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MATHEMATICAL FOUNDATIONS OF NAVIGATION PARAMETERS OF THE AIR TRANSPORT BY MAGNETOMINAL CIRCULATION BY USING THE METHOD OF LEAST SQUARES

Сучасні процеси управління повітряним рухом реалізуються на єдиній інформаційній платформі – глобально-локальних інформаційних кластерах, що об'єднані в єдине інформаційне середовище. Головною інформаційною складовою для такої системи є навігаційна інформація про місцеположення кожного повітряного судна, яка містить динамічно оновлюванні дані про його поточні координати та параметри руху. За таких умов постає задача комплексування навігаційної інформації, отриманої від різних інформаційних джерел, що складають авіоніку повітряного судна. Традиційні підходи не забезпечують сумісне згладжування координатної інформації інваріантно до синхронізації вимірів, тому об'єктом досліджень визначено статистичний аналіз експериментальних вибірок навігаційних параметрів.

У роботі розглядається математичний апарат для забезпечення багатовимірного згладжування навігаційних параметрів повітряного судна з використанням методу найменших квадратів. Для реалізації запропонованого підходу використано метод афінного відображення координатного простору. Це забезпечує отримання незалежних функціональних залежностей координатного поля літака для їх сумісної обробки методом найменших квадратів.

Розроблене математичне забезпечення багатомірного згладжування навігаційних параметрів повітряного судна може бути використане для дослідження процесів еволюції та супроводження програмного забезпечення бортового обчислювального комплексу повітряного судна. Особливістю підходу є часове узгодження отримуваних вимірів на рівні моделей, що знижує вимоги до синхронності первинних навігаційних параметрів та розширює можливості до переліку параметрів, що обробляються сумісно. Зазначене забезпечує підвищення точності визначення поточного положення літака порівняно з традиційними підходами за рахунок залучення додаткової надмірності даних.

Запропонований підхід може бути розповсюджений на широке коло прикладних галузей, пов'язаних з вимірюванням, оцінюванням і прогнозуванням зміни у часі взаємозалежних процесів.

Ключові слова: *процеси управління повітряним рухом, авіоніка повітряного судна, навігаційна інформація, метод найменших квадратів.*

1. Introduction

The development of information technologies in a decisive way forms the development trend of the aviation industry. Modern processes of air traffic control are realized on a single information platform, named globally-localized information clusters, united in a single information environment. The elements and participants of such a process (subjects and objects of management) include:

– land-based systems and air control systems;
– land-based air traffic control complexes – land-based cluster;
– air cluster, which includes aircrafts of different types and destination.

An air cluster should be considered at the local level of an individual aircraft. Then the local information environment of the aircraft should be regarded as being formed by systems and avionics complexes.

Modern avionics systems are based on the principle of integrated systems, which is a local element of a distributed information system that functions on the general information environment of the air traffic control system. The main information component for such a system is navigational information about the location of each aircraft, which contains dynamically updated data on its current coordinates and traffic parameters. Traditionally, in the avionics of each aircraft there are three classes of navigation systems: autonomous navigation systems; stand-alone radio-navigation systems; non-autonomous radionavigation systems [1]. Each of these three classes can potentially be represented by up to four types of discrete radio navigation devices. Such presence of navigation systems and devices is stipulated by strict requirements to the safety of aircraft flights, which is achieved by elementary reservation and dubbing sources of trajectory information. At the same time, the resulting excessive data can be applied to improve the accuracy of determining the navigational parameters of the aircraft when they are jointly processed.

Implementation of such an idea requires the creation of a unified information environment and the development of mathematical methods for integrated (multidimensional) processing of navigation information from a set of independent information sources – navigation systems of different classes and types located on board a specific type of aircraft.

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From a practical point of view, such an approach requires the application of the evolution of the software of the airborne computing complex with the introduction of network technologies for collecting, storing and processing navigation information [2].

The relevance of the research is in development of a mathematical background for multidimensional smoothing of navigational parameters of an aircraft for the implementation of software evolution of the onboard aircraft computer complex.

2. The object of research and its technological audit

The object of research is a statistical analysis of experimental sampling of aircraft navigational parameters. These parameters characterize the aircraft path trajectory taking into account the parametric and temporal excesses to improve the accuracy of determining the navigational parameters of the aircraft using the least squares method.

The subject of research is the process of processing the accumulated sampling of aircraft navigational parameters. To do this, it is necessary to determine their functional relationship, as the quantities characterizing a single object of a particular aircraft.

One of the most problematic places is the realization of the process of obtaining functional interfaces of the coordinates of aircraft with the preservation of the parameters and properties of the path trajectory of a dynamic object, regardless of the transformation of the coordinate space.

3. The aim and objectives of research

The aim of the research is to develop a mathematical background for multidimensional smoothing of aircraft navigational parameters using the least squares method.

Achievement of research goal is provided by solving interrelated partial scientific problems:

1. Mathematical formulation of the task of multidimensional smoothing of navigational parameters.
2. Solution of the formulated problem by the method of least squares using the approach of affine mapping of the coordinate space.

4. Research of existing solutions of the problem

Multivariate smoothing of navigation parameters belongs to the class of problems of statistical processing of experimental data. This is realized by finding the parameters of the a priori given model, consistent with the stochastic sampling of measurements [3, 4]. At present, all known approaches to constructing mathematical models based on experimental data are based on the maximum likelihood method and its derivatives in the form of recursive algorithms, or smoothing of the accumulated sample.

It is known solution of the problems of multidimensional filtration of stochastic processes using recurrent filters [3, 5]. However, recurrent evaluation procedures are usually based on simplified models of investigated processes, low accuracy of estimation and adequate forecast data for 3–4 dimensions of the investigated process. Recurrence of filtration processes is also characterized by the phenomenon of «filter difference», but the popularity of their application is justified by the real time obtaining estimates of the studied dynamic processes.

According to disadvantages of recurrent approaches for certain tasks of a posteriori evaluation, preference is given to smoothing algorithms based on the accumulated sample. In this sense, the problem of multidimensional smoothing of the accumulated sample using method of least squares remains unresolved, which provides high-precision estimation of the aircraft navigational parameters in the integrated avionics.

Evidence of this is a series of periodicals.

Thus in [6] the solution of the problem of multidimensional recurrent filtration of navigational parameters of a ship is proposed. The authors give a solution to the problem of the non-synchronous measurements of the joint evaluation. At the same time, such an approach to the implementation of multidimensional processing of the accumulated sample is accumulated

In [5], a multimodal Kalman filter was used to smooth navigation parameters of aerodynamic targets using nonlinear models. Multidimensionality is achieved by using block-diagonal matrices in matrix form of a filter. This does not provide an increase in the interval accuracy of the resulting estimation of navigational parameters, due to the failure to take into account the correlation between them.

In [7] it is proposed to implement navigation of an unmanned aircraft using intelligent technologies for constructing an integrated trajectory based on data from several navigation sources. The methods of recurrent filtration are selected based on the procedure for evaluating primary navigation parameters. Data integration is implemented as a decisive algorithm for the neural network. At the same time, statistical dependencies and asynchronous measurements were not taken into account.

In [8] a multidimensional recurrent filtration of three navigation parameters was implemented. Measurements are taken synchronously, without taking into account the correlation between them, which is reflected in the formation of the diagonal correlation error matrix. The question of asynchronous a posteriori smoothing of correlated navigation parameters for the non-defined composition of primary sources remains unresolved.

In [9], it was proposed to use a three-level scalar Kalman filter to estimate the parameters of the motion of aerodynamic objects. The three-level filter structure does not provide multidimensional smoothing, the more such a solution is not suitable for a posterior smoothing.

In [10], the concept of integrated statistical operational analysis of aircraft trajectories is discussed. Statistical processing is based on recurrent expanded algorithms. Such an approach does not solve the problem of multidimensional a posteriori assessment of aircraft navigational parameters.

In [11] the task of multidimensional estimation of statistical error of processing of experimental data is solved. The given results allow estimate accuracy of multidimensional approaches of complex navigation parameters. However, the linear models used in this case do not allow the results obtained for accumulated samples to be significantly affected by the nonlinearities of the propulsion system.

In [11] the method of multidimensional interpolation based on method of least squares is proposed. In essence, the approach reflects the block-diagonal expansion of least squares method matrices, followed by the scalar replacement of the algorithm for calculating the estimates. The disadvantages of the approach are: loss of the invariance of the algorithm to the type of model of the investigated process; failure to take into account the correlation dependence of the measured parameters; linearization of models of change of investigated processes.

Finally, in [12] it is considered the possibility of implementing a multidimensional statistical analysis. The basic method of evaluation is least squares method. The unity of the model of the investigated process for the formation of basic matrices is realized through the mechanisms of spatial transformations. However, the procedure for the formation of least squares method matrices is not specified in the work, the specificity of combining the non-synchronous navigation parameters of the PS and the nonlinear nature of the models of its motion are not taken into account.

Consequently, the well-known approaches to multidimensional smoothing of the aircraft navigational parameters in the integrated avionics are not provided: accumulated multidimensional smoothing of asynchronous measurements; taking into account the correlation of navigation parameters for increasing the interval accuracy of the assessment of the location of the aircraft; the use of nonlinear parameters of models that adequately describe the aircraft path.

5. Methods of research

The development of a mathematical apparatus to provide multidimensional smoothing of navigational parameters of an aircraft is carried out using the least squares method. To implement the proposed approach, the method of affine mapping of coordinate space is used. This provides the obtaining of independent functional dependencies of the coordinate field of the aircraft for their joint processing by the least squares method.

6. Research results

Let's suppose aircraft navigation parameters are received from a hypothetical navigation system on a certain flight segment represent a sampling in the topocentric polar (point, radar) coordinate system (RCS):

$$\begin{aligned} &r=\{r_1,r_2,r_3,...,r_i,...,r_n\},\\ &\varepsilon=\{\varepsilon_1,\varepsilon_2,\varepsilon_3,...,\varepsilon_i,...,\varepsilon_n\},\\ &\beta=\{\beta_1,\beta_2,\beta_3,...,\beta_i,...,\beta_n\},\\ &\dot r=\{\dot r_1,\dot r_2,\dot r_3,...,\dot r_i,...,\dot r_n\},\\ &\ddot r=\{\ddot r_1,\ddot r_2,\ddot r_3,...,\ddot r_i,...,\ddot r_n\}, \end{aligned} \quad (1)$$

where r, ε, β are sample measurements, r_i, ε_i, β_i are radial distance, azimuth and angle of aircraft location in space with their change in discrete moments of time $i=1...n;$ $\dot r$, $\ddot r$ are ample measurements, $\dot r_i$, $\ddot r_i$ are speed and acceleration along the radial distance, respectively.

The measurement samples (1) are parametric (by the type of measured coordinates and rates of their change), as well as the time (on time parameter) redundancy of the output data.

It is necessary to realize the common (multidimensional) smoothing of the samplers of aircraft navigation parameters (1), which characterize the path trajectory, taking into account parametric and temporal excesses using method of least squares.

It is possible to simplify the change of coordinates of aircraft in space in the RCS by the form of polynomial models:

$$\begin{aligned} &r(t)=r_0+r_1t+r_2t^2+...,\\ &\varepsilon(t)=\varepsilon_0+\varepsilon_1t+\varepsilon_2t^2+...,\\ &\beta(t)=\beta_0+\beta_1t+\beta_2t^2+...,\\ &\dot r(t)=\frac{dr(t)}{dt}=r_1+2r_2t+...,\\ &\dot\varepsilon(t)=\frac{d\varepsilon(t)}{dt}=\varepsilon_1+2\varepsilon_2t+...,\\ &\dot\beta(t)=\frac{d\beta(t)}{dt}=\beta_1+2\beta_2t+..., \end{aligned} \quad (2)$$

where r_0, ε_0, β_0, r_1, ε_1, β_1, r_2, ε_2, β_2, ... are coordinate, speed of change and acceleration according to the parameters of aircraft path trajectory.

In order to realize the process of common smoothing of the measured parameters of aircraft traffic using the method of least squares, it is necessary to determine their functional relationship as the values characterizing a single object of a particular aircraft. The realization of the process of obtaining functional interrelations of the coordinates of aircraft is proposed using the properties of affine mappings in the part concerning the preservation of the parameters and properties of the traffic trajectory of a dynamic object, regardless of the transformation of the coordinate space [14, 15]. Thus, the interconnection of the parameters (1) and models (2) can be found by converting them from the RCS to the geocentric absolute coordinate system (GACS) according to the sequence shown in the diagram (Fig. 1).

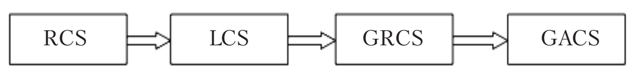

Fig. 1. Transformation scheme of RSC in GKCS: RCS is radar coordinate system; LCS is local (topocentric Cartesian) coordinate system; GRCS is geocentric relative coordinate system; GACS is geocentric absolute coordinate system

Operations of Fig. 1 are provided by the formation and use of transformation matrices, which in an analytical form are characterized by expression [16]:

$$\vec B_{CACS}=F_{CACS}\left[F_{GRCS}\left[F_{LCS}\vec A_{RCS}\right]\right], \quad (3)$$

where $\vec A_{RCS}$ is a vector of the instantaneous coordinates of aircraft and their rates of change (the base six parameters) in the RCS; $\vec B_{CACS}$ is a vector of coordinates of aircraft and speeds of their changes in GACS; F_{CACS}, F_{GRCS}, F_{LCS} are matrices of transformation of corresponding coordinates, the form of which is known and widely used [14].

Expression (3) is also true for the transformation of models (2). Then, as a result of the application of the transformation (3) to the models (2), it is obtained an unambiguous relationship between the measured parameters in the type of the GACS:

$$\begin{aligned}
&x(t)=f_x\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right),\\
&y(t)=f_y\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right),\\
&z(t)=f_z\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right),\\
&\dot{x}(t)=f_{\dot{x}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right),\\
&\dot{y}(t)=f_{\dot{y}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right),\\
&\dot{z}(t)=f_{\dot{z}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,...\right).
\end{aligned} \tag{4}$$

The dependencies thus obtained (4) have a complex nonlinear character with more transcendental transpose operations. To simplify their subsequent analysis and use, apply differential-Taylor transformation (DTT) to them, which will ensure that the polynomial form is obtained without loss of the main one, for the property of the problem under consideration, the functional dependence of the measured coordinates.

Differential transformation [17] is an operational method based on the translation of originals into an image area using a differentiation operation. Differential transformations in the general case are functional transformations of the form:

$$Z(K)=P\left\{z(t)\right\}_i=\frac{H^K}{K!}\left[\frac{d^K z(t)}{dt^K}\right]_i, \tag{5}$$

$$z(t)=f(t,c), \tag{6}$$

where t^{*n} is an argument value, which is used for transformation; $Z(K)$ is discrete function with real arguments $K=0,1,2,...$; H is segment of the argument, on which the function is considered $z(t)$, $f(t,c)$, is restoring, or approximating function; c is a set of free coefficients c.

Expression (5) defines a direct transformation that allows for the original $z(t)$ to find mapping $Z(K)$. Inverse conversion that restores the original $z(t)$ in the form of an approximating function, is determined by the expression (6). Differential mapping $Z(K)$ is called differential spectrum (DS), and values of function $Z(K)$ with the corresponding values of argument K are called discretes of the differential spectrum.

In the simplest case, the function, which restores $f(t,c)$, has the form of a polynomial and the task of restoring the original reduces to discretes summing in the form of a segment of the Taylor series. Differential transformations in this case are called differential-Taylor [17].

Applying DTT (5), (6) to transcendental dependencies (4), it is obtained polynomial forms of the form:

$$\begin{aligned}
&x(t)=K_{0x}+K_{1x}t+K_{2x}t^2+...,\\
&y(t)=K_{0y}+K_{1y}t+K_{2y}t^2+...,\\
&z(t)=K_{0z}+K_{1z}t+K_{2z}t^2+...,\\
&\dot{x}(t)=K_{0\dot{x}}+K_{1\dot{x}}t+K_{2\dot{x}}t^2+...,\\
&\dot{y}(t)=K_{0\dot{y}}+K_{1\dot{y}}t+K_{2\dot{y}}t^2+...,\\
&\dot{z}(t)=K_{0\dot{z}}+K_{1\dot{z}}t+K_{2\dot{z}}t^2+....
\end{aligned} \tag{7}$$

In expressions (7), the coefficients of polynomials (discretes of models (4)) characterize the instantaneous value of the coordinate, the speed of their change and acceleration, respectively, for each coordinate, which characterizes the location of aircraft in GACS. However, the obtained polynomial coefficients have a unique functional dependence on the measured coordinates of aircraft in RCS, namely:

$$\begin{aligned}
&K_{0x}=f_{0x}\left(r_0,\varepsilon_0,\beta_0\right),\\
&K_{1x}=f_{1x}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1\right),\\
&K_{2x}=f_{2x}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,r_2,\varepsilon_2,\beta_2\right),\\
&K_{0\dot{x}}=f_{0\dot{x}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1\right),\\
&K_{1\dot{x}}=f_{1\dot{x}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1\right),\\
&K_{2\dot{x}}=f_{2\dot{x}}\left(r_0,\varepsilon_0,\beta_0,r_1,\varepsilon_1,\beta_1,r_2,\varepsilon_2,\beta_2\right).
\end{aligned} \tag{8}$$

A similar form of dependence of the measured coordinates in the RCS has the coefficients of the polynomials (7) for the coordinates of aircraft in GACS and their derivatives according to $y(t)$, $z(t)$, $\dot{y}(t)$, $\dot{z}(t)$. The question of the dependence formation of aircraft coordinates of GACS from the measured coordinates, in addition to the mapping properties (3), is determined by the form of representation of the initial data (instantaneous coordinate values (1) or analogical models (2)) and determines the order of polynomial forms (7), as well as dependencies (8). In the future, this circumstance should be taken into account when forming the matrices of least squares method for the implementation of common smoothing of measured experimental samples.

Thus, in obtaining the analytic dependencies of the form (7), it is necessary to take into account the set of output (measured) data (1) for the formation of models (2) and the transformations (3)–(8) for the purpose of joint smoothing of coordinates.

In essence, the task of general smoothing of the measured parameters was reduced to least squares method approximation with multiple and (or) triple nodes [18]. It is possible to demonstrate this when forming the matrixes of the least squares method in the example of triple nodes in this way. In general, the algorithm of least squares method with triple nodes can be represented in the matrix form [4]:

$$\begin{aligned}
&B=Q^{1/2}\Phi,\ \hat{R}=B^T RB,\\
&\hat{\bar{K}}=\hat{R}^{n-1}B^T\left(Q^{1/2}\Omega\bar{K}\right),\ \hat{\bar{X}}=F\hat{\bar{K}}.
\end{aligned} \tag{9}$$

The matrices of least squares method algorithm in the form (9) have the following names and dimensions for experimental samples of measurements and polynomials of the m-th degree, taking into account the transformations of the initial parameters (1) or in the form (2) to the form (7):

– $\hat{\bar{K}}[m\times 1]$ – vector of coefficients of smoothing polynomial type (7);
– $F[n\times m]$ – matrix of values of basic functions for a coordinate;
– $\hat{\bar{X}}[n\times 1]$ – vector of smoothing (estimations) of the parameters of aircraft trajectory in GACS;
– $\bar{K}[3n\times 1]$ – vector of coordinates and its derivatives in GACS;
– $\Phi[3n\times m]$ – matrix of values of basic functions for a coordinate and its derivatives;
– $Q[3n\times 3n]$ – weight matrix for the coordinate and its derivatives;
– $R[m\times m]$ – correlation error matrix (CEM) for determining the coordinates of aircraft in GACS associated with the measured parameters in RCS in the form (8);
– $\hat{R}[m\times m]$ – correlation matrix of estimation errors of aircraft coordinates in GACS;

– $\Omega[3n\times 3n]$ – correction matrix;
– $B[3n\times m]$ – matrix of weighted values of the basis functions of the coordinate and its derivatives.

The indicated matrices in the general form taking into account transformations of the initial data (1), (2) to the form (7) are formed as follows:

$$\bar{K}=\begin{pmatrix} K_{0X_1}\ K_{1X_1}\ K_{2X_1}\ K_{0X_2}\ K_{1X_2}\ K_{2X_2}\ \ldots \\ \ldots K_{0X_i}\ K_{1X_i}\ K_{2X_i}\ \ldots K_{0X_n}\ K_{1X_n}\ K_{2X_n} \end{pmatrix}^{\mathrm{T}},$$

$$\hat{\bar{K}}=\left(\hat{K}_0\ \hat{K}_1\ \hat{K}_2 \ldots \hat{K}_m\right)^T, \hat{\bar{X}}=\left(\hat{X}_1\ \hat{X}_2 \ldots \hat{X}_n\right)^{\mathrm{T}},$$

$$R=\begin{pmatrix} \sigma^2_{K_0} & \rho_{K_0K_1} & \ldots & \rho_{K_0K_m} \\ \rho_{K_1K_0} & \sigma^2_{K_1} & \ldots & \rho_{K_1K_m} \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{K_mK_0} & \rho_{K_mK_1} & \ldots & \sigma^2_{K_m} \end{pmatrix},$$

$$\hat{R}=\begin{pmatrix} \sigma^2_{\hat{K}_0} & \rho_{\hat{K}_0\hat{K}_1} & \ldots & \rho_{\hat{K}_0\hat{K}_m} \\ \rho_{\hat{K}_1\hat{K}_0} & \sigma^2_{\hat{K}_1} & \ldots & \rho_{\hat{K}_1\hat{K}_m} \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{\hat{K}_m\hat{K}_0} & \rho_{\hat{K}_m\hat{K}_1} & \ldots & \sigma^2_{\hat{K}_m} \end{pmatrix}, \quad (10)$$

$$F=\begin{pmatrix} \phi_0(x_1) & \phi_1(x_1) & \ldots & \phi_m(x_1) \\ \phi_0(x_2) & \phi_1(x_2) & \ldots & \phi_m(x_2) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_0(x_n) & \phi_1(x_n) & \ldots & \phi_m(x_n) \end{pmatrix},$$

$$\Phi=\begin{pmatrix} \phi_1(x_1) & \phi_2(x_1) & \ldots & \phi_m(x_1) \\ \phi'_1(x_1) & \phi'_2(x_1) & \ldots & \phi'_m(x_1) \\ \phi''_1(x_1) & \phi''_2(x_1) & \ldots & \phi''_m(x_1) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_1(x_1) & \phi_2(x_1) & \ldots & \phi_m(x_1) \\ \phi'_1(x_1) & \phi'_2(x_1) & \ldots & \phi_m(x_1) \\ \phi''_1(x_1) & \phi''_2(x_1) & \ldots & \phi''_m(x_1) \end{pmatrix}, \quad (11)$$

$$Q=\begin{pmatrix} q_1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & q'_1 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & q''_1 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & q_n & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & q'_n & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & q''_n \end{pmatrix},$$

$$\Omega=\begin{pmatrix} 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & \Omega_1 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \Omega_1^2 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & \Omega_n & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & \Omega_n^2 \end{pmatrix}. \quad (12)$$

In (10) denotations $q_i=1/\sigma^2_{K_{0X_i}}$, $q_i=1/\sigma^2_{K_{0X_i}}$, $q_i=1/\sigma^2_{K_{0X_i}}$ characterize the quantities inversely proportional to the dispersion of the measurement of the corresponding coordinates and its derivatives, and the parameter Ω_i is equal to the time interval for obtaining measurements of velocity and acceleration in coordinate T_0. Parameter x_i is a digitization of the elements of an experimental sample of even-discrete sequence – a grid of measurements for scaling and temporal binding on the argument of approximating polynomials of the form (7). If it is necessary to obtain, in addition to estimates, the coordinates of its derivatives should be replaced by a matrix of basic functions F its extended analogue Φ.

Definition of basis functions for coordinate $\phi_m(x)$ and its derivatives $\phi'_m(x)$, $\phi''_m(x_i)$ is realized from the condition of their mutual (within the same name parameter) of orthogonality in a polynomial form on the basis of a power series.

Calculations of CEM determination of the aircraft coordinates in GACS are realized on the source CEM of the aircraft coordinates in RCS – R_{RCS}. The output matrix is formed by a diagonal form according to known information about the potential accuracy of aircraft coordinate meter. Next, the properties of the affine mapping of the coordinates of the substation with the RCS in the GACS (referring to the scheme of Fig. 1) for obtaining R in GACS according to the expression [9]:

$$R=J^T_{CACS}\left[J^T_{GRCS}\left[J^T_{GRCS}R_{RCS}J_{LCS}\right]J_{GRCS}\right]J_{CACS}, \quad (13)$$

where $J_{LCS}=(\partial\bar{B}_{LCS}/\partial\bar{A}_{RCS})$, $J_{GRCS}=(\partial\bar{B}_{GRCS}/\partial\bar{B}_{LCS})$, $J_{CACS}=(\partial\bar{B}_{CACS}/\partial\bar{B}_{GRCS})$ are Jacobi matrices composed of the Jacobians transforming the source vector from RCS into GACS with intermediate transformations according to the scheme of Fig. 1 in the coordinate vectors of aircraft in LCS – $\bar{B}_{LCS}$ and GRCS – $\bar{B}_{GRCS}$; $R_{RCS}=diag\left(\sigma^2_r\ \sigma^2_\varepsilon\ \sigma^2_\beta\ \sigma^2_{\dot{r}}\ \sigma^2_{\dot{\varepsilon}}\ \sigma^2_{\dot{\beta}}\right)$ is a diagonal matrix of dispersion of errors of measurement of corresponding coordinates and speeds of their changes.

Formation of the vector of measurements of the coordinate and its derivatives in GACS is realized by transferring the received experimental samples of aircraft coordinates (1) to GRCS in accordance with the scheme of Fig. 1.

Formulated using the (11)–(13) matrix (10) of least squares method algorithm (9), one can obtain polynomial models of the form (7), estimating the aircraft coordinates and the velocities of their changes in the GACS taking into account the measurements of the coordinates and velocities of the observation object in the RCS. In other words, take into account simultaneously the parametric and temporary redundancy of the initial data in the estimation of a single coordinate and its derivatives. The mechanism of accounting for time and parametric excesses is the essence of the formed expressions (8) for models (7), where, in the form of functions with separated variables, one-valued functional dependences of the evaluated coordinates on the measured parameters are obtained.

7. SWOT analysis of research results

Strengths. Multidimensional smoothing provides an increase in the accuracy of the estimation of the position of the aircraft in space due to the joint statistical processing of parametric and time redundancy of the measured navigational parameters of the aircraft. It also provides increased spatial accuracy of the estimation of the aircraft's position by increasing the interval accuracy of the assessment of navigational parameters.

Weaknesses. Method of least squares, which is used in the work, implements a common (multidimensional) smoothing of samples of navigational parameters of the aircraft, whose functional dependencies have a complex nonlinear character with a large number of transposed transpose operations.

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Opportunities. The proposed approach to multidimensional smoothing can be extended to a wide range of applied branches related to measurement, estimation and forecasting of time-varying interdependent processes.

Threats. The process of multidimensional smoothing of samples of navigational parameters of aircraft by the proposed method is independent of other processes of processing of navigation information, therefore there is no threat of negative impact on the object of study of external factors.

Implementation of the proposed approach does not require additional costs for the company.

8. Conclusions

1. Mathematical statement of the problem of multidimensional smoothing of navigational parameters of the substation is formulated. These parameters characterize the aircraft trajectory taking into account the parametric and temporal excesses to improve the accuracy of determining the navigational parameters of the aircraft using the least squares method.

2. The method of least squares used to implement the common smoothing of measured parameters of the propulsion system was further developed. For this purpose, a functional relationship between these parameters was defined as the values characterizing a single object of a particular aircraft. The realization of the process of obtaining functional interactions of coordinates of aircraft is proposed by using the properties of affine mappings in the part concerning the preservation of the parameters and properties of the trajectory of the motion of a dynamic object irrespective of the transformation of the coordinate space. Differential-Taylors transformations were used to facilitate their subsequent analysis and use.

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