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DEVELOPMENT OF A METHOD FOR FORECASTING RANDOM EVENTS DURING INSTABILITY PERIODS

Об'єктом дослідження є випадкові події при формуванні нових економічних та фінансових моделей, зокрема, при кардинальних змінах економічної та соціальної стратегій. Область застосування та різноманітність методів, використовуваних в завданнях прогнозування випадкових процесів, велика. Перспективним математичним апаратом вирішення проблеми є статистичні методи аналізу. На сьогоднішній день існує багато методів прогнозування випадкових процесів, проте більшість існуючих моделей не є придатними для прогнозування нестаціонарних процесів. Одним з найбільш проблемних місць в прогнозуванні часових рядів є те, що єдиної методології, за якою можна було б аналізувати характеристики нестаціонарного випадкового процесу, не існує. Тому необхідно розробляти спеціальні методи аналізу, які можливо застосовувати до окремих випадків нестаціонарних процесів. Оптимальним варіантом вирішення проблеми може стати апроксимація часового ряду дрібно-раціональними функціями або так звана апроксимація Паде. Такий підхід повинен мати перевагу від поліноміальної апроксимації. При поліноміальній апроксимації поліном не може мати горизонтальної асимптоти, що не дає можливості робити довгострокові прогнози. Раціональна апроксимація гарантовано прагне до горизонтальної асимптоти, при цьому усі полюси дрібно-раціональної функції повинні лежати у лівій частині p-площини, тобто площини перетворення Лапласа. Запропоновано метод прогнозування нестаціонарних часових рядів з високою точністю оцінювання та гнучкістю параметрів. Для забезпечення стійкості методу та стабільності отриманих результатів запропоновано примусове введення полюсів апроксимуючої функції в зону стійкості – одиничне коло z-площини з дотриманням правил конформного перетворення. А саме – трансформацією лінійних розмірів та зі збереженням кутів між ортогональними координатами на нескінченно малих околицях координатної площини (так званий консерватизм кутів). Показано, що при дотриманні конформності запропонованого перетворення зберігаються динамічні характеристики системи оцінювання та прогнозування. Цей метод особливо успішно може застосовуватися при наявності нестаціонарностей самої різної природи.

Ключові слова: *випадкові процеси, нестаціонарні процеси, часові ряди, апроксимація Паде, довгостроковий прогноз, перетворення Лапласа.*

1. Introduction

In periods of instability that are characteristic of transition economies, changes in economic strategy, and especially during periods of global critical changes, incidents can occur that will affect fundamental operating principles. For prediction, and if possible, to prevent such events, powerful statistical methods of modeling and forecasting are needed.

A feature of the changes that are considered is precisely their suddenness, which occurs due to the lack of a priori information. A model such as a flash of white noise could serve as an adequate model, but as a result of an ultra-wide band of such a process, it does not provide practical conclusions for making operational and correct management decisions. The desire to build a model that most closely matches the real situation, the immediate and adequate reaction of decision makers, and the need to improve the quality of forecasts lead to modifications of existing models and the emergence of new classes of models. But at the same time, it is necessary to carefully analyze the weaknesses of such models that are not obvious at first glance.

This work is devoted to the search for answers to these pressing questions.

2. The object of research and its technological audit

In the formation of economic strategies from the point of view of mathematical modeling, the problem arises of forecasting time series of various origins, since the influence of many factors. In this case, it is necessary to analyze many factors that determine the behavior of an object. When forecasting economic indicators, the requirements for forecasting accuracy are rather rigidly put forward; this leads to a constant search for new simple and adequate, as a rule, formal mathematical models. Thus, *the object of the study* is random events in the formation of new economic and financial models, in particular, with cardinal changes in economic and social strategy.

One of the most problematic places in forecasting time series is that a single methodology by which it would be possible to analyze the characteristics of an unsteady random process does not exist in principle. Therefore, it is necessary to develop special methods of analysis that can be applied to individual cases of unsteady processes.

Since a unified approach to the analysis of the characteristics of a non-stationary random process does not exist in principle, it is necessary to look for ways to solve the problem. Alternative approaches can be used for this. The first is to develop a classification of non-stationary random processes with an exhaustive mathematical description of all the characteristics of each class. Based on this mathematical model, it is theoretically possible to synthesize a general method and an estimation algorithm. The second is to develop special analysis methods that can be applied only to individual classes of non-stationary processes.

According to the author of this work, the general processing method and algorithm, which would cover all possible classes of non-stationary random processes, is too complex to obtain results of acceptable quality. As always, the optimum between the two alternative approaches – to serve all random processes with zero quality or to serve an empty set of random processes with ideal quality – lies somewhere in between.

3. The aim and objectives of research

The aim of this research is development of a powerful and accurate method for predicting non-stationary time series with the ability to evaluate the parameters of random processes.

To achieve this aim, it is necessary to complete the following objectives:

1. To analyze existing approaches to forecasting sudden events.

2. To justify and choose an approach to forecasting random processes.

3. To evaluate the local characteristics of the process non-stationarity by the current implementation and the synthesis of the corresponding approximation methods and algorithms.

4. Research of existing solutions of the problem

Today, many models for predicting sudden events have been developed. Mathematical models of sudden changes can be built, for example, on the basis of the theory of emissions of random processes [1] or methods based on the theory of Markov processes [2]. However, these theories, with a sufficiently high degree of abstraction, rarely give practical results that could be applied to achieve real goals.

Recently, neural network forecasting models have gained popularity. The main disadvantage of the class of neural network models is the inaccessibility of intermediate calculations performed by the system, and, as a result, the complexity of interpreting the simulation results. In addition, the weakness of this class of models is the difficulty in choosing an algorithm for training a neural network [3, 4].

The role of statistical methods of analysis is not losing significance and is growing steadily. The scope and variety of methods used in the tasks of forecasting time series is great. Moreover, most of the proposed methods [5] relate to the analysis of stationary time series. But the question remains, if the series is unsteady, then these methods do not give a reliable result. Most often, by the nature of origin, the studied time series are the result of observing the behavior of complex systems whose deterministic description is impossible [6].

Literary sources describe various approaches to the analysis of time series. This is primarily due to the diversity of the origin of time series. Among the main works devoted to solving this problem, one can single out works aimed at constructing adequate mathematical models of time series, in particular, financial time series [7, 8]. These models are constructed using the results of systems analysis theory and conflict theory [9, 10].

To predict non-stationary processes in most of the analyzed works, regression models are used taking into account the trend of various types [11, 12]. However, this does not take into account the significant limitations of trend-based forecasting results. In particular, in cases where the process involves a slow trend or jumps in mathematical expectation. A polynomial model is usually taken as a trend. Designing a trend for a long term into the future is impractical, because sooner or later the variable must stabilize, and the polynomial can't have a horizontal asymptote. In addition, the selection of the trend and the seasonal component should be carried out using an iterative process, it provides at least two estimates of each component. As a result, the amount of computation will, as a rule, be significant even for high-speed computers [13].

A polynomial of even a high degree does not give a good prognosis. In any case, it can be used in this capacity only for a fairly near future. For values located at a considerable distance, this polynomial grows, and its derivative also grows. Accordingly, the forecast error also increases [14, 15].

Thus, the results of the analysis allow to conclude that approximation of a number of finely rational functions or the so-called Padé approximation can give good results [16, 17]. Such an approach should take advantage of polynomial approximation. In polynomial approximation, polynomial can't have horizontal asymptotes, which makes it impossible to make long-term forecasts. A rational approximation is guaranteed to tend to horizontal asymptotes, with all the poles of the finely rational function lying on the left side of the p-plane, that is, the Laplace transform plane.

5. Methods of research

The Padé approximation [18] represents a function in the form of the ratio of two polynomials. Using the Padé approximation with the help of a rational (more precisely, finely rational) function, it is possible to get rid of the restrictions associated with the expansion in a Taylor series.

While let's consider the approximated function as the function of a real variable. Following Baker [19], let's specify the coefficients of the polynomials. Obviously, they are determined by the coefficients of the expansion of the function in a Taylor series. Thus, if an expansion in a power series of the form is given:

$$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots = \sum_{i=0}^{\infty} c_i x^i, \qquad (1)$$

then the Padé approximation is a rational function of the form:

$$\frac{a_0 + a_1 x + \ldots + a_L x^L}{b_0 + b_1 x + \ldots + b_M x^M}, \tag{2}$$

the expansion of which in the Taylor series coincides with the expansion (1).

Function (2) has $L+1$ coefficients in the numerator and $M+1$ coefficients in the denominator. The entire set of coefficients is determined accurate to a common factor. To simplify, it is possible to put one of the constant terms (a_0 or b_0) equal to unity, since this does not affect the dynamic properties of the process, which is subject to approximation. Let's put for definiteness $b_0 = 1$. Then let's have $L+1$ free terms in the numerator and M in the denominator of fraction (2), that is $L+M+1$ free terms. Then the coefficients of the expansion of the function $[L/M]$ in a Taylor series at powers $1, x, x^2, \ldots, x^{L+M}$ must coincide with the corresponding coefficients of the series (1). The obvious relation follows from this:

$$\sum_{i=0}^{\infty} c_i x^i = \frac{a_0 + a_1 x + \ldots + a_L x^L}{b_0 + b_1 x + \ldots + b_M x^M} + O\left(x^{L+M+1}\right). \tag{3}$$

Let's multiply function (3) by the denominator of the fraction and find that:

$$\left(b_0 + b_1 x + \ldots + b_M x^M\right) \cdot \sum_{i=0}^{\infty} c_i x^i =$$
$$= \left(b_0 + b_1 x + \ldots + b_M x^M\right)\left(c_0 + c_1 x + c_2 x^2 + \ldots\right) =$$
$$= a_0 + a_1 x + \ldots + a_L x^L + O\left(x^{L+M+1}\right). \tag{4}$$

Equating the coefficients of $x^{L+1}, x^{L+2}, \ldots, x^{L+M}$, let's obtain the obvious equalities:

$$b_M c_{L-M+1} + b_{M-1} c_{L-M+2} + \ldots b_0 c_{L+1} = 0;$$
$$b_M c_{L-M+2} + b_{M-1} c_{L-M+3} + \ldots b_0 c_{L+2} = 0;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{5}$$
$$b_M c_L + b_{M-1} c_{L+1} + \ldots b_0 c_{L+M} = 0.$$

To generalize, let $c_j \equiv 0$ at $j < 0$.

Recall that by definition $b_0 = 1$. Then it is possible to rewrite equalities (5) in the form of a system of M linear equations with M unknown coefficients for the denominator:

$$\begin{pmatrix} c_{L-M+1} & c_{L-M+2} & c_{L-M+3} & \cdots & c_L \\ c_{L-M+2} & c_{L-M+3} & c_{L-M+4} & \cdots & c_{L+1} \\ c_{L-M+3} & c_{L-M+4} & c_{L-M+5} & \cdots & c_{L+2} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ c_L & c_{L+1} & c_{L+2} & \cdots & c_{L+M-1} \end{pmatrix} \begin{pmatrix} b_M \\ b_{M-1} \\ b_{M-2} \\ \cdot \\ \cdot \\ \cdot \\ b_1 \end{pmatrix} = - \begin{pmatrix} c_{L+1} \\ c_{L+2} \\ c_{L+3} \\ \cdot \\ \cdot \\ \cdot \\ c_{L+M} \end{pmatrix}. \tag{6}$$

Comparing the coefficients in the formula (4) at the same degrees $x - 1, x, x^2, \ldots, x^L$, let's find the coefficients of the numerator:

$$a_0 = c_0;$$
$$a_1 = c_1 + b_1 c_0;$$
$$\vdots \tag{7}$$
$$a_L = c_L + \sum_{k=1}^{\min[L,M]} b_k c_{L-k}.$$

As can be seen from expression (7), in the general case, the minimum number of terms in the sum $\sum_{k=1}^{\min[L,M]} b_k c_{L-k}$ is selected from a pair $[L,M]$: if $L > M$, a number M is selected, if $L < M$ – a number L is selected. However, in the approximation problem there are additional considerations on the ratio of powers of L and M, which will be discussed later.

Formulas (6), (7) are a pair of Padé equations. The coefficients of the numerator and denominator of the Padé approximation $[L/M]$ are determined from these equations (in the case when the system of equations (6) has a stable solution. The conditions for obtaining a stable solution are also discussed below).

The coefficients of the terms of the Taylor series in which the function $f(x)$ is expanded in powers $1, x, x^2, \ldots, x^{L+M}$ coincide with the corresponding coefficients of the series (1).

If consider a function $f(z)$ as an analytic function, it is uniquely determined on the entire plane of the complex variable, this function can be extended to the complex region. Let a continuous function $f(x)$ of a real variable be given on a segment $[x_1, x_2]$ of the real axis X. Then, in a certain domain $\wp$ of the complex plane containing a segment $[x_1, x_2]$ of the real axis, there can exist only one analytic function $f(z)$ of a complex variable z that takes these values $f(x)$ on the segment $[x_1, x_2]$. Function $f(z)$ is analytic extension of the function $f(x)$ of a real variable x into a complex domain $\wp$. In this case, there is a natural transition from the expansion of a function $f(x)$ in a Taylor series to the expansion of a function $f(z)$ in a Laurent series [20, 21]:

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z - z_0)^n = \sum_{n=0}^{\infty} c_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{c_n}{(z - z_0)^n}, \tag{8}$$

where z_0 – the fixed point of the z plane.

Let the Laurent series have a finite number m of terms with negative powers. Then the isolated face point of the function $f(z)$ is the pole of the m-th order. Using the Padé approximation of the Laurent series $\sum_{n=-\infty}^{\infty} c_n (z - z_0)^n$, it is possible to obtain approximations of the function $f(z)$ with an error, tends to zero unlimited, but here two problems arise: the construction of the Padé approximation and its convergence to the function is approximated. To construct the Padé approximation by expressions (6) and (7), it is necessary to have only the coefficients of the series (8). Let's consider the problem of stability and convergence of the obtained solutions in the next section.

6. Research results

As it is known, any step series has its own region of convergence – the circle of convergence of radius R. The series coincides at $|z| < R$, when $|z| > R$ – diverges. If $R \to \infty$, the series is a function analytic everywhere in the complex plane [18]. The value of the function at an arbitrary point z can be approximately obtained by direct summation of the series, and the error of approximation with an unlimited increase in the number of terms of the series monotonously tends to zero:

$$\varepsilon_{appr} = \left| f(z) - \sum_{k=-K}^{K} c_k z^k \right| \to 0 \quad \text{at} \quad K \to \infty.$$

If the sequence of Padé approximations of the formal power series (8) converges to a function $f(z)$ in the circle of convergence $\mathbb{R}$, $z \in \mathbb{R}$, in practical applications it is possible to safely assume that the Laurent series of type (8) corresponds to a function $f(z)$. If the Laurent series converges to a function $f(z)$ in a circle $|z| < R$, $0 < R < \infty$, then theoretically the sequence of Padé approximants can coincide in a wider region. But for practical applications (and it is for them!) Of fundamental interest is the problem of stability of the solution: under what conditions is a polynomial:

$$\frac{a_0 + a_1 z + a_2 z^2 + \ldots + a_L z^L}{b_0 + b_1 z + b_2 z^2 + \ldots + b_M z^M},\qquad(9)$$

which, in essence, plays the role of analytic elongation of polynomial (2) onto the plane of the complex variable z, will be stable. A positive answer to this question will take place when the convergence condition for the Padé approximation in the unit circle of the z-plane is satisfied [17]. This condition is satisfied relatively simply.

In accordance with the main theorem of higher algebra [17], a polynomial of arbitrary degree (say, N degree) with real coefficients has exactly N roots that are either real or create complex conjugate pairs. Then the denominator of polynomial (2) and the purely formal denominator of polynomial (9) have exactly M roots. These roots are the poles of the function of a complex variable (9). If these poles do not go beyond the limits of a single z-plane, the object (digital filter, difference equation, etc.) is stable. In other words, it returns to a stable state after completion of arousal.

Thus, when applying the Padé approximation, it is necessary to control the absolute values of the poles of the approximating polynomial. This is a kind of fee that it is possible to pay for the high accuracy of the Padé approximations and their convergence to horizontal asymptotes even with a low order of the approximating polynomial [17, 19]. In this regard, let's note that the approximating polynomial will retain its properties, the desire for horizontal asymptotes (in particular, the abscissa axis) when a simple condition is met. In expressions of type (9), the rule $L < M$ must always be fulfilled; there is a degree of the numerator of function (9) should always be to a lesser extent the denominator.

At the end of this section, let's consider another subtle point in the construction of the Padé approximation. If in the approximating polynomial (9) one or several poles go beyond the limits of a single z-plane, then the Laurent series is such that it diverges everywhere except for a point $z = 0$, and its application, in fact, becomes useless.

To force a polynomial of the form (9) to return to a stable state, it is necessary to return the «unreliable» poles inside the unit circle. To do this, it is necessary to make the pole modules less than unity, but so that the angular position of these poles does not change, that is, observe the rules of conservatism of angles [21]. The easiest way to do this is by representing the pole as a complex number in exponential form. Let the

m-th pole $p_m = |p_m| \cdot \exp(j\varphi_{pm})$, moreover $|p_m| > 1$. Let's take the new pole:

$$p_{mnew} = \big[1/|p_m|\big] \cdot \exp(j\varphi_{pm}).$$

Fig. 1 shows the location diagram of the initial pole p_m, which is modulo more than 1, which leads to instability of the function $f(z)$, and the modified pole p_{mnew}, which module is $|p_{mnew}| = 1/|p_m|$ by definition less than 1.

When a mirror reflection of the pole inside the unit circle, observing the conservatism of the angles, the dynamics of changes in the state of the object is not violated [22]. In Fig. 2, 3 as an example, the amplitude-frequency and pulse characteristics of a second-order digital filter are shown; stable – impulse response tends to zero (Fig. 2) and unstable – impulse response increases unlimited (Fig. 3).
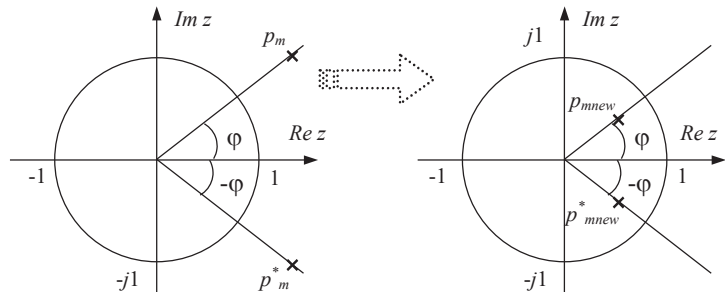

Fig. 1. The diagram of the transformation of the poles to enter a digital object in the stability zone $|p_{mnew}| = 1/|p_m|$, $|p^*_{mnew}| = 1/|p^*_m|$, $\varphi = \text{const}$, $-\varphi = \text{const}$
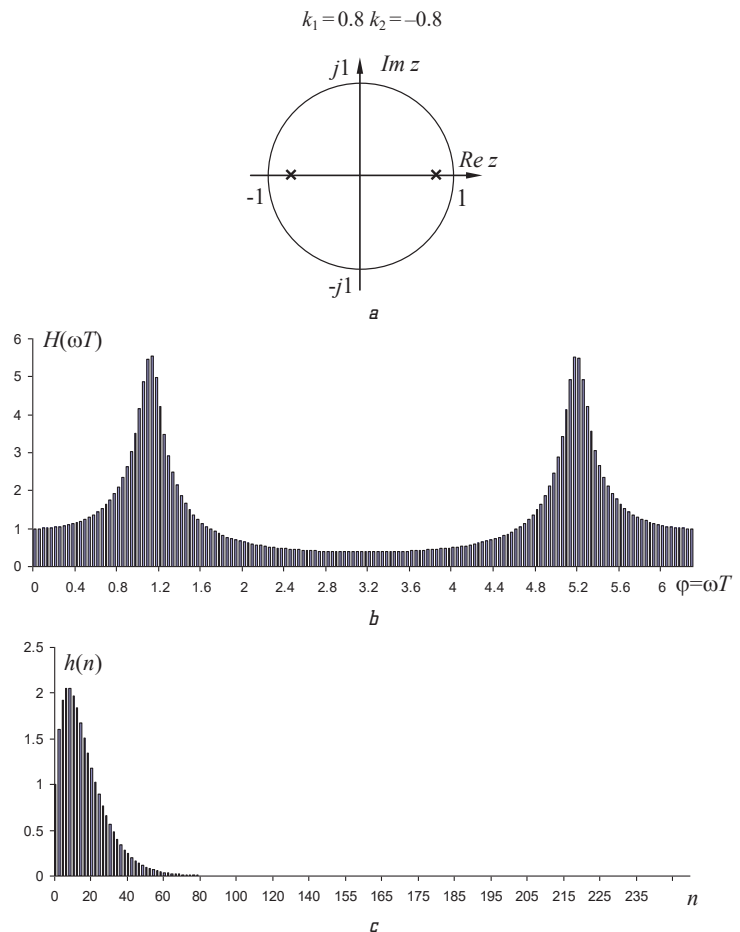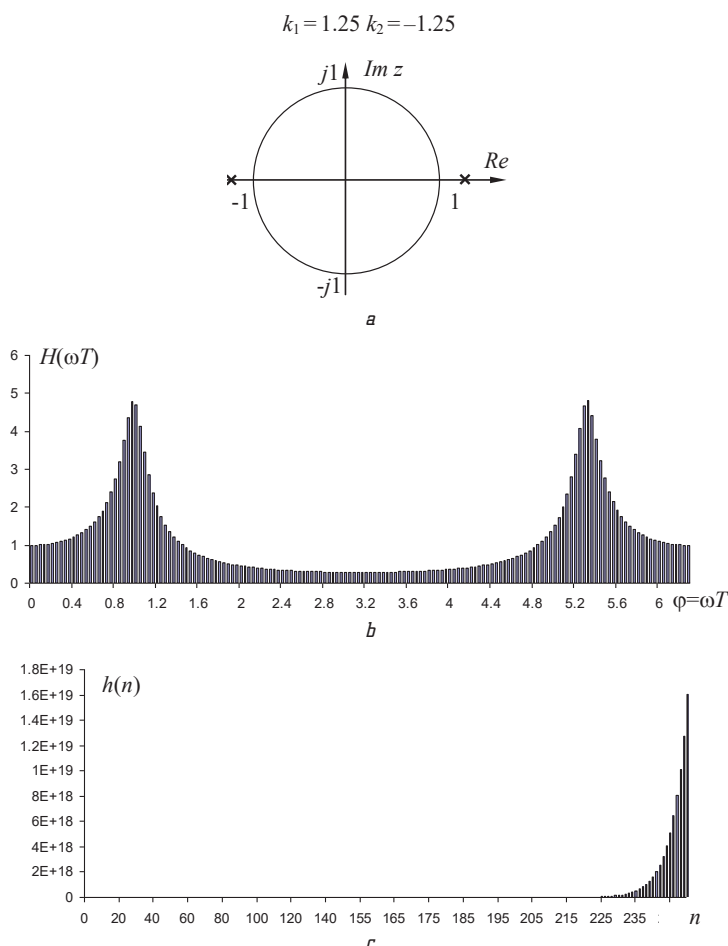
$$k_1 = 0.8 \quad k_2 = -0.8$$

Fig. 2. Stable object: a – poles inside a unit circle; b – amplitude-frequency characteristic; c – impulse response

$k_1 = 1.25 \quad k_2 = -1.25$

Fig. 3. An unstable object: a – poles outside the unit circle; b – amplitude-frequency characteristic; c – impulse response

Let's pay attention to the identity of the amplitude-frequency characteristics in contrast to the difference in the impulse characteristics. Therefore, it can be argued that the proposed transformation is conformal.

7. SWOT analysis of research results

Strengths. To obtain the most accurate forecast results in the proposed method, an approach based on the Padé approximation is used. Using this method of identifying the parameters of random processes, the maximum forecast accuracy will be ensured.

Weaknesses. The negative side when applying the Padé approximation is that it is necessary to control the absolute values of the poles of the approximating polynomial. However, this inconvenience is compensated by the high accuracy of the Padé approximations and their convergence to horizontal asymptotes, even with a low order of the approximating polynomial.

Opportunities. In the future, it is planned to develop a method for simplifying the analysis by decomposing a high-order approximating function into an additive composition of elementary units of the first or second order with real coefficients. To calculate the weight coefficients of elementary links, it is advisable to use the method of residue theory – a powerful and effective method of the theory of functions of a complex variable.

Threats. It is practically impossible for statistical models to be «comprehensive» in the sense of including all

relevant variables that affect the process parameters. The imperfect nature of statistical models is reinforced by real evidence that the output is often incomplete, inconsistently encoded, and generally «raw». The main factors influencing the characteristics of the studied processes are so different and unpredictable that the results of each new experiment are almost unique. Naturally, statistical analysis needs to be carried out only on one sample-implementation of a random process.

8. Conclusions

1. An analysis of existing approaches to forecasting sudden events has been carried out, which shows that today there are many methods for predicting random processes, including methods based on the theory of Markov chains, theories of random emissions, neural network models, etc.

2. It is shown that most of the existing models are not suitable for predicting non-stationary processes. This is primarily due to the diversity of the origin of time series. It is shown that the optimal solution to the problem may be the approximation of the time series by finely rational functions or the so-called Padé approximation.

3. For quick and accurate prediction of global events during periods of instability, a new powerful method of statistical processing and forecasting time series is proposed in the work, taking into account the specifics of non-stationarity of the process, assessing its local characteristics. The method for forecasting non-stationary time series is developed with high accuracy of estimation and flexibility of settings.

This method can be especially successfully applied in the presence of non-stationarity of various natures. To ensure the stability of the method and the stability of the results obtained, it is proposed that the poles of the approximating function be introduced into the stability zone – the unit circle of the z-plane in compliance with the rules of conformal transformation. Namely, by transforming linear dimensions and preserving the angles between the orthogonal coordinates on infinitely small neighborhoods of the coordinate plane (the so-called conservatism of angles). It is shown that, subject to the conformity of the proposed transformation, the dynamic characteristics of the estimation and forecasting system are stored.

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