 ISSN 2226-3780

UDC 004.942:519
DOI: 10.15587/2312-8372.2019.179170

Kovalyov A.

DEVELOPMENT OF THE PHYSICAL AND MATHEMATICAL MODEL OF THE BAKING PROCESS OF THE DOUGH PIECES IN BAKERY OVENS

Об'єктом дослідження є фізична та математична моделі, призначені для опису тепломасоперенесення всередині пористого матеріалу під час випікання. З метою підвищення якості та одночасного зниження енергоспоживання у виробництві, а також покращання техніко-економічних показників роботи печей, тривалості і безпеки їхньої експлуатації ведеться удосконалення конструкцій пічних агрегатів, розробка нових і оптимізація теплових режимів їх роботи. Однією з найбільших проблем є задача по заміні застарілих конструкцій печей новими, з автоматичним регулюванням теплового режиму випікання, що забезпечить високу якість хліба при зниженні витрат палива, пари, електроенергії та людських ресурсів. Оскільки якість виробленої продукції, зокрема смак, аромат, пористість, глянець, зовнішній вигляд та інші показники хлібобулочних виробів в значній мірі залежать від конструкції пічного агрегату, теплового та гігротермічного режимів робочої камери, а також правильної його експлуатації. Ці фактори впливають на втрати при випіканні, які можуть змінюватись від 6 до 12 %, що впливає на вихід хліба. У даній роботі наведено фізичну і математичну модель процесу випікання тістових заготовок в хлібопекарських печах на прикладі розробленої автором промислової печі К-ПХМ-25 (Україна).

Приведена математична модель процесу випікання хліба в газових каналах пекарної камери з врахуванням радіаційно-конвективного теплообміну, масообміну з врахуванням введення водяної пари для зволожування тістових заготовок та турбулентності багатофазного потоку. Залежність турбулентності багатофазного потоку сформульована на основі осереднених за Рейнольдсом системи рівнянь Ейлера. Дана модель дозволяє з достатньою точністю і детальністю враховувати технологічні режими та конструктивні особливості сучасних конвеєрних хлібопекарських печей. А також дозволяє проводити широкі параметричні дослідження сполученого теплообміну в них з виходом на кінцевий показник – якість готових виробів.

Ключові слова: *промислова піч К-ПХМ-25, радіаційно-конвективний тепломасообмін, математична модель процесу випікання хліба.*

Received date: 13.11.2018
Accepted date: 05.12.2018
Published date: 30.06.2019

Copyright © 2019, Kovalyov A.
This is an open access article under the CC BY license
(http://creativecommons.org/licenses/by/4.0)

1. Introduction

Baking bread is the most important process in the production of bakery products. Taste, aroma, porosity, gloss and other quality indicators of finished products is the result of a number of physicochemical changes inside the product during baking, which depend primarily on the thermal regime of baking and steam humidification [1–3]. The oven is the main equipment of the bakery, it determines the type and capacity of the enterprise, the range and quality of products. The oven is not only a thermal, but primarily a technological unit, the main purpose of which is producing high-quality products while ensuring high technical and economic indicators – product output with minimal energy costs. To ensure high-quality performance of the furnace, it is necessary to use models of a real system and conduct experiments based on a mathematical model for analysis, design or redesign, control and forecasting of a specific real process. This is relevant, so many researchers have analyzed the baking process in order to obtain an accurate mathematical model for modeling bread baking. So, the authors of [4] developed a model for the process of baking a yeast dough and assessing the influence of technological parameters. And in [5], the authors made an analysis of heat and mass transfer and changes in product quality during continuous baking of cookies based on modeling of inductive furnaces. The authors of [6] developed a model of bread baking using the direct 3D numerical method at the micro scale based on the analysis of the heat flux field during baking. But the authors of [7] obtained the mathematical dependence of the process of baking a dough biscuit. The authors of [8] described the dependence of the heating of a cylindrical dough piece. Thus, *the object of this research* is the physical and mathematical models designed to describe the heat and mass transfer inside the porous material during baking. And *the aim of research* is development of a mathematical model of the process of baking bread in the gas channels of the baking chamber, taking into account radiation-convective heat transfer, mass transfer, taking into account the introduction of water vapor to moisten the dough pieces and turbulence of the multiphase flow.

2. Methods of research

The design of the K-BOM-25 baking oven developed by the author (Fig. 1) includes the following components:

– an all-metal construction assembled from separate modules and insulated from the outside with mineral wool;
– gas channels in which bread is baked;
– a movable bucket conveyor for moving dough pieces (DP) in gas channels;
– input and output nodes of the furnace, including the gas duct for the supply of green gas;
– steam humidification system;
– loading and unloading device.

That is, in the physical model of the baking oven under consideration, the furnace-igniter unit is not considered, and in accordance with this, the combustion processes of the fuel also.

The environment of the gas channels of the baking oven is considered to be two-phase, selective emitting and absorbing, and is surrounded by diffuse boundaries and consists of green gases and water vapor, which is used to moisten the dough pieces.

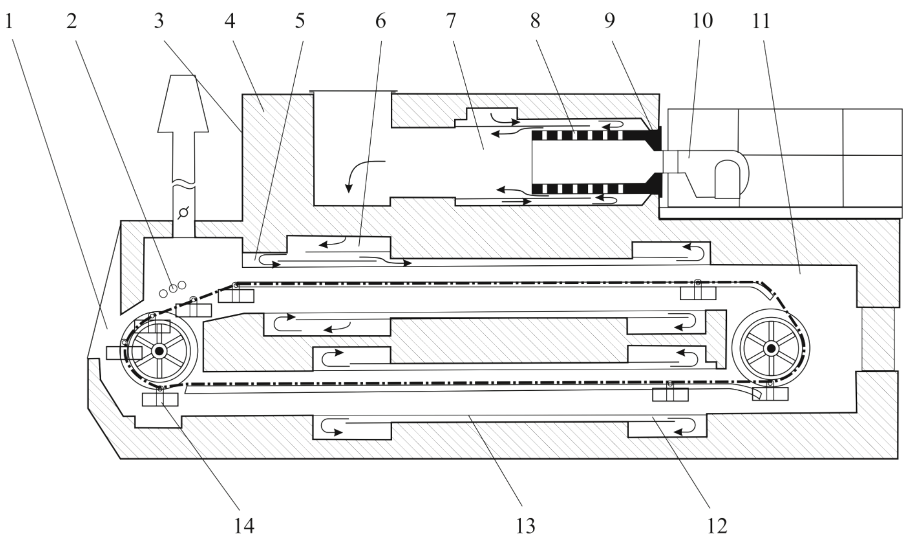

Fig. 1. K-BOM-25 bakery oven:
1 – loading and unloading device; 2 – steam humidification system; 3 – full-metal construction; 4 – thermal insulation; 5, 13 – upper and lower heating channels; 6, 12 – upper and lower flues; 7 – firebox; 8 – mixing chamber; 9 – muffle; 10 – burner; 11 – baking chamber; 14 – bucket conveyor

Dough pieces and bread are considered wet capillary-porous media with effective physical properties.

Dough pieces move with the bucket conveyor.

3. Research results and discussion

In the work, a mathematical model of the process of baking bread in the gas channels of the oven's baking chamber, taking into account radiation-convective heat transfer, mass transfer by introducing water vapor (with water droplets) to moisten the dough pieces, is baked, the multiphase flow turbulences can be formulated using the system of Euler equations averaged over Reynolds [9, 10]. The equations written for each of the phases: continuity, conservation of momentum, kinetic turbulent energy and its dissipation and energy conservation:

$$\frac{1}{\rho_{ri}}\left\{\left[\frac{\partial(\alpha_i\rho_i)}{\partial\tau}+\nabla\cdot\left(\alpha_i\rho_i\bar{\mathbf{V}}_i\right)\right]=\sum_{j=1}^{n}\left(\dot m_{ji}-\dot m_{ij}\right)\right\}, \qquad (1)$$

where ρ_{ri} – the average density of the i-th phase, kg/m^3; α_i – the volume fraction of the i-th phase in the flow $\left(\sum_{i=1}^{n}\alpha_i=1\right)$; ρ_i – the density of the i-th phase, kg/m^3; τ – the time, s; $\bar{\mathbf{V}}_i$ – the vector of the speed of the i-th phase averaged beyond Reynolds or Favre, m/s; ∇ – Hamilton operator, m^{-1}; $\dot m_{ji}$ and $\dot m_{ij}$ – mass transfer rate from phase j to phase i and vice versa, respectively (moreover), kg/(s·m^3); n – the number of phases in the stream.

$$\frac{\partial\left(\alpha_i\rho_i\bar{\mathbf{V}}_i\right)}{\partial\tau}+\left(\alpha_i\rho_i\bar{\mathbf{V}}_i\cdot\nabla\right)\bar{\mathbf{V}}_i=-\alpha_i\nabla p_i+\nabla\cdot\tau_i+\alpha_i\rho_i\mathbf{g}+$$
$$+\sum_{j=1}^{n}\left[K_{ji}\left(\bar{\mathbf{V}}_j-\bar{\mathbf{V}}_i\right)+\dot m_{ji}\bar{\mathbf{V}}_{ji}-\dot m_{ij}\bar{\mathbf{V}}_{ij}\right]+\mathbf{F}_i+\mathbf{F}_{lift,i}+\mathbf{F}_{vm,i}, \qquad (2)$$

where p_i – the partial pressure of the i-th phase, Pa; μ_i and λ_i – the shear and bulk viscosity of phase i, respectively, Pa·s;

$$\tau_i=\alpha_i\mu_i\left[\nabla\bar{\mathbf{V}}_i+\nabla\bar{\mathbf{V}}_i^T\right]+$$
$$+\alpha_i\left(\lambda_i-\frac{2}{3}\mu_i\right)\nabla\cdot\bar{\mathbf{V}}_i\mathbf{I}$$

– stress tensor of the 2nd rank (or the physical equation of state of the medium that relates stress to strain rate), Pa; $\mathbf{I}$ – the unit tensor of the 2nd rank; g – the acceleration vector associated with gravity, m/s^2; $K_{ji}=K_{ij}$ – coefficient of exchange of momentum between phases, depends on friction, pressure and other factors, kg/(m^3·s); $\bar{\mathbf{V}}_{ji}\left(\bar{\mathbf{V}}_j \text{ at } \dot m_{ji}>0, \bar{\mathbf{V}}_i \text{ at } \dot m_{ji}<0\right)$ – interphase surface velocity, m/s; $\mathbf{F}_i$ – the external mass force related to the volume, N/m^3;

$$\mathbf{F}_{lift,i}=-0.5\rho_g\alpha_p\left(\bar{\mathbf{V}}_g-\bar{\mathbf{V}}_p\right)\times$$
$$\times\left(\nabla\times\bar{\mathbf{V}}_g\right)$$

– lifting volumetric force, N/m^3; g – index of heating gases; p – water vapor index with the inclusion of water droplets;

$$\mathbf{F}_{vm,i}=0.5\alpha_p\rho_s\left(\frac{d\bar{\mathbf{V}}_g}{d\tau}-\frac{d\bar{\mathbf{V}}_p}{d\tau}\right)$$

– attached volumetric force, N/m^3;

$$\frac{\partial(\alpha_i\rho_i k_i)}{\partial\tau}+\nabla\cdot\left(\alpha_i\rho_i\bar{\mathbf{V}}_i k_i\right)=\nabla\cdot\left[\alpha_i\left(\mu_i+\frac{\mu_{t,i}}{\sigma_k}\right)\nabla k_i\right]+$$
$$+\left(\alpha_i G_{k,i}-\alpha_i\rho_i\varepsilon_i\right)+\sum_{j=1}^{n}K_{ji}\left(C_{ji}k_j-C_{ij}k_i\right)-$$
$$-\sum_{j=1}^{n}K_{ji}\left(\bar{\mathbf{V}}_j-\bar{\mathbf{V}}_i\right)\left(\frac{\mu_{t,j}}{\alpha_j\sigma_k}\nabla\alpha_j-\frac{\mu_{t,i}}{\alpha_i\sigma_k}\nabla\alpha_i\right), \qquad (3)$$

$$\frac{\partial(\alpha_i\rho_i\varepsilon_i)}{\partial\tau}+\nabla\cdot\left(\alpha_i\rho_i\bar{\mathbf{V}}_i\varepsilon_i\right)=$$
$$=\nabla\cdot\left(\alpha_i\frac{\mu_{t,i}}{\sigma_\varepsilon}\nabla\varepsilon_i\right)+\frac{\varepsilon_i}{k_i}\left(C_{1\varepsilon}\alpha_iG_{k,i}-C_{2\varepsilon}\alpha_i\rho_i\varepsilon_i\right)+$$
$$+C_{3\varepsilon}\frac{\varepsilon_i}{k_i}\left[\begin{array}{l}\sum\limits_{j=1}^{n}K_{ji}\left(C_{ji}k_i-C_{ij}k_i\right)-\\-\sum\limits_{j=1}^{n}K_{ji}\left(\bar{\mathbf{V}}_j-\bar{\mathbf{V}}_i\right)\left(\frac{\mu_{t,j}}{\alpha_j\sigma_\varepsilon}\nabla\alpha_j-\frac{\mu_{t,i}}{\alpha_i\sigma_\varepsilon}\nabla\alpha_i\right)\end{array}\right],\quad(4)$$

where k_i – the mass turbulent kinetic energy of the i-th phase, J/kg; $\mu_{t,i}=\rho_iC_\mu k^2_i/\varepsilon_i$ – turbulent viscosity of the i-th phase, Pa·s; ε_i – the dissipation rate of the turbulent kinetic energy of the i-th phase, J/(kg·s); $G_{k,i}=\mu_{t,i}\nabla\bar{\mathbf{V}}_i\left(\nabla\bar{\mathbf{V}}_i+\bar{\mathbf{V}}_i\nabla\right)$, Pa/s; $C_{ji}=2$, $C_{ij}=2\eta_{ji}/(1+\eta_j)$ – coefficient associated with the dispersion of drops [2]; σ_k – a constant; σ_k, σ_ε, C_μ, $C_{1\varepsilon}$, $C_{2\varepsilon}$, $C_{3\varepsilon}$ – the parameters (constants) of the k–ε model;

$$\frac{\partial(\alpha_i\rho_ih_i)}{\partial\tau}+\nabla\cdot\left(\alpha_i\rho_i\bar{\mathbf{V}}_ih_i\right)=\alpha_i\frac{\partial p_i}{\partial\tau}+\tau_i:\nabla\bar{\mathbf{V}}_i-\nabla\cdot\mathbf{q}_i+$$
$$+\sum_{j=1}^{n}\left(Q_{ji}+\dot m_{ji}h_{ji}-\dot m_{ij}h_{ij}\right)+\alpha_iE_i(T)+S_i,\quad(5)$$

where (:) – the double scalar product operator;

$$h_i=\int_{T_{ref}}^{T}c_{p(i)}\mathrm{d}T$$

– mass enthalpy of phase i, J/kg; $c_{p(i)}$ – the mass isobaric heat capacity, J/(kg·K); T_{ref} – reference temperature, K; $\mathbf{q}_i=-\alpha_i[\mathrm{l}_i/c_{p(i)}+\mu_{t,i}]\nabla h_i(T)$ – the heat flux density vector of phase i (or the physical equation of state of the medium connecting $\mathbf{q}$ with ∇T), which takes into account heat transfer for flow turbulence account, W/m^2; λ_i – the thermal conductivity of phase i, W/(m·K); $Q_{ji}=-Q_{ij}$ – the heat transfer intensity between phases j and i (with $Q_{ii}=0$), W/m^3;

$$h_{ji}=\begin{cases}h_j\ \text{at}\ \dot m_{ji}>0;\\h_i\ \text{at}\ \dot m_{ji}<0\end{cases}$$

– enthalpy on the interphase surface (during evaporation, this may be the vapor enthalpy at the temperature of water droplets), J/kg; S_i – volumetric heat source from chemical reactions, W/m^3;

$$E_i(T)=\int_{\nu=0}^{\infty}K_{\nu i}\left(\int_{\Omega=4\pi}I_{\nu i}d\Omega-4\pi n_{\nu i}^2I_{0\nu i}(T_i)\right)\mathrm{d}\nu$$

– bulk density of the radiation heat flux of the selective emitting and absorbing medium of the i-th phase, W/m^3; ν – the radiation frequency, Hz; $K_{\nu i}$ and $n_{\nu i}$ – the selective absorption coefficient (m^{-1}) and the refractive index of the i-th phase, respectively; Ω – solid angle, sr; $I_{0\nu i}$ – Planck function, W·s/(m^2·sr).

The spectral radiation intensity of the i-th phase (W·s/(m^2·sr)) $I_{\nu i}$ for the direction s in the solid $d\Omega$ can be represented as the dependence:

$$I_{vi}(s)=I_{vi}(s_0)\exp\left(-\int_{s0}^{s}K_{vi}\mathrm{d}s\right)+$$
$$+\int_{s0}^{s}n_{vi}^2I_{0vi}K_{vi}\exp\left(-\int_{s'}^{s}K_{vi}\mathrm{d}s''\right)\mathrm{d}s',\quad(6)$$

where s_0 corresponds to the boundary of the medium.

For solid structural elements of the furnace and dough pieces move together with a bucket conveyor, the system of equations (1)–(5) is simplified to the heat equation of the form:

$$\frac{\partial\rho h}{\partial\tau}=\nabla\cdot\left[\lambda(T)\nabla T(X)\right]+q_v,$$
$$X_{mov}(x,y,z,\tau)\cup X_{im\,mov}(x,y,z)\in X(x,y,z)\in\Omega,\quad(7)$$

where q_v – the volumetric heat source associated with loading the DP and unloading the baked bread (BB) from the oven, W/m^3, the specific power of which is determined from the system of equations:

$$q_v=\begin{cases}\dot m_{DP}\dfrac{\int_0^{T_{DP}}c_{p(DP)}(T)\mathrm{d}T}{V_{DP}},\\-\dot m_{BB}\dfrac{\int_0^{T_{BB}}c_{p(BB)}(T)\mathrm{d}T}{V_{BB}},\end{cases}\quad(8)$$

where $\dot m_{DP},\dot m_{BB}$ – the mass flow rate of the DP/BB in the loading and unloading unit, respectively, kg/s; $c_{p(DP)}$, $c_{p(BB)}$ – mass heat capacity of the DP/BB, respectively, J/(kg·K); V_{DP}, V_{BB} – volume of DP/BB, respectively, m^3; X (x, y, z) – the Cartesian coordinate system of the solid elements of the furnace, including the fixed – $X_{im\,mov}$ (x, y, z) and the moving part, which refers to the bucket conveyor and dough pieces – $X_{im\,mov}$ (x, y, z, τ); $\mathbf{V}_{conv}$ – the conveyor speed vector along with the bread that is baked, m/s; Ω – the calculated area of the baking oven.

The coefficient of exchange of momentum between the phases in equations (2)–(5) depends on the selected model of hydraulic resistance, Reynolds number, viscosity, etc. and is determined depending on the physical state of the phases interacting: liquid-liquid or liquid-gas, or gas-gas. So, for example, the coefficient of exchange of momentum between water droplets p and gas g can be determined as [4]:

$$K_{pg}=\frac{\alpha_g\alpha_p\rho_pf}{\tau_p},\quad(9)$$

where

$$\tau_p=\frac{\rho_pd_p^2}{18\mu_{eff}}$$

– the relaxation time of water droplets, s; d_p – diameter of water droplets, m; $\mu_{eff}=\mu_g/(1-\alpha_p)^{2.5}$ – effective dynamic viscosity of a gas-liquid droplets, Pa·s; $\mu_{eff}=\mu_g/(1-\alpha_p)^{2.5}$ – hydraulic resistance function; C_D – the coefficient of hydraulic resistance.

The Reynolds number of a two-phase medium is determined by the ratio:

$$\mathrm{Re}=\frac{\rho_p\left|\bar{\mathbf{V}}_p-\bar{\mathbf{V}}_g\right|d_p}{\mu_{eff}}. \quad (10)$$

The coefficient of hydraulic resistance is determined depending on the Reynolds number:

$$C_D=\begin{cases}\dfrac{24}{\mathrm{Re}} \text{ at } \mathrm{Re}<1;\\ \dfrac{24}{\mathrm{Re}}\left(1+0.1\mathrm{Re}^{\frac{3}{4}}\right) \text{ at } 1\le\mathrm{Re}\le1000;\\ \dfrac{2}{3}\dfrac{d_p}{\lambda_{RT}}\left[\dfrac{1+17.67\left(f^*\right)^{\frac{6}{7}}}{18.67f^*}\right]^2 \text{ at } \mathrm{Re}>1000,\end{cases} \quad (11)$$

where $f^*=(1-\alpha_p)^3$; $\lambda_{RT}=(\sigma/(g\Delta\rho_{pg}))^{0.5}$ – wavelength of the Rayleigh-Taylor instability, m; σ – the coefficient of surface tension, N/m; g – the acceleration of gravity, m/s^2; $\Delta\rho_{pg}$ – the density difference between the phases p and g, kg/m^3.

The initial conditions for the system of equations (1)–(6) at $\tau=0$:

$$\begin{cases}\bar{\mathbf{V}}_i=0\ ;\ p_i=0;\ T_i=T_i^0;\\ k_i=k_i^0;\ \varepsilon_i=\varepsilon_i^0;\\ L_p=L^0;\ \alpha_p=\alpha^0,\end{cases} \quad (12)$$

where $i=\overline{1,n}$ – the phase index in the stream; T_i^0, k_i^0, ε_i^0 – initial temperature (K), mass turbulent kinetic energy (J/kg) and its dissipation rate (J/(kg s)); L^0 – the initial length of the zone of water vapor in the gas channels of the baking oven, m; α^0 – the volume fraction of water vapor together with droplets of length L^0.

The boundary conditions for the system of equations (1)–(8) with:

– in the inlet sections of the furnace, parameters are set for green gas ($i=g$) and water vapor ($i=p$), mass flow rate and DP temperature:

$$\begin{cases}\mathbf{n}\cdot\bar{\mathbf{V}}_i=V_i^{in};\ T_i=T_i^{in};\ \dot{m}_{DP}=\dot{m}_{DP}^{in};\ T_{DP}=T_{DP}^{in};\\ k_i=k_i^{in};\ \varepsilon_i=\varepsilon_i^{in};\ \alpha_i=\alpha_i^{in},\end{cases} \quad (13)$$

where $\mathbf{n}$ – the vector of the external normal to the surface of the input furnace sections; V_i^{in} – normal speed, m/s; T_i^{in} – temperature of gas phases, K; $\dot{m}_{DP}^{in}$ – DP mass flow rate, kg/s; T_{DP}^{in} – DP temperature, K; k_i^{in} – mass turbulent kinetic energy, J/kg; ε_i^{in} – rate of its dissipation, J/(kg·s); α_i^{in} – volumetric part of the i-th phase at the entrance to the furnace;

– in the outgoing sections of the oven, parameters are set for the mixture (14), gas phases and mass flow rate, as well as the temperature of the baked bread (15):

$$p_{mix}=0, \quad (14)$$

$$\begin{cases}T_i=T_i^{out};\ \ \alpha_i=\alpha_i^{out};\dot{m}_{BB}=\dot{m}_{BB}^{out};\ T_{BB}=T_{BB}^{out};\\ k_i=k_i^{out};\ \varepsilon_i=\varepsilon_i^{out},\end{cases} \quad (15)$$

where p_{mix} – the overpressure of the gas mixture, Pa; T_i^{out}, α_i^{out}, $\dot{m}_{BB}^{out}$, T_{BB}^{out}, k_i^{out}, ε_i^{out} – temperature (K), volume fraction of the i-th phase, BB mass flow rate (kg/s), BB temperature (K), mass turbulent kinetic energy (J/kg) and its dissipation rate (J/(kg s));

– at the contact boundary between solids, the conditions for absolute contact are set:

$$\begin{cases}\{T\}=0;\\ \{\mathbf{n}\cdot\mathbf{q}\}=0,\end{cases} \quad (16)$$

where $\{T\}=T^+-T^-$; $\{\mathbf{n}\cdot\mathbf{q}\}=\mathbf{n}^+\cdot\mathbf{q}^+-\mathbf{n}^-\cdot\mathbf{q}^-$; «+» and «–» means to the left and right of the contact boundary; $\mathbf{q}=-\lambda(\mathrm{T})\nabla\cdot\mathrm{T}$ – the heat flux density vector; $\mathbf{n}$ – the vector of the external normal to the surface of the body;

– at the contact boundary between the multiphase medium of the furnace, DP and the fencing of the gas channels, the adhesion conditions for each of the gas phases and the interface conditions in terms of temperature and heat flux density are set:

$$\begin{cases}\bar{\mathbf{V}}_i=0;\\ \{T\}=0;\\ \sum_{i=1}^{n}\left[\mathbf{n}\cdot\left(-\alpha_i\lambda_i\left(T_i\right)\nabla T_i\right)-\mathbf{n}\cdot\mathbf{q}_{ri}\right]-q_{lh}=\mathbf{n}\cdot\left(-\lambda_{eff}\left(T\right)\nabla T\right),\end{cases} \quad (17)$$

where n – the number of phases in the stream;

$$\mathbf{q}_{ri}=\alpha_i\int_{v=0}^{\infty}\left[n_{vi}^2\varepsilon_v I_{0vi}-\varepsilon_v\int_{\mathbf{s}\cdot\mathbf{n}>0}I_{in,vi}\left(\mathbf{s}\cdot\mathbf{n}\right)d\Omega\right]\mathrm{d}v$$

– the resulting radiation flux of the i-th phase, W/m^2; ε_v – the spectral blackness of the surface of the gas channels of the furnace;

$$q_{lh}=\frac{h_{lh}\left(\dot{m}_{cond}\left(\bar{T}_F\right)-\dot{m}_{evap}\left(\bar{T}_F\right)\right)}{F_{DP(BB)}}$$

– surface heat flux density associated with mass transfer on the DP surface (moisture exchange due to condensation/evaporation of moisture), W/m^2; h_{lh} – the heat of the first-order phase transition for water (condensation/evaporation), J/kg; $\dot{m}_{cond}$, $\dot{m}_{evap}$ – mass flow rate of water during condensation and evaporation, respectively, kg/s; $\bar{T}_F$ – average integral DP/BB surface temperature, K; $F_{DP(BB)}$ – DP/BB surface area, m^2; l_{eff} – the effective thermal conductivity of the DP/BB, W/(m·K);

– at the boundary of the contact of the fencing of the furnace with the surrounding air, boundary conditions of convective type are set:

$$\mathbf{n}\cdot\left[-\lambda(T)\nabla T\right]=\alpha(T)\left(T-T_p\right), \quad (18)$$

where α – the heat transfer coefficient; T_p – the ambient temperature.

4. Conclusions

A mathematical model of the process of baking bread in the gas channels of the baking chamber is developed taking into account radiation-convective heat transfer, mass transfer taking into account the introduction of water vapor to moisten the dough pieces and turbulence of the multiphase flow. It is theoretically grounded that this model will allow with sufficient accuracy and detail to take into account all the operational and design features of modern conveyor baking ovens. And it will also allow for extensive parametric studies of conjugate heat transfer in them with access to the final indicator – the quality of finished products. But this theoretical justification requires empirical evidence.

References

1. Vanin, F. M., Lucas, T., Trystram, G. (2009). Crust formation and its role during bread baking. *Trends in Food Science & Technology, 20 (8),* 333–343. doi: http://doi.org/10.1016/j.tifs.2009.04.001
2. Purlis, E., Salvadori, V. O. (2009). Modelling the browning of bread during baking. *Food Research International, 42 (7),* 865–870. doi: http://doi.org/10.1016/j.foodres.2009.03.007
3. Purlis, E. (2010). Browning development in bakery products – A review. *Journal of Food Engineering, 99 (3),* 239–249. doi: http://doi.org/10.1016/j.jfoodeng.2010.03.008
4. Lostie, M., Peczalski, R., Andrieu, J., Laurent, M. (2002). Study of sponge cake batter baking process. II. Modeling and parameter estimation. *Journal of Food Engineering, 55 (4),* 349–357. doi: http://doi.org/10.1016/s0260-8774(02)00132-2
5. Bikard, J., Coupez, T., Della Valle, G., Vergnes, B. (2008). Simulation of bread making process using a direct 3D numerical method at microscale: Analysis of foaming phase during proofing. *Journal of Food Engineering, 85 (2),* 259–267. doi: http://doi.org/10.1016/j.jfoodeng.2007.07.027
6. Broyart, B., Trystram, G. (2003). Modelling of Heat and Mass Transfer Phenomena and Quality Changes During Continuous Biscuit Baking Using Both Deductive and Inductive (Neural Network) Modelling Principles. *Food and Bioproducts Processing, 81 (4),* 316–326. doi: http://doi.org/10.1205/096030803322756402
7. Bikard, J., Coupez, T., Della Valle, G., Vergnes, B. (2010). Simulation of bread making process using a direct 3D numerical method at microscale: analysis of baking step. *International Journal of Material Forming, 5 (1),* 11–24. doi: http://doi.org/10.1007/s12289-010-1018-3
8. Baldino, N., Gabriele, D., Lupi, F. R., de Cindio, B., Cicerelli, L. (2014). Modeling of baking behavior of semi-sweet short dough biscuits. *Innovative Food Science & Emerging Technologies, 25,* 40–52. doi: http://doi.org/10.1016/j.ifset.2013.12.022
9. Desyk, M. G., Telychkun, Yu. S., Lytovchenko, I. M., Telychkun, V. I. (2016). Mathematical modelling of heating the dough pieces of cylindrical shape. *Scientific Works o f NUFT, 22 (4),* 134–140.
10. Cornejo, P., Farias, O. (2011). Mathematical Modeling of Coal Gasification in a Fluidized Bed Reactor Using a Eulerian Granular Description. *International Journal of Chemical Reactor Engineering, 9 (1),* 1515–1542. doi: http://doi.org/10.1515/1542-6580.2288

Kovalyov Alexander, *PhD, Associate Professor, Department of Machines and Apparatus of Food and Pharmaceutical Manufacture, National University of Food Technologies, Kyiv, Ukraine, ORCID: http://orcid.org/0000-0002-4646-2919, e-mail: rait2006@ukr.net*