Альрефаі
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МАТЕМАТИЧНА МОДЕЛЬ ХАОСУ, ВИКЛИКАНОГО МІЖНАРОДНОЮ ТОРГІВЛЕЮ

Основні ефекти і закономірності, які характеризують кейнсіанські моделі ділового циклу, були змодельовані за допомогою системи лінійних диференціальних рівнянь. Знайдені розв'язки системи, яка описує взаємодію двох або трьох держав, і досліджувана стабільність цієї системи.

Ключові слова: *кейнсіанські моделі, проблеми стійкості, фазовий простір, аттрактор, хаос.*

1. Introduction

The majority of economic processes proceed in time. Owing to this, corresponding mathematical models are basically dynamic and nonlinear.

Till now all new «synergetic» concepts and objects have appeared and investigated in nonlinear systems with rather small dimension ($n \leq 4$). Research of multivariate systems leads to complexities, both theoretical, and computing. The model of economic system of 6-th order is considered below. However it is assumed to be linear, it shows the complex behavior, similar to determined chaos, known in nonlinear systems. Similar results for economic system in 2006 were actively discussed on the Internet. In the given work model parameters of economies of 3 states are offered, at which the collaboration between economies leads to the phase portraits, that are similar to determined chaos of resonant torus type.

The given system is considered with the purpose of increasing the management efficiency of the union, that is the increase of the income for all or most of the countries from the union.

2. Analysis of publications

The general problem is to model relationships of several countries at a level of macroeconomic streams with the objective to define stability of the total system.

Now growth of the international attitudes, expansion of import-export streams thereof makes it necessary to analyze and forecast its development [1—10]. Thus, the task in view is actual.

Consider dynamic economic system, i.e., continuously developing in time.

At a level of economic categories it has been offered by Keynes. Its general model for three countries consists of six differential equations and describes international trade. The economy of each country is described by two equations reflecting dynamics of the national income and the rate of percent.

According to Keynes, the general model for i-th country ($i = 1, 2, 3$) looks as follows:

$$\begin{cases} \dfrac{dY_i}{dt} = A_i(I_i - S_i) + Ex_i - \text{Im}_i, \\ \dfrac{dR_i}{dt} = B_i\left(L_i - \dfrac{M_i}{P_i}\right), \end{cases}$$

where A_i, B_i, ($i = \overline{1,3}$) are some positive constants; Y_i, ($i = \overline{1,3}$) is national income of i-th country (phase variable); R_i, ($i = \overline{1,3}$) is the current rate of percent in i-th country (a phase variable); I_i is the function of demand for investments in i-th country which can be written as:

$$I_i = DY_i \cdot Y_i + DL_i \cdot (RR_i - R_i),$$

S_i is the function of savings of i-th country which can be written down as:

$$S_i = SY_i \cdot Y_i + SL_i \cdot R_i - CA_i.$$

L_i is the function of demand for money of i-th country which can be taken as follows:

$$L_i = LY_i \cdot Y_i + LI_i \cdot (R_{\max,i} - R_i).$$

M_i is the amount of money supply of i-th country; it is regarded as constant; P_i is price level of i-th country. It is constant (thus, a constant M_i/P_i represents the offer of money in i-th country); Ex_i is the function of export for i-th country which can be written down, for example, so:

$$Ex_i = E_{0i} + U_i \cdot Y_j + V_i \cdot Y_k.$$

Im_i is linear function of import for i-th country as:

$$\text{Im}_i = Z_{0i} + Z_i \cdot Y_i.$$

Instability in economic system can play even positive role (economic growth).

This case also was considered. Let's note, that stationary points and limiting cycles are considered as classical attractors. To «strange» attractor we concern the limited sets on phase space which are points and cycles in a broad sense. Resonant torus holds intermediate position in this hierarchy.

According to the theory, there are no sources of dynamic chaos in linear systems. But at the certain combination of constants of this system of the differential equations the pattern shows chaotic behavior at separate intervals of time, and in some cases, phase trajectories leave on torus strange attractor, the objective of the present work is research of a phase portrait of the given system on a randomness.

3. The research results

For the given models the following analysis has been made:

— found stationary conditions were found;

— the analysis of stability has been made. The Lyapunov parameters for the further estimation of system's stability and trajectories of development were obtained. So, identification of the type of system's dynamics was made;

— the numerical analysis for investigated system at the various initial parameters describing various types of interaction of subsystems (for example when two countries have terminated the attitudes — they become independent) was made.

In this work the solutions of the system is made using two methods — numerically-analytical (i. e. eigenvalues of a matrix of 6-th order are obtained numerically, and then the known formula for the solution of linear system with constant factors is written out) and purely numerical, using Runge-Kutta 4-th order method.

Comparison of solutions has shown, that digitization is lawful, and is not the reason of chaotic behavior of the system. Such behavior, in full conformity with the theory, appears for the combination of parameters when real parts, at least of two eigenvalues are equal 0, and imaginary ones, responsible for the period of fluctuations are not comparable, i. e. their quotient is irrational, or close to it. Elements of a matrix are chosen so that these parities were carried out, but parameters of model were within the limits of economic sense. For example, the majority of them should be inside (0, 1) interval.

Various figures of projections and sections of phase portraits are presented by means of Mathematica package. They allow to present the torus-kind attractor on a plane in 6 dimensional space. From phase portraits it is clearly seen, that the drawing set is limited, and trajectories on it have no circles, so, this fits the basic criterion of «strangeness» for attractor.

The similar phenomena were observed in 4 dimensional models for economies of two states.

4. Conclusions

Thus the required combination of parameters has been found, and the hypothesis of chaos as a result of globalization has been proved to be true, even for the elementary models.

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Основные эффекты и закономерности, которые характеризуют кейнсианские модели делового цикла были смоделированы с помощью системы линейных дифференциальных уравнений. Найдены решения системы, которая описывает взаимодействие двух или трех государств, и исследована стабильность этой системы.

Ключевые слова: кейнсианские модели, проблемы устойчивости, фазовое пространство, аттрактор, хаос.

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