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## OPTIMIZATION OF AMPLITUDE-FREQUENCY CHARACTERISTIC OF BROADBAND VOLTAGE DIVIDER INTENDED FOR MEASUREMENT OF POWER QUALITY PARAMETERS

Об'єктом дослідження є схема широкосмугового ємнісно-омічного подільника напруги з послідовно-паралельним з'єднанням його резистивних і ємнісних елементів. Довгі роки застосування подільників напруги обмежувалася вимірюванням різних напруг в умовах високовольтних лабораторій. Однак подільники напруги, у порівнянні із трансформаторами напруги, характеризуються більш широкою смугою пропускання, тому вони стали розглядатися як один з основних засобів вимірювання напруг у високовольтних електричних мережах. Одним з каталізаторів впровадження цього рішення може стати інтенсивний розвиток концепції *Smart Grid*, яка вимагає нових, більш досконалих засобів моніторингу якості електроенергії. Тому експериментальні й теоретичні дослідження, спрямовані на зниження похибки широкосмугових подільників напруги, є важливими.

Завдання оптимального коректування низьковольтного плеча подільника напруги було вирішено за допомогою застосування елементів лінійного програмування для дослідження функції систематичної похибки.

У даній роботі представлені результати дослідження коректування амплітудно-частотної характеристики подільника напруги, які спрямовані на зниження його похибки. Для цього було знайдено такий параметр оптимізації значення ємності низьковольтного плеча, при якому абсолютне значення позитивного й негативного максимуму систематичної похибки ємнісно-омічного подільника напруги будуть однаковими. Розрахунки були виконані для різних значень коефіцієнта ділення подільника напруги. Отримані масиви даних узагальнено у вигляді тривимірних графіків.

Робота вносить вклад у подальший розвиток теорії високовольтних подільників напруги. У результаті проведених досліджень показана можливість оптимізації амплітудно-частотної характеристики широкосмугового ємнісно-омічного подільника напруги шляхом варіювання значення ємності його низьковольтного плеча. Проведені дослідження є перспективними у зв'язку з тим, що дана категорія високовольтних масштабних перетворювачів має можливість стати обов'язковою для визначення показників якості електричної енергії безпосередньо у високовольтних мережах.

**Ключові слова:** подільник напруги, амплітудно-частотна характеристика, якість електричної енергії, високовольтний масштабний перетворювач.

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### 1. Introduction

Measurement of electric power quality parameters is necessary during energy generation, distribution and consump-

tion. In addition, measurements are necessary to ensure the possibility of controlling the quality of electricity as a type of product. For this purpose, technical and organizational measures are being implemented to achieve compliance of

electric power quality parameters with the requirements of international standards, for example, IEC 61000-4-30:2015. Under modern conditions, attention to the electric power quality is constantly growing, since electric power quality determines in many cases the ability to function of many complicated devices, critical equipment and entire systems. This task requires the development of measuring instruments with an extremely low error and the ability to measure voltages over a wide frequency range. Among high-voltage scale transducers, one should mention voltage transformers and voltage dividers. For many years, the use of voltage dividers was limited to measuring various voltages [1, 2] in high-voltage laboratories. However, voltage dividers, compared to voltage transformers, are characterized by a wider bandwidth. Therefore, many researchers began consider them as one of the main means for voltage measurement in high-voltage electric networks [3–5]. The authors also believe that voltage dividers have much greater potential for improvement than instrument transformers. In this connection, studies on the possibility of using voltage dividers to measure electric power quality parameters [6–8] were started at the Department of Theoretical Electrical Engineering of the National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (Ukraine). The work being performed contains both experimental and theoretical studies on the possibility of using voltage dividers instead of voltage transformers for measuring the electric power quality parameters.

Thus, *the object of research* is the circuit diagram of a broadband capacitive-resistive voltage divider with a series-parallel connection of its resistive and capacitive components. In its turn, *the main aim of the article* is to study the adjustment of the amplitude-frequency characteristics of the voltage divider, which is aimed at reducing its measurement error.

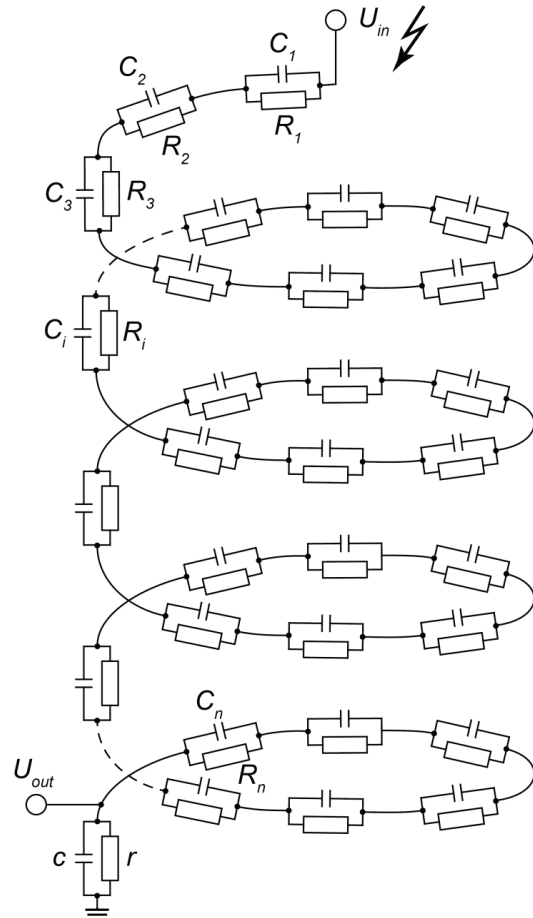
## 2. Methods of research

The voltage divider, constructed according to principle of voltage distribution over the complex impedances, is usually called a mixed capacitive-resistive voltage divider with a series-parallel connection of its resistive and capacitive components. A generalized equivalent circuit for a voltage divider of this type, which also gives a general idea of the voltage divider component layout in space, is shown in Fig. 1.

In Fig. 1:  $U_{in}$  – divider’s input voltage;  $U_{out}$  – divider’s output voltage;  $C_1, C_2, \dots, C_n$  – capacitive components of high-voltage arm;  $R_1, R_2, \dots, R_n$  – resistive components of high-voltage arm;  $c$  – capacitance of low-voltage arm;  $r$  – resistance of low-voltage arm.

There is no doubt that a careful selection of resistors and capacitors the high-voltage arm is assembled of, can improve the transfer properties of the voltage divider. In addition to the fact that such an approach is very labour-consuming, even after the most careful selection of components, there will always be some kind of non-identity of the components. For this reason, it can’t be considered that the voltage divider will consist of the same components. Hence, all components of the voltage divider will be characterized by different thermal stresses and different dependencies on atmospheric conditions (humidity, pressure). As a result of the non-identity of the high-voltage arm components, the divider’s voltage ratio, in addition to the dependence on the frequency and amplitude of the applied

voltage, also becomes dependent on the temperature, humidity, and ambient pressure. This is especially true, since modern voltage dividers are quite dimensional devices with a vertical arrangement of components. However, it is possible to reduce the negative effect of the non-identity of the high-voltage arm components by making some adjustments to the low-voltage arm components of the voltage divider [6].



**Fig. 1.** Circuit diagram for a broadband capacitive-resistive voltage divider, a lumped-component model

According to the theory of voltage dividers [9], the amplitude-frequency characteristic of the voltage divider is determined by the expression:

$$A(\gamma) = \frac{1}{K} A_1(\gamma), \% \quad (1)$$

where  $A_1(\gamma)$  – the reduced amplitude-frequency characteristic, which in its turn is determined by the following expressions:

$$A_1(\gamma) = \frac{K}{K-1} \sqrt{\frac{1}{1 + \left(\frac{\gamma}{1 + \delta' \cdot \Theta}\right)^2}} \frac{1}{\sqrt{a^2 + \gamma^2 b^2}}, \quad (2)$$

$$a = \frac{1+f}{1+\gamma^2} + \frac{1}{K-1} \frac{1}{1 + \left(\frac{\gamma}{1 + \delta' \cdot \Theta}\right)^2}, \quad (3)$$

$$b = \frac{1+\delta}{1+\gamma^2} + \frac{1}{K-1} \frac{1}{1+\frac{\delta' \cdot \Theta}{\gamma^2}} \quad (4)$$

In expressions (1)–(4):  $K$  – divider’s nominal voltage ratio;  $\gamma$  – dimensionless parameter of angular frequency  $\omega$ . Parameter  $\gamma$  is determined by the following expressions:

$$\gamma = \omega R_0 C_0; \quad R_0 = \frac{1}{n} \sum_{i=1}^n R_i; \quad C_0 = \frac{1}{n} \sum_{i=1}^n C_i,$$

where  $R_0$  and  $C_0$  – average values of resistive and capacitive components of the high-voltage arm of the voltage divider, respectively;  $n$  – the total number of these components.

It is shown in [6] that the non-identity of the resistive components of the high-voltage arm of the voltage divider is negligible compared to the non-identity of the capacitive components.

Then in the expression (3) the function  $f$  is determined by the ratio:

$$f = \frac{\gamma^2}{1+\gamma^2} \frac{1}{n} \sum_{i=1}^n \frac{\alpha_i^2 (\gamma^2 (3+2\alpha_i) - 1)}{1+\gamma^2 (1+\alpha_i)^2} \quad (5)$$

In its turn, in expression (4), the function  $\delta$  is determined by the expression:

$$\delta = \frac{\gamma^2}{1+\gamma^2} \frac{1}{n} \sum_{i=1}^n \frac{\alpha_i^2 (\gamma^2 (1+\alpha_i) - 3 - \alpha_i)}{1+\gamma^2 (1+\alpha_i)^2} \quad (6)$$

In the two above expressions parameter  $\alpha_i$  depends on the capacitance values of the high-voltage arm of the voltage divider (refer to Fig. 1) as follows:

$$\alpha_i = \frac{C_i - C_0}{C_0}.$$

The parameter  $\delta'$  in (2)–(4) corresponds to the maximum value of  $\delta$  (6) when  $\gamma \rightarrow \infty$ , that is:

$$\delta' = \frac{1}{n} \sum_{i=1}^n \frac{\alpha_i^2}{1+\alpha_i}.$$

The selection of the low-voltage arm components is being performed according to the common expressions:

$$r = \frac{nR_0}{K-1}, \quad c = \frac{C_0}{n}(K-1), \quad (7)$$

what corresponds to the value of  $\Theta = 0$  in formulas (2)–(4). Herewith, this case corresponds to the absence of the low-voltage arm adjustment of the voltage divider.

The value of  $\Theta = 1$  in formulas (2)–(4) corresponds to the maximum (or, in other words, ultimate) adjustment of the low-voltage arm capacitance of the voltage divider, which takes the value:

$$c' = \frac{C_0}{n} \frac{K-1}{1+\delta'}. \quad (8)$$

As preliminary calculations show that the use of the maximum adjustment of the low-voltage arm capacitance allows reducing the maximum value of the systematic error of the voltage divider by more than 2 times:

$$\Delta_A = (A_1 - 1) \cdot 100, \quad \% \quad (9)$$

However, this maximum value  $\Delta_A$ , in its turn, can again be reduced by almost 2 times by choosing the optimal parameter value  $0 < \Theta_{opt} < 1$ . The task of the work is to search for this optimal value  $\Theta_{opt}$ , as well as to study the dependence of the «minimized» error of the voltage divider on a number of factors.

It should be noted that the optimized value of the low-voltage arm capacitance of the voltage divider can be determined by the ratio:

$$C_{opt} = \frac{C_0}{n} \frac{K-1}{1+\Theta_{opt} \delta'}, \quad (10)$$

and its amplitude-frequency characteristic is determined by the expressions (1)–(4) when substituting  $\Theta = \Theta_{opt}$ .

As a model of capacitive components’ non-identity of the high-voltage arm of the voltage divider, a symmetric «triangular» distribution is used [6]. For this distribution the maximum deviations of the capacitances  $C_i$  from  $C_0$  are characterized by the ratio:

$$C_i = C_0 (1 \pm \Delta_C), \quad (11)$$

where  $\Delta_C$  – the parameter specified in this research, which can take values in the range from 0 to 0.2.

All the above formulas form a mathematical background for the research. In general, the graph of the systematic error of the capacitive-resistive voltage divider is a multimodal function containing both a positive and a negative maximum. The task of the research is to find a value of the parameter  $\Theta$ , at which the absolute value of the positive and negative maximum will be the same. Such an adjustment of the low-voltage arm is called optimal.

This problem is solved using linear programming. The key fragment of the program for finding the optimal parameter value  $\Theta$  is shown in Fig. 2.

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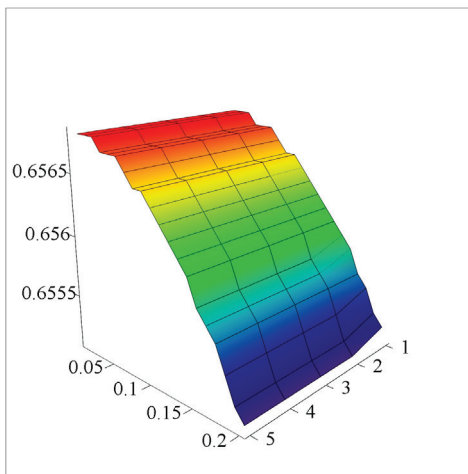
Θ := K1 ← 101
maxA ← 1
minA ← 0.1
Θ ← 0.63
while  $\frac{-\max_A}{\min_A} < 0.9999$ 
  for i ∈ 1..10000
    | γi ← 0.01 · i
    | Aresi ← (A(K1, γi, δs · Θ) - 1) · 100
  maxA ← Ares1
  for i ∈ 1..10000
    maxA ← Aresi if maxA < Aresi
  minA ← Ares1
  for i ∈ 1..10000
    minA ← Aresi if minA > Aresi
  Θ ← Θ + 1 · 10-4
return Θ
    
```

**Fig. 2.** Program for optimizing the amplitude-frequency characteristic

Programming is performed with a help of the Mathcad software [10]. The program works as follows. First, the program searches for the maximum of the systematic error curve. Then, the program searches for the minimum of the systematic error curve. The program then compares the absolute values of the maximum and minimum. If these values are different, the variable  $\Theta$  increments 0.0001 and the search loop repeats. The loop will end when equality is reached between the absolute value of the maximum and minimum of the studied function. The program allows finding the value of  $\Theta_{opt}$  accurate to the fourth digit after the decimal point. The results of calculations obtained with a help of this program are given in the next section.

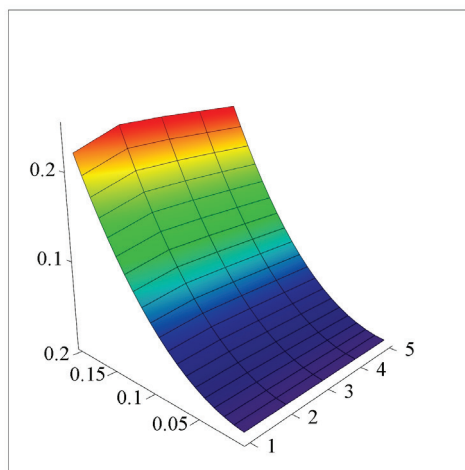
### 3. Research results and discussion

The calculations were performed for various values of the divider's voltage ratio ( $K=10^1; 10^2; 10^3; 10^4; 10^5$ ) and for various values of the maximum deviation of the high-voltage arm capacitances from the average value ( $\Delta_C = 0.01 \dots 0.20$ ). Under such conditions, authors obtained surface graphs of the variable  $\Theta_{opt}$  (refer to Fig. 3) and the systematic error  $\Delta_A$  (refer to Fig. 4).



$(K, \Delta_C, \Theta_{opt})$

**Fig. 3.** Three-dimensional graph of the variable  $\Theta_{opt}$



$(K, \Delta_C, \Delta_A)$

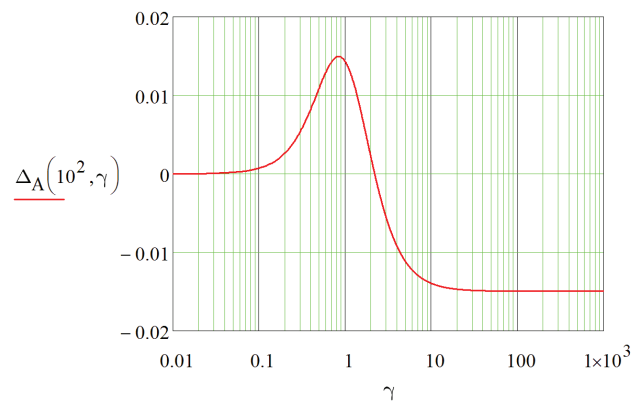
**Fig. 4.** Three-dimensional graph of the systematic error  $\Delta_A$  (%) of the capacitive-resistive voltage divider

In Fig. 3 and Fig. 4, for the divider's voltage ratios  $K$  the logarithmic scale is used (the orders of magnitude of the divider's voltage ratio are plotted along the axis).

The surface graph in Fig. 3 depicts what value a variable  $\Theta_{opt}$  should have so that for given  $K$  and  $\Delta_C$  the absolute values of the maximum and minimum of the systematic error are the same.

The surface graph in Fig. 4 depicts that with an increase in the  $\Delta_C$  parameter value, almost independently of the value of the divider's voltage ratio  $K$ , the value of the amplitude error  $\Delta_A$  increases in a parabolic dependence on  $\Delta_C$ . The graphs in Fig. 3 and Fig. 4 summarize a huge array of computational data.

Let's show in more details one of the results of optimizing the amplitude-frequency characteristics of the voltage divider in Fig. 5. This graph shows the dependence of the systematic error  $\Delta_A$  (%) on the generalized parameter  $\gamma$ . The shape of the curve is practically the same in the entire studied range of parameters  $\Delta_C, K, \Theta_{opt}$ . Only the absolute values of the maximum and minimum differ.



**Fig. 5.** Example of the systematic error graph  $\Delta_A$  (%) obtained after optimization by adjusting the low-voltage arm of the voltage divider

Functional dependence in Fig. 5 is obtained for  $\Delta_C = 0.05$  and  $K = 10^2$ . Using the above search algorithm, a value  $\Theta_{opt} = 0.656$  is obtained. With this optimal value the absolute values of the maximum and minimum amplitude errors are the same and equal to 0.01495 %.

The further development of the theory of voltage dividers is promising due to the fact that this category of high-voltage scale transducers has the potential to become mandatory for determining the quality parameters of electric energy directly at high voltage. One of the catalysts for this may be the intensive development of the Smart Grid concept, which requires new, more advanced means of monitoring the quality of electric power [11, 12]. Therefore, experimental and theoretical studies aimed at reducing the error of broadband voltage dividers are important.

### 4. Conclusions

As a result of the performed research, the possibility of optimizing the amplitude-frequency characteristics of a broadband capacitive-resistive voltage divider by varying the value of its low-voltage arm capacitance is shown.

The optimization parameter of the low-voltage arm capacitance value of the voltage divider in the entire range of the studied parameters  $K=10^1-10^5, \Delta_C=0-0.2$  can be characterized by a constant value  $\Theta_{opt} = 0.656$ .

The dependence of the systematic error of the optimized voltage divider on the dimensionless frequency parameter  $\gamma$  is universal in form with the difference in the maximum ultimate values of  $\Delta_A$ .

For the values of divider's voltage ratio in the range  $1 < K < 10$ , an additional research is required to optimize the amplitude-frequency characteristic.

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