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OPTIMIZATION OF THE PRODUCTION PLAN BY THREE-CRITERION MODELING

Об'єктом дослідження є процеси оптимізації плану виробництва продукції за певними критеріями шляхом моделювання. Одним з найбільш проблемних місць є складність взаємоузгодження і врахування впливу критеріїв на оптимальний план виробництва. З точки зору математики пошук оптимального результату можна отримати при різних закладених умовах, але з економічної точки зору важливо вибрати ті, які мають визначальне значення. Тобто їх вагомість важлива для споживача в ухваленні рішення про покупку і для виробника – з точки зору можливостей виробництва певних видів продукції та результатів діяльності (ефективності виробництва). Дану проблему вдалося вирішити шляхом розв'язання трикритеріальної задачі планування виробництва продукції. Пошук компромісної альтернативи досягнутий за допомогою покрокового рішення запропонованої математичної моделі оптимізації плану виробництва продукції згідно найважливіших для виробника і для споживача критеріїв: прибуток, якість і попит на продукцію кожного виду з урахуванням відомої кількості одиниць кожного ресурсу.

В ході дослідження використовувалися метод ідеальної точки, симплекс метод та метод множників Лагранжа. На тестовому прикладі наводиться алгоритм вирішення поставленого завдання оптимізації. Отриманий результат – вирішена трикритеріальна задача планування виробництва продукції, яка дає можливість максимізувати прибуток від виробництва, якість продукції та попит на продукцію при відомих вихідних ресурсних складових. Важливість/значимість розробленого науково-методичного підходу підтверджується/аргументується тим, що досягнення ефективних результатів діяльності підприємства безпосередньо залежить від оптимального плану виробництва продукції.

З огляду на результати проведеного дослідження, з безлічі можливих альтернатив підприємство зможе максимально ефективно здійснювати випуск необхідної для споживача продукції за видами і якістю. Тобто, досягається при відомих ресурсних параметрах максимально можливий позитивний результат для виробника і для споживача.

Запропонований економіко-математичний інструментарій можна використовувати у вирішенні задач оптимізації виробництва в різних галузях економіки.

Ключові слова: *економіко-математичний інструментарій, планування виробництва продукції, трикритеріальна задача, діяльність підприємства.*

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1. Introduction

The basis (center) of the functioning of any enterprise is the production program (production and sales plan). The main task of the production plan is maximization of customer satisfaction in high-quality products manufactured by the enterprise, on the one hand, with optimal use of resources by the enterprise and making it maximize profits, on the other. An important tool for planning and solving many economic problems is modeling.

In 1906, V. Pareto [1] graphically presented production functions in the form of oblique curves and founded a subsequent study of the theory of production based on equal opportunity curves. Followers of Pareto's ideas continued to study the characteristics of the production function [2, 3]. Mathematical methods of organizing and planning production, as well as ideas of optimality in the economy, are described in [4]. The development of the optimization of the production plan in order to maximize profits was reflected in [5]. Today, modeling is used in various areas of economic research, including in the study of production processes [6–8]. In particular, in [9] the process of modeling production capacities of engineering enterprises of Ukraine is investigated. The multicriteria task of making managerial decisions regarding the formation of the energy security of the enterprise was the subject of research [10]. In general, it should be noted that the multivariance of the tools for the analysis of economic problems provided by models allows us to achieve the necessary alternative and flexibility. Given the competitive conditions for the functioning of enterprises in modern realities, the urgent task is to produce certain types of products that maximize consumer demand and enterprise profits. Thus, the object of research is the processes of optimizing the production plan based on certain criteria by modeling. The aim of research is optimization of the production plan according to the criteria: profit, quality and demand for products of each type, taking into account the known number of units of each resource, which are used to produce units of production of each type.

2. Methods of research

Achieving effective results directly depends on the optimal production plan.

The most important thing in determining the optimal production plan is the selection of modeling criteria.

From the point of view of mathematics, the optimal result can be obtained with various criteria laid down, but from an economic point of view, it is important to choose those that are of decisive importance. That is, their weight is important for the consumer in making a purchasing decision and for the manufacturer – in terms of the production capabilities of certain types and results (production efficiency).

Therefore, we consider the three-criterion task of planning production, in which it is necessary to maximize profits from production, product quality and demand for products.

3. Research results and discussion

For the production of certain types of products various resources are used (raw materials, means of labor, labor, etc.). It is known how many units of each resource are used to produce units of each type of product, the quality of products of each type and the demand for products of each type.

The task is drawing up a production plan with available resources, which ensures maximum profit, maximum product quality and maximum demand for products.

Let's denote the parameters for the model:

n – the number of different types of products that can be manufactured from available resources;

m – the amount of resources used in production;

a_{ij} – the number of units of the i-th resource, which is used to produce a unit of production of the j-th type;

b_i – the maximum number of units of the i-th resource that can be used in production;

c_j – the profit from the sale of a unit of production of the j-th type;

r_j – indicators (level) of product quality of the j-th type;

p_j – the indicator of demand for products of the j-th type;

x_j – the number of production units of the j-th type that is planned to be manufactured.

Then the mathematical model of the problem will look like:

$$L_1 = \sum_{j=1}^{n} c_j x_j \to \max, \quad (1)$$

$$L_2 = \sum_{j=1}^{n} r_j x_j \to \max, \quad (2)$$

$$L_3 = \sum_{j=1}^{n} p_j x_j \to \max, \quad (3)$$

under condition:

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2\ldots m, \quad (4)$$

$$x_j \ge 0, j = 1,2,\ldots n. \quad (5)$$

To solve the problem, let's use the ideal point method, which was also applied in solving the two-criterion problem of production planning [11].

In this case, a compromise alternative to x^* is solution of the scalarization problem:

$$L = \left(\left(\sum_{j=1}^{n} c_j x_j - a_1 \right)^2 + \left(\sum_{j=1}^{n} r_j x_j - a_2 \right)^2 + + \left(\sum_{j=1}^{n} p_j x_j - a_3 \right)^2 \right) \to \min,$$

where a_1 – the solution to problem (1), (4), (5); a_2 – the solution to problem (2), (4), (5) and a_3 – the solution to problem (3)–(5).

Example. Find a solution to the three-criterion problem:

$$L_1 = 5x_1 + 2x_2 + 4x_3 \to \max, \quad (6)$$

$$L_2 = 4x_1 + 2x_2 + 3x_3 \to \max, \quad (7)$$

$$L_3 = 5x_1 + 3x_2 + 4x_3 \to \max, \quad (8)$$

under conditions:

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 \le 120, \\ 2x_1 + x_2 + 4x_3 \le 100, \\ 4x_1 + 2x_2 + 2x_3 \le 150; \end{cases} \quad (9)$$

$$x_j \ge 0, j = 1,2,3. \quad (10)$$

Fig. 1 shows a set of alternatives.

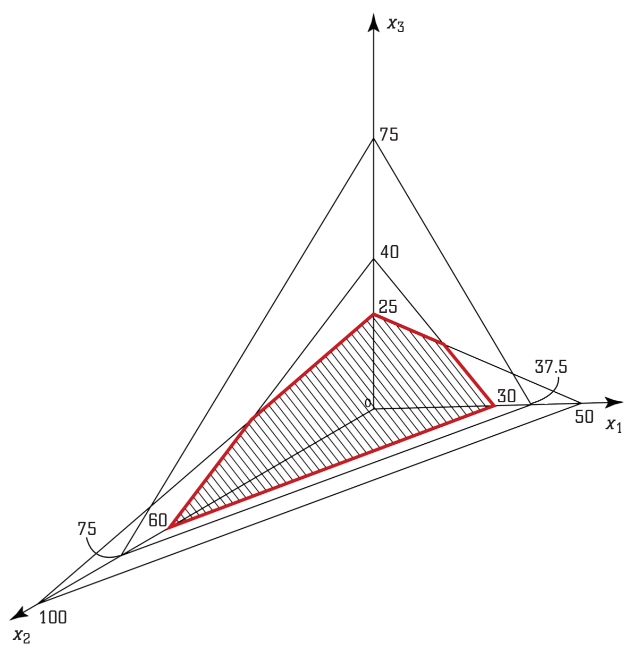

Fig. 1. An admissible set of alternatives X_j

As it is possible to see from Fig. 1, the admissible set of alternatives is not affected by the inequality:

$$4x_1 + 2x_2 + 2x_3 \le 150.$$

Therefore, the admissible set of alternatives is determined by the system of inequalities:

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 \le 120, \\ 2x_1 + x_2 + 4x_3 \le 100. \end{cases}$$

Solving the problem:

$$L_1 = 5x_1 + 2x_2 + 4x_3 \to \max,$$

under conditions:

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 \le 120, \\ 2x_1 + x_2 + 4x_3 \le 100; \end{cases}$$

$$x_j \ge 0, j = 1,2,3,$$

by the simplex method let's obtain the results (Table 1).

Table 1

The results of solving the problem $L_1 = 5x_1 + 2x_2 + 4x_3 \to \max$ by the simplex method

i	B	S	P_0	5	2	4	0	0
				P_1	P_2	P_3	P_4	P_5
1	P_4	0	120	**4**	2	3	1	0
2	P_5	0	100	2	1	4	0	1
3			0	–5	–2	–4	0	0
1	P_1	5	30	1	1/2	3/4		0
2	P_5	0	40	0	0	**5/2**	–	1
3			150	0	1/2	–1/4		0
1	P_1	5	18	1	1/2	0		
2	P_3	4	16	0	0	1	–	–
3			154	0	1/2	0		

Note: $i=1,2$; $B=P_4$, P_5; $S=0,0$; $P_0=120,100,0$, $P_1=5$, $P_2=2$, $P_3=4$, $P_4=0$, $P_5=0$

The data in Table 1 indicate that max $L_1=154$ at $X=(18, 0, 16)$. Therefore, $a_1=154$.

Having solved the problem:

$$L_2 = 4x_1 + 2x_2 + 3x_3 \to \max,$$

under conditions:

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 \le 120, \\ 2x_1 + x_2 + 4x_3 \le 100; \end{cases}$$

$$x_j \ge 0, j = 1,2,3,$$

by the simplex method let's obtain the results (Table 2).

The data in Table 2 indicate that $L_2=120$ at $X=(18, 0, 16)$. Therefore, $a_2=120$.

Having solved the problem:

$$L_3 = 5x_1 + 3x_2 + 4x_3 \to \max,$$

under conditions:

$$\begin{cases} 4x_1 + 2x_2 + 3x_3 \le 120, \\ 2x_1 + x_2 + 4x_3 \le 100; \end{cases}$$

$$x_j \ge 0, j = 1,2,3,$$

by the simplex method let's obtain the results (Table 3).

Table 2

The results of solving the problem $L_2 = 4x_1 + 2x_2 + 3x_3 \to \max$ by the simplex method

i	B	S	P_0	4	2	3	0	0
				P_1	P_2	P_3	P_4	P_5
1	P_4	0	120	**4**	2	3	1	0
2	P_5	0	100	2	1	4	0	1
3			0	–4	–2	–3	0	0
1	P_1	4	30	1	1/2	3/4		0
2	P_5	0	40	0	0	**5/2**	–	1
3			120	0	0	0		0
1	P_1	4	18	1	1/2	0		
2	P_3	3	16	0	0	1	–	–
3			120	0	0	0		

Note: $i=1,2$; $B=P_4$, P_5; $S=0,0$; $P_0=120,100,0$, $P_1=4$, $P_2=2$, $P_3=3$, $P_4=0$, $P_5=0$

Table 3

The results of solving the problem $L_3 = 5x_1 + 3x_2 + 4x_3 \to \max$ by the simplex method

i	B	S	P_0	5	3	4	0	0
				P_1	P_2	P_3	P_4	P_5
1	P_4	0	120	**4**	2	3	1	0
2	P_5	0	100	2	1	4	0	1
3			0	–5	–3	–4	0	0
1	P_1	5	30	1	1/2	3/4		0
2	P_5	0	40	0	0	**5/2**	–	1
3			150	0	–1/2	–1/4		0
1	P_1	5	18	1	**1/2**	0		
2	P_3	4	16	0	0	1	–	–
3			154	0	–1/2	0		
1	P_1	3	36	2	1	0		
2	P_3	4	16	0	0	1	–	–
3			172	1	0	0		

Note: $i=1,2$; $B=P_4$, P_5; $S=0,0$; $P_0=120,100,0$, $P_1=5$, $P_2=3$, $P_3=4$, $P_4=0$, $P_5=0$

The data in Table 3 indicate that $L_3=172$ at $X=(0, 3, 4)$. Therefore, $a_3=172$.

In order to find a compromise alternative, it is necessary to solve the scalarization problem:

$$(5x_1 + 2x_2 + 4x_3 - 154)^2 + (4x_1 + 2x_2 + 3x_3 - 120)^2 + \\ + (5x_1 + 3x_2 + 4x_3 - 172)^2 \to \min, \quad (11)$$

under conditions:

$$\begin{cases} 4x_1+2x_2+3x_3\le 120, \\ 2x_1+x_2+4x_3\le 100; \end{cases} \quad (12)$$

$$x_j\ge 0, j=1,2,3.$$

The level lines of the objective function of the scalarization problem are concentric ellipsoids centered at the point $O=(154,\ 120,\ 172)$, which is the point of the unconditional minimum of this function and is the solution to the system of equations:

$$\begin{cases} 5x_1+2x_2+4x_3=154, \\ 4x_1+2x_2+3x_3=120, \\ 5x_1+3x_2+4x_3=172. \end{cases} \quad (13)$$

To solve the quadratic programming problem (11)–(13), let's use the Lagrange multiplier method.

Let's compose the Lagrange function:

$$L(x_1,x_2,x_3,\lambda_1,\lambda_2)=(5x_1+2x_2+4x_3-154)^2+$$
$$+(4x_1+2x_2+3x_3-120)^2+(5x_1+3x_2+4x_3-172)^2+$$
$$+\lambda_1(4x_1+2x_2+3x_3-120)+\lambda_2(2x_1+x_2+4x_3-100).$$

Then, a necessary and sufficient condition for a saddle point of a function $L(x_1,x_2,x_3,\lambda_1,\lambda_2)$ is:

$$\frac{\partial L}{\partial x_1}=10(5x_1+2x_2+4x_3-154)+$$
$$+8(4x_1+2x_2+3x_3-120)+10(5x_1+3x_2+4x_3-172)+$$
$$+4\lambda_1+2\lambda_1\ge 0,$$

$$\frac{\partial L}{\partial x_2}=4(5x_1+2x_2+4x_3-154)+$$
$$+4(4x_1+2x_2+3x_3-120)+5(5x_1+3x_2+4x_3-172)+$$
$$+2\lambda_1+4\lambda_1\ge 0,$$

$$\frac{\partial L}{\partial x_3}=8(5x_1+2x_2+4x_3-154)+$$
$$+6(4x_1+2x_2+3x_3-120)+8(5x_1+3x_2+4x_3-172)+$$
$$+3\lambda_1+4\lambda_1\ge 0,$$

$$\frac{\partial L}{\partial \lambda_1}=4x_1+2x_2+3x_3-120\le 0,$$

$$\frac{\partial L}{\partial \lambda_2}=2x_1+x_2+4x_3-100\le 0.$$

$$x_2\frac{\partial L}{\partial x_2}=x_2(66x_1+34x_2+52x_3-2128+2\lambda_1+4\lambda_1)=0,$$

$$x_3\frac{\partial L}{\partial x_3}=x_3(104x_1+52x_2+82x_3-3328+3\lambda_1+4\lambda_1)=0,$$

$$\lambda_1\frac{\partial L}{\partial \lambda_1}=\lambda_1(4x_1+2x_2+3x_3-120)=0,$$

$$\lambda_2\frac{\partial L}{\partial \lambda_2}=\lambda_2(2x_1+x_2+4x_3-100)=0;$$

$$x_j\ge 0,\ \ j=1,2,3;\ \lambda_i\ge 0,\ i=1,2$$

or

$$\begin{cases} 132x_1+66x_2+104x_3+4\lambda_1+2\lambda_1\ge 4220, \\ 66x_1+34x_2+52x_3+2\lambda_1+4\lambda_1\ge 2128, \\ 104x_1+52x_2+82x_3+3\lambda_1+4\lambda_1\ge 3328, \\ 4x_1+2x_2+3x_3\le 120, \\ 2x_1+x_2+4\le 100, \\ x_1(132x_1+66x_2+104x_3-4220+4\lambda_1+2\lambda_1)=0, \\ x_2(66x_1+34x_2+52x_3-2128+2\lambda_1+4\lambda_1)=0, \\ x_3(104x_1+52x_2+82x_3-3328+3\lambda_1+4\lambda_1)=0, \\ \lambda_1(4x_1+2x_2+3x_3-120)=0, \\ \lambda_2(2x_1+x_2+4x_3-100)=0; \end{cases}$$

$$x_j\ge 0,\ \ j=1,2,3;\ \lambda_i\ge 0,\ i=1,2.$$

Introducing additional non-negative variables y_1, y_2, y_3, z_1, z_2, let's obtain the following relations:

$$\begin{cases} 132x_1+66x_2+104x_3+4\lambda_1+2\lambda_2-y_1=4220, \\ 66x_1+34x_2+52x_3+2\lambda_1+4\lambda_2-y_2=2128, \\ 104x_1+52x_2+82x_3+3\lambda_1+4\lambda_2-y_3=3328, \\ 4x_1+2x_2+3x_3+y_1=120, \\ 2x_1+x_2+4x_3+y_2=100, \end{cases} \quad (14)$$

$$x_jy_j\ge 0,\ \ j=1,2,3;\ \lambda_iz_i\ge 0,\ i=1,2; \quad (15)$$

$$x_j\ge 0,\ y_j\ge 0,\ j=1,2,3;\ \lambda_i\ge 0,\ z_i\ge 0,\ i=1,2. \quad (16)$$

To find a solution to problems (6)–(10), it is necessary to solve the system of linear equations (14) under conditions (15), (16). To do this, it is enough to find a solution to the linear programming problem:

$$V=v_1+v_2+v_3\to \min,$$

under conditions:

$$\begin{cases} 132x_1+66x_2+104x_3+4\lambda_1+2\lambda_1-y_1+v_1=4220, \\ 66x_1+34x_2+52x_3+2\lambda_1+4\lambda_1-y_2+v_2=2128, \\ 104x_1+52x_2+82x_3+3\lambda_1+4\lambda_1-y_3+v_3=3328, \\ 4x_1+2x_2+3x_3+y_1=120, \\ 2x_1+x_2+4+y_2=100, \end{cases}$$

$$x_j\ge 0,\ y_j\ge 0,\ j=1,2,3;$$

$$\lambda_i\ge 0,\ z_i\ge 0,\ i=1,2;\ v_j\ge 0,\ j=1,2,3.$$

The process of solving this problem is presented in Table 4.

The results in the Table 4 indicate that min $L=0$ at $X=(0,\ 3,\ 4)$. Thus $x_1=18$, $x_2=0$, $x_3=16$, $\lambda_1=42$, $\lambda_2=6$, $y_1=0$, $y_2=0$, $y_3=6$, $z_1=0$, $z_2=0$, $v_1=0$, $v_2=0$, $v_3=0$. Since x_3, $y_3\ne 0$, condition (15) is not satisfied. Therefore, let's derive the vector P_8 from the basis. Let's obtain the Table 5.

The results in the Table 5 indicate that $x_1=14.4$, $x_2=7.2$, $x_3=16$, $y_1=y_2=y_3=0$, $\lambda_1=43.2$, $\lambda_2=3.6$, $z_1=z_2=0$. Condition (15) is satisfied.

Table 4

The process of solving the linear programming problem

i	B	S	P_0	0	0	0	0	0	0	0	0	0	0	1	1	1
				P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}
1	P_{11}	1	4220	132	66	104	4	2	–1	0	0	0	0	1	0	0
2	P_{12}	1	2128	66	34	52	2	4	0	–1	0	0	0	0	1	0
3	P_{13}	1	3328	104	52	82	3	4	0	0	–1	0	0	0	0	1
4	P_9	0	120	**4**	2	3	0	0	0	0	0	1	0	0	0	0
5	P_{10}	0	100	2	1	4	0	0	0	0	0	0	1	0	0	0
6	–	–	9676	302	152	238	9	10	–1	–1	–1	0	0	0	0	0
1	P_{11}	1	260	0	0	5	4	2	–1	0	0	–	0	1	0	0
2	P_{12}	1	148	0	1	5/2	2	4	0	–1	0	–	0	0	1	0
3	P_{13}	1	208	0	0	4	3	4	0	0	–1	–	0	0	0	1
4	P_1	0	30	1	1/2	3/4	0	0	0	0	0	–	0	0	0	0
5	P_{10}	0	40	0	0	**5/2**	0	0	0	0	0	–	1	0	0	0
6	–	–	616	0	1	23/2	9	10	–1	–1	–1	–	0	0	0	0
1	P_{11}	1	180	0	0	0	4	2	–1	0	0	–	–	1	0	0
2	P_{12}	1	108	0	1	0	2	**4**	0	–1	0	–	–	0	1	0
3	P_{13}	1	144	0	0	0	3	4	0	0	–1	–	–	0	0	1
4	P_1	0	18	1	1/2	0	0	0	0	0	0	–	–	0	0	0
5	P_3	0	16	0	0	1	0	0	0	0	0	–	–	0	0	0
6	–	–	432	0	1	0	9	10	–1	–1	–1	–	–	0	0	0
1	P_{11}	1	126	0	–1/2	0	3	0	–1	1/2	0	–	–	1	–	0
2	P_5	0	27	0	1/4	0	1/2	1	0	–1/4	0	–	–	0	–	0
3	P_{13}	1	36	0	–1	0	**1**	0	0	1	–1	–	–	0	–	1
4	P_1	0	18	1	1/2	0	0	0	0	0	0	–	–	0	–	0
5	P_3	0	16	0	0	1	0	0	0	0	0	–	–	0	–	0
6	–	–	162	0	–3/2	0	4	0	–1	3/2	–1	–	–	0	–	0
1	P_{11}	1	18	0	5/2	0	0	0	–1	–5/2	**3**	–	–	1	–	–
2	P_5	0	9	0	3/4	0	0	1	0	–1/4	1/2	–	–	0	–	–
3	P_4	0	36	0	–1	0	1	0	0	1	–1	–	–	0	–	–
4	P_1	0	18	1	1/2	0	0	0	0	0	0	–	–	0	–	–
5	P_3	0	16	0	0	1	0	0	0	0	0	–	–	0	–	–
6	–	–	18	0	5/2	0	0	0	–1	–5/2	3	–	–	0	–	–
1	P_2	0	6	0	**5/6**	0	0	0	–1/3	–5/6	1	–	–	–	–	–
2	P_5	0	6	0	1/3	0	0	1	1/6	1/6	0	–	–	–	–	–
3	P_4	0	42	0	–1/6	0	1	0	–1/3	1/6	0	–	–	–	–	–
4	P_1	0	18	1	1/2	0	0	0	0	0	0	–	–	–	–	–
5	P_3	0	16	0	0	1	0	0	0	0	0	–	–	–	–	–
6	–	–	0	0	0	0	0	0	–2	0	0	–	–	–	–	–

Note: $i=1,2,3,4,5$; $B=P_{11},P_{12},P_{13},P_9,P_{10}$; $S=1,1,1,0,0$; $P_0=4220,2128,3328,120,100$; $P_1=0$, $P_2=0$, $P_3=0$, $P_4=0$, $P_5=0$, $P_6=0$, $P_7=0$, $P_8=0$, $P_9=0$, $P_{10}=0$, $P_{11}=1$, $P_{12}=1$, $P_{13}=1$

Table 5

The process of finding the best result

i	B	S	P_0	0	0	0	0	0	0	0	0	0
				P_1	P_2	P_3	P_4	P_5	P_6	P_1	P_3	P_5
1	P_2	0	7.2	0	1	0	0	0	–2/5	–1	6/5	–
2	P_5	0	3.6	0	0	0	0	1	3/10	1/3	–2/5	–
3	P_4	0	43.2	0	0	0	1	0	–2/5	1/3	–2/5	–
4	P_1	0	14.4	0	0	0	0	0	0	0	0	–
5	P_3	0	16	0	0	1	0	0	0	0	0	–
6	–	–	0	0	0	0	0	0	–2	0	0	–

Note: $i=1,2,3,4,5$; $B=P_2$, P_5, P_4,P_1, P_3; $C=0,0,0,0,0$; $P_0=7.2,3.6,43.2$, $14.4,16$; $P_1=0$, $P_2=0$, $P_3=0$, $P_4=0$, $P_5=0$, $P_6=0$

Therefore, a compromise alternative is $x^*=(14.4;\ 7.2;\ 16)$. For this alternative, $L_1=150.4$, $L_2=120$, $L_3=157.6$.

4. Conclusions

A mathematical model for optimizing a production plan according to criteria is presented: profit, quality and demand for products of each type, taking into account the known number of units of each resource. Using the ideal point method, the simplex method and the Lagrange multiplier method, an algorithm for solving the optimization problem is given on a test example.

The proposed scientific and methodological approach makes it possible to draw up an optimal production plan taking into account the maximization of criteria: profit, quality and demand for products with given/known resource parameters. The proposed three-criterion model for optimizing the production plan can be used in various sectors of the economy.

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