,0}\right), \\ h_1^{-1}\left(p^{n+1}_{3,1}-p^{n+1}_{3,0}\right)=h_1^{-1}\left(p^{n+1}_{2,1}-p^{n+1}_{2,0}\right), \end{cases} \qquad (10)$$

$i=0,\ 0<n\mathrm{J}N,$

$$\begin{cases} h_M^{-1}\left(p^{n+1}_{1,M}-p^{n+1}_{1,M-1}\right)=0, \\ p^{n+1}_{2,M}=\varphi^{n+1}_2, \\ h_M^{-1}\left(p^{n+1}_{3,M}-p^{n+1}_{3,M-1}\right)=0. \end{cases} \qquad (11)$$

$i=M,\ 0<n\le N,$

Here

$$\hbar=\frac{1}{2}(h_i+h_{i+1}),\ (r\rho_\alpha\lambda_\alpha)_{i\pm1/2}=\frac{(r\rho_\alpha\lambda_\alpha)_i+(r\rho_\alpha\lambda_\alpha)_{i\pm1}}{2},$$

$$f_{11,i}=\left[\rho_1'(1-s_2-s_3)-\rho_1 s_2'+\rho_1 s_3'\right]_i,\ f_{21}=\rho_1 s_2',\ f_{13}=-\rho_1 s_2',$$

$$f_{21}=\rho_2 s_2',\ f_{21}=\rho_2' s_2-\rho_2 s_2',\ f_{21}=0,$$

$$f_{31}=-\rho_3 s_3',\ f_{32}=0,\ f_{32}=\rho_3' s_3-\rho_3 s_3',$$

$$\rho_\alpha'=\frac{d\rho_\alpha}{dp_\alpha},\ \alpha=1,2,3;\ s_2=s_2(p_{k1}),\ s_3=s_3(p_{k2}),$$

$$s_1=1-s_2(p_{k1})-s_3(p_{k2}),$$

$$s_2'=\frac{ds_2}{dp_{k1}},\ s_3'=\frac{ds_3}{dp_{k2}},\ s_1'=\frac{ds_2}{dp_{k1}}-\frac{ds_3}{dp_{k2}}.$$

By approximating the derivatives $\partial p_\alpha/\partial t$ on the right side of system (8) as follows:

$$\frac{\partial p_\alpha}{\partial t}=\frac{p^{n+1}_{\alpha,i}-p^n_{\alpha,i}}{\tau},\ \alpha=1,2,3,$$

and having carried out some elementary transformations, let's reduce (8) to the following form:

$$\hbar^{-1}h_i^{-1}(r\rho_1\lambda_1 n_1)^{l+1}_{i-1/2}\,p^{l+1}_{1,i-1}-\Big[\hbar^{-1}h_{i+1}^{-1}(r\rho_1\lambda_1 n_1)^{l+1}_{i+1/2}+$$
$$+\hbar^{-1}h_i^{-1}(r\rho_1\lambda_1 n_1)^{l+1}_{i-1/2}+r_i m\tau^{-1}f_{11,i}\Big]p^{l+1}_{1,i}+$$
$$+\hbar^{-1}h_{i+1}^{-1}(r\rho_1\lambda_1)^{l+1}_{i+1/2}\,p^{l+1}_{1,i+1}-r_i m\tau^{-1}f_{12,i}p^{l+1}_{2,i}+$$
$$+r_i m\tau^{-1}f_{13,i}p^{l+1}_{3,i}=-r_i m\tau^{-1}\left(f_{11,i}p^n_{1,i}+f_{12,i}p^n_{2,i}-f_{13,i}p^n_{3,i}\right),$$

$$\hbar^{-1}h_i^{-1}(r\rho_2\lambda_2)^{l+1}_{i-1/2}\,p^{l+1}_{2,i-1}-\hbar^{-1}\Big[h_{i+1}^{-1}(r\rho_2\lambda_2)^{l+1}_{i+1/2}+$$
$$+h_i^{-1}(r\rho_2\lambda_2)^{l+1}_{i-1/2}+r_i m\tau^{-1}f_{22,i}\Big]p^{l+1}_{2,i}+$$
$$+\hbar^{-1}h_{i+1}^{-1}(r\rho_2\lambda_2)^{l+1}_{i+1/2}\,p^{l+1}_{2,i\mp1}-r_i m\tau^{-1}f_{21,i}p^{l+1}_{1,i}p^{l+1}_{3,i}=$$
$$=-r_i m\tau^{-1}\left(f_{21,i}p^n_{1,i}+f_{22,i}p^n_{2,i}\right),$$

$$\hbar^{-1}h_i^{-1}(r\rho_3\lambda_3)^{l+1}_{i-1/2}\,p^{l+1}_{3,i-1}-\hbar^{-1}\Big[h_{i+1}^{-1}(r\rho_3\lambda_3)^{l+1}_{i+1/2}+$$
$$+h_i^{-1}(r\rho_3\lambda_3)^{l+1}_{i-1/2}+r_i m\tau^{-1}f_{33,i}\Big]p^{l+1}_{3,i}+$$
$$+\hbar^{-1}h_{i+1}^{-1}(r\rho_3\lambda_3)^{l+1}_{i+1/2}\,p^{l+1}_{3,i\mp1}-r_i m\tau^{-1}f_{31,i}p^{l+1}_{1,i}p^{l+1}_{3,i}=$$
$$=-r_i m\tau^{-1}\left(f_{31,i}p^n_{1,i}+f_{33,i}p^n_{3,i}\right). \qquad (12)$$

Thus, replacing $p^{n+1}_{\alpha,i}=\widehat{Y}_{\alpha,i},\ p^n_{\alpha,i}=Y_{\alpha,i},$ let's introduce the following matrices and vectors:

$$B_0=\begin{bmatrix}-1&0&0\\1&-1&0\\0&1&-1\end{bmatrix},\ C_0=\begin{bmatrix}0&0&0\\1&-1&0\\0&1&-1\end{bmatrix},\ \overset{n}{F}_0=\begin{bmatrix}-\widehat{\varphi}_1\\0\\0\end{bmatrix},$$

$$A=\begin{bmatrix}A_1&0&0\\0&A_2&0\\0&0&A_3\end{bmatrix},\ \overset{n}{Y}=\begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix},\ \overset{n}{F}_i=\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix},$$

$$C=\begin{bmatrix}C_1&0&0\\0&C_2&0\\0&0&C_3\end{bmatrix},\ B=\begin{bmatrix}b_1&0&0\\0&b_2&0\\0&0&b_3\end{bmatrix},$$

$$A_M=\begin{bmatrix}1&0&0\\0&0&0\\0&0&1\end{bmatrix},\ B_M=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\ \overset{n}{F}_0=\begin{bmatrix}0\\-\widehat{\varphi}_2\\0\end{bmatrix}.$$

System (12) is non-linear, the defined functions are included in the coefficients of the system. In order to linearize nonlinear coefficients, let's use the simple iteration method, i. e., at each new time point, these coefficients are considered known from the previous time point [13]:

$$T^{l+1}(r,p)\approx T^l(r,p).$$

Taking into account the above, the difference problem (12) can be written in the following three-diagonal canonical form:

$$\begin{cases} -\overset{n}{B}{}^{(l)}_0\,\widehat{Y}^{(l+1)}_0+\overset{n}{C}{}^{(l)}_0\,\overset{n}{Y}{}^{(l+1)}_1=-\overset{n}{F}{}^{(l)}_0,\ i=0, \\ \overset{n}{A}{}^{(l)}_i\,\widehat{Y}^{(l+1)}_{i-1}-\overset{n}{B}{}^{(l)}_i\,\overset{n}{Y}{}^{(l+1)}_i+\overset{n}{C}{}^{(l)}_i\,\overset{n}{Y}{}^{(l+1)}_{i+1}=-\overset{n}{F}{}^{(l)}_i,\ 1\mathrm{J}i\mathrm{J}M-1, \\ \overset{n}{A}{}^{(l)}_M\,\overset{n}{Y}{}^{(l+1)}_{M-1}-\overset{n}{B}{}^{(l)}_M\,\overset{n}{Y}{}^{(l+1)}_M=-\overset{n}{F}{}^{(l)}_M,\ i=M. \end{cases} \qquad (13)$$

The elements of the matrices A, B, and C, as well as the vector $\overset{n}{F}$, have the following form:

$$A_{1,i}=\hbar^{-1}h_i^{-1}(r\rho_1\lambda_1 n_1)^{l+1}_{i-1/2};$$

$$A_{2,i}=\hbar^{-1}h_i^{-1}(r\rho_2\lambda_2)^{l+1}_{i-1/2};$$

$$A_{3,i}=\hbar^{-1}h_i^{-1}(r\rho_3\lambda_3)^{l+1}_{i-1/2};$$

$$F^{(l)}_{1,i}=-r_i m\tau^{-1}\left(f_{11,i}p^n_{1,i}+f_{12,i}p^n_{2,i}-f_{13,i}p^n_{3,i}\right);$$

$$F^{(l)}_{2,i}=-r_i m\tau^{-1}\left(f_{21,i}p^n_{1,i}+f_{22,i}p^n_{2,i}\right);$$

$$F^{(l)}_{3,i}=-r_i m\tau^{-1}\left(-f_{31,i}p^n_{1,i}+f_{33,i}p^n_{3,i}\right);$$

$$C_{1,i}=\hbar^{-1}h_{i+1}^{-1}(r\rho_1\lambda_1 n_1)^{(l)}_{i+1/2};$$

$$C_{2,i}=\hbar^{-1}h_{i+1}^{-1}(r\rho_2\lambda_2)^{(l)}_{i+1/2};$$

$$C_{3,i}=\hbar^{-1}h_{i+1}^{-1}(r\rho_3\lambda_3)^{(l)}_{i+1/2};$$

$$b_{11,i}=\hbar^{-1}\begin{bmatrix}h_{i+1}^{-1}(r\rho_1\lambda_1 n_1)^{(l)}_{i+1/2}+\\+h_i^{-1}(r\rho_1\lambda_1 n_1)^{(l)}_{i-1/2}+r_i m\tau^{-1}f_{11,i}{}^{(l)}\end{bmatrix};$$

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$$b_{12,i} = r_i m \tau^{-1} f_{12,i};$$

$$b_{13,i} = -r_i m \tau^{-1} f_{32,i};$$

$$b_{21,i} = r_i m \tau^{-1} f_{21,i};$$

$$b_{22,i} = \hbar^{-1} \begin{bmatrix} h_{i+1}^{-1} \left(r \rho_2 \lambda_2 \right)_{i+1/2}^{l+1} + \\ + h_i^{-1} \left(r \rho_2 \lambda_2 \right)_{i-1/2}^{l+1} + r_i m \tau^{-1} f_{22,i} \end{bmatrix};$$

$$b_{23,i} = 0; \quad b_{31,i} = -r_i m \tau^{-1} f_{31,i};$$

$$b_{32,i} = \hbar^{-1} \begin{bmatrix} h_{i+1}^{-1} \left(r \rho_3 \lambda_3 \right)_{i+1/2}^{l+1} + \\ + h_i^{-1} \left(r \rho_3 \lambda_3 \right)_{i-1/2}^{l+1} + r_i m \tau^{-1} f_{33,i} \end{bmatrix};$$

$$b_{33,i} = 0.$$

Thus, specifying the initial approximations in the form of (9) in order to find the l+1-st approximation from the system (13), let's use the matrix version of the sweep method (Thomas algorithm) [13, 14]. At each moment of time, the iterative process is continued until such a value of l_0 that the conditions are:

$$\max_{0 \le i \le M} \left| Y_{1,i}^{(l_0+1)} - Y_{1,i}^{(l_0)} \right| \le \varepsilon_1,$$

$$\max_{0 \le i \le M} \left| Y_{2,i}^{(l_0+1)} - Y_{2,i}^{(l_0)} \right| \le \varepsilon_2,$$

$$\max_{0 \le i \le M} \left| Y_{3,i}^{(l_0+1)} - Y_{3,i}^{(l_0)} \right| \le \varepsilon_3,$$

would be satisfied at the same time. Here, ε_1, ε_2, ε_3 – the predetermined accuracy for finding the i-th component of the approximate solution.

6. Research results

The iterative-difference method proposed above was applied to the numerical simulation of one three-phase filtration process. The following initial data were used for the calculations:

$$R = 100 \text{ m}, \; H = 10 \text{ m}, \; r_c = 0.1 \text{ m}; \; m = 0.2; \; k = 10^{-12} \text{ m}^2;$$

$$\mu_1 = 0.3 \text{ poise}; \; \mu_2 = 0.01 \text{ poise}; \; \mu_3 = 0.0013 \text{ poise};$$

$$\varphi_1(t) = 139.5 \text{ atm}; \; \varphi_2(t) = 140 \text{ atm}; \; p_2^0(r) = 140 \text{ atm};$$

$$S_2^0(r) = 0.25; \; S_1^0(r) = 0.45; \; S_3^0(r) = 0.3; \; G_1 = 100 \text{ dyne/cm}^3;$$

$$R_0 = 100 \text{ m}; \; k_0 = 10^{-12} \text{ m}^2; \; \mu_0 = 0.01 \text{ poise}; \; T_0 = 10^8 \text{ s};$$

$$p_0 = 140 \text{ atm}; \; \rho_0 = 1 \text{ g/cm}^3.$$

Here R_0, k_0, μ_0, T_0, p_0, ρ_0 – the characteristic measurement values used to bring the original problem to a dimensionless form. The equations of state of the phases are given in the following form [15]:

$$\rho_1(p_1) = 0.000853 p_1 + 0.82592,$$

$$\rho_2(p_2) = 0.01035 p_2 + 0.99989,$$

$$\rho_3(p_3) = 0.063 p_3.$$

Prior to the beginning of the process, the phase saturation was $S_1 = 0.45$, $S_2 = 0.25$, $S_3 = 0.30$, respectively. The dependences of relative permeabilities and capillary forces on saturations were taken following [15].

The conducted numerical experiments show that at the beginning of the impact process, an enlarged zone of oil saturation begins to form in the reservoir – an oil shaft, which dimensions increase and which, displacing the gas, moves towards the production well. The displacement of oil occurs almost after the oil shaft. Under the «width» and «increase» of «oil shaft» profile refers to the following values:

$$l_v = mes\left\{ r: \; r \in [r_c, R], \; S_1(r) > S_1^0 \right\},$$

$$h_v = \max S_1(r) - S_1^0,$$

$$r \in [r_c, R].$$

Fig. 1 shows the results of calculations at $\varphi_2 - \varphi_1 = 0.5$ kgf/cm^2 in various times for different values of oil viscosity using an adaptive grid. Here, the solid line shows the graphs reflecting the state of the oil shaft at different points in time, and the dashed lines indicate graphs showing water saturation.

It should be noted the following feature of the process. When the oil shaft approaches the production well, only gas escapes from the reservoir. Fig. 1 shows that as the viscosity of the oil decreases, the time for the oil shaft to approach the production well decreases.

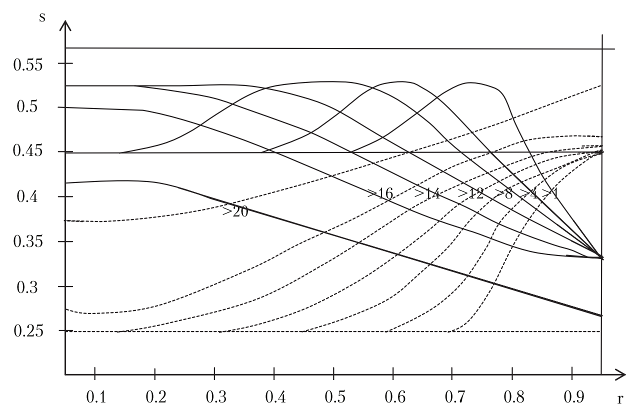

Fig. 1. The state of the oil shaft at different points in time

Fig. 2 shows the effect of oil viscosity on the size and «growth» of the oil shaft. Here, the solid line shows the graph of oil saturation and the dashed line graph of gas saturation at time t=3 months ($\varphi_2 - \varphi_1 = 0.7$ kgf/cm^2).

In Fig. 2, the initial distribution of oil and gas saturation is shown by dots. Curves marked with numbers 1, 2, 3, 4, 5, 6 are the results of the calculations, respectively, at $\mu_1 = 0.01$; 0.02; 0.03; 0.1; 0.2; 0.4 poise.

As can be seen from Fig. 2, with an increase in oil viscosity, the length l_v and the «increase» h_v of the oil shaft decrease, and the decrease in l_v compared to h_v occurs at a higher rate. For example, from Table 1 that when increasing the value of μ_1 by 2, 10 and 40 times, l_v decreases by about 41 %, 73 %, 86 %, and h_v, respectively, decreases by 5 %, 36 %, 65 %.

Thus, based on the above, in the presence of a free gas phase, the geometrical dimensions of the oil shaft characterize the completeness of the oil displacement, so that an increase in the shaft dimensions is the result of the oil being displaced by water.

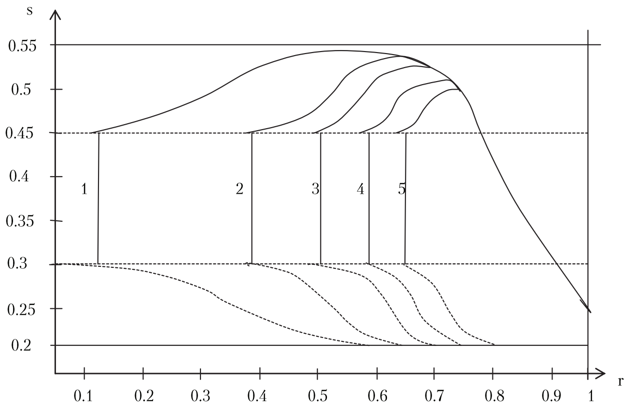

Fig. 2. The effect of oil viscosity on the size and «growth» of the oil shaft

Table 1

The effect of oil viscosity on the size of the oil shaft

t=5 month; φ_2–φ_1=0.7 kgf/cm^2, k=2·10^{-8} cm^2, S_3^* = 0.2						
μ_1, poise	0.01	0.02	0.03	0.1	0.2	0.4
l_v(m)	60.01	33.02	25.29	14.49	11.21	9.40
h_v	0.0079	0.0739	0.0701	0.0501	0.0427	0.0269

Let's consider the effect of the displacement rate on the process of three-phase displacement (Table 2).

Table 2

The effect of displacement rate on the filtering process

$n\varphi$	0.5 kgf/cm^2			1 kgf/cm^2		
r	S_1	S_2	S_3	S_1	S_2	S_3
0.1	0.4500	0.2500	0.3000	0.4511	0.2500	0.2980
0.2	0.4500	0.2500	0.3000	0.4750	0.2503	0.2747
0.3	0.4500	0.2500	0.3000	0.5446	0.2506	0.2048
0.4	0.4525	0.2502	0.2974	0.5446	0.2506	0.2048
0.5	0.5199	0.2505	0.2296	0.5320	0.2632	0.2048
0.6	0.5370	0.2560	0.2070	0.3936	0.4016	0.2048
0.7	0.4329	0.3624	0.2048	0.3197	0.4755	0.2048
0.8	0.3617	0.4336	0.2048	0.2579	0.5373	0.2048
0.9	0.3139	0.4813	0.2048	0.2150	0.5802	0.2048
1.0	0.2726	0.5226	0.2048	0.1804	0.6148	0.2048

The results of calculations show that with an increase in the filtration rate, the geometric dimensions and the «increment» of the oil shaft increase dramatically. As can be seen from the Table 2, with an increase in the pressure difference to 0.4 kgf/cm^2, the shaft length increases by 35.4 %, and the increase by 9.6 %.

7. SWOT analysis of research results

Strengths. The proposed algorithm can be used to develop the definition of the optimal mode and time of the processes of thermal conductivity in inhomogeneous media.

Weaknesses. The weaknesses of research include the complexity of the calculation.

Opportunities. Using the obtained research, it is possible to increase the oil recovery of the field.

Threats. To implement the research it is necessary to purchase additional equipment, which means cash costs.

8. Conclusions

1. A mathematical model of a one-dimensional three-phase filtration process in a porous medium is constructed, which more adequately describes the process under consideration. A feature of this model is that it takes into account many of the above properties of filtered liquids and gas, for example, compressibility, viscoplasticity, the dependence of viscosity on pressure, relative phase permeability and capillary forces (the Rapoport-Lias model).

2. An algorithm for constructing an adaptive grid is given, which takes into account the refinement of steps in areas of abrupt change in the solution, and which makes it possible to more accurately approximate the derivatives of the desired functions.

3. The algorithm of the difference-iterative method in moving grids is developed, which takes into account the features of the solution and produces automatic thickening in the areas of abrupt solution change.

4. On the basis of numerical experiments, it is established that at the beginning of the impact process, an oil shaft begins to form in the reservoir, the dimensions of which increase and which, displacing the gas, moves towards the production well. A displacement of oil occurs almost after the oil shaft.

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