UDC 338.26:519.863 JEL Classification: C44, R58 DOI: 10.15587/2706-5448.2020.210848

Domina O. SELECTION OF ALTERNATIVE SOLUTIONS IN THE OPTIMIZATION PROBLEM OF NETWORK DIAGRAMS OF PROJECT IMPLEMENTATION

The object of research is a model network diagram of the project implementation in the form of a diagram, in which individual operations are represented by arcs, and vertices are considered as events corresponding to the beginning and end of operations. The term «model» is understood as an arbitrary network diagram, in relation to the parameters of which the solution is already known.

One of the most problematic areas is the lack of substantiated methods of formalizing the task of optimizing network diagrams, allowing for a targeted selection of early start dates for individual operations within the project, ensuring optimal solutions for the selected criteria. In this study, such criteria are the possibility of shifting the operation with the maximum intensity in a given time interval, and the uniformity of the workload of personnel, assessed by the ratio of the maximum and minimum intensities $y=q_{max}/q_{min}$ for the analyzed version of the network diagram.

Methods of network planning and management, mathematical experimental planning and optimization methods are used.

The results obtained confirm the possibility of using the proposed methods for solving optimization problems with respect to arbitrary network diagrams. This is due to the fact that the proposed methods for optimizing network diagrams allow one to obtain regression equations that serve as mathematical models for making targeted decisions on the choice of early start dates for operations that provide the best results in relation to the selected optimization criteria for network diagram.

The features of the proposed solutions are: conditions for the selection of input variables, a procedure for the targeted selection of early start dates for operations with the highest load intensity of personnel, and providing optimal solutions for the criterion of minimizing uneven load. Thanks to these features, it is possible to optimize network diagrams with arbitrary source data. To do this, it is enough to apply the proposed procedure for your version of the network diagram, having previously estimated possible alternatives with respect to the choice of significantly influencing input variables and the intervals of their variation.

Keywords: *network diagram, alternative solutions, central compositional orthogonal design, peak load intensity,* uneven workload of personnel.

Received date: 30.04.2020 Accepted date: 26.05.2020 Published date: 31.08.2020

1. Introduction

Methods of network planning have great potential in the design of processes for the creation and implementation of innovative products, which is explained by their adaptability and the possibility of widespread use of computer equipment and technologies [1]. Among the advantages of these methods can be noted the possibility of information support for investment decisions, which allows optimal management of investment projects at all stages of their implementation. In turn, effective planning and management of the investment design process reduces the total duration of the entire complex of work on the project [2]. The use of the network planning method for the goals and objectives of an investment industrial project is justified by insufficient elaboration of the adaptation of methodological issues of Copyright © 2020, Domina O. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0)

organizing technological processes to the socio-economic measures of the investment project. The scientific and practical significance of the results of constructing network models of the project is associated with the possibility of practical application of the graphical analytical method in the preparation of technical and economic documentation for an investment project [3].

The use of network planning for managing investment projects allows to optimize the management of the investment projecting process [4], and the development of a universal mechanism for making managerial decisions in investment planning makes it possible to reduce the implementation period and obtain savings in financial resources [5].

In addition to managing investment projects, network planning methods are used today to address emergency response issues, in particular when carrying out preventive measures, solving operational problems and training specialists in the field of emergency protection [6]. The planning of educational processes is also the domain of these methods. The use of a network model with a deterministic structure and duration of disciplines in planning the educational process allows to draw up a calendar plan and obtain operational data on possible intersections of disciplines and software in the educational process [7].

However, networking methods are not limited to project management. They are also used in the design of automated control systems (ACS) for organizing information transfer by minimizing the time characteristics and cost of the telecommunication data exchange network (TDEN) in order to obtain the maximum effect [8]. In this case, as the authors of this work note, the TDEN is represented by a network of events and operations, therefore, it can be considered as a model of the information transfer process that visualizes the relationship of all its elements. Considering network methods from such positions, it can be assumed that they become important in the implementation of the Industry 4.0 concept in the part that concerns the exchange of information and the timing of production processes. In this way, cyber-physical systems (CPS) design tools are added as objects of the core Industry 4.0 technology, in particular, planning algorithms for time-synchronized information flows. In particular, it is possible to talk about time-sensitive networks (TSN) in the context of the implementation of the concept of the Industrial Internet of Things (IIoT) [9]. So, for example, in [10] in the context of Industry 4.0, the concept of network related industries is used, the schemes of which are represented by the links of individual industries, functioning synchronously and adapting to external conditions. The use of such a network view is illustrated by several examples of three business models for the stock industry [11] and e-commerce systems in the distribution channel of the wine industry with a focus on the North American market [12].

Finally, it is important to note the possibility of considering network methods as part of information modeling technology, with the peculiarity that all elements of a complex of models must be connected by certain dependencies. This concept is implemented, for example, within the framework of information modeling in construction (BIM) technology [13]. The approach outlined in this work reveals the possibilities of using the MSProject program for calculating network diagrams of linear ob-

jects, variant design of construction organization, calculating resource provision throughout the construction period, reflecting the real progress of work and operational management of construction.

The application of this concept is multifaceted and may include scheduling construction using activity execution workspaces (AEWs). At the same time, AEW is managed by integrating workspace management with the current planning process and BIM data in a 4D/5D environment, providing visualization of the management process [14, 15]. In [16], this idea was developed by developing a system for checking workspace conflicts based on the integration capabilities of special algorithms. The use of such algorithms includes automatic creation

- 10

of workspace models and automatic check of workspace conflict in the 4D simulator. This allows the project manager to identify a workspace conflict using the described system, and overlapping activities can be redesigned to minimize conflict.

Therefore, in the context of such approaches, it is relevant to obtain the dependencies of individual elements of network models, which make it possible to evaluate the main functional indicators and solve the problems of optimal planning and organization of project implementation processes.

2. The object of research and its technological audit

The object of research is a model network diagram of the project implementation, represented by a diagram, in which individual operations are represented by arcs, and the vertices are considered as events corresponding to the beginning and end of operations. In this case, the term «model» means an arbitrary network diagram, in relation to the parameters of which the solution is already known.

The subject of research is optimization procedures. Therefore, an arbitrary network diagram can be taken as the initial one. Initial data for determining its parameters are formed according to the technological audit, if to talk about a project of a technological nature or a production system.

If to talk about any other project, including an educational one, the initial data are formed on the basis of the judgments of the experts participating in the project. The diagram given in [17] is chosen as such a network diagram. It is considered as the starting point for the implementation of the optimization procedure, since the early start dates for each subsequent operation correspond to the early end dates for the previous operation. This condition is equivalent to the following equality:

$$t_k^0 = t_k' - \tau_k,\tag{1}$$

where k – index of the current operation; t_k^0 – early start date of the current operation; t'_k – early completion date of the current operation; τ_k – duration of the current operation.

The model network diagram is shown in Fig. 1. Initial data and calculated parameters are given in Tables 1, 2.



Fig. 1. Model network diagram

Table 1

Table 2

Initial data for calculating the parameters of the model network diagram

-	4	(U
Operation identifier	Duration, time unit	Immediately preceding operations	Intensity, person/time unit
A	1	_	8
В	4	А, С, D	13
С	1	-	20
D	1	-	20
E	2	-	4
F	3	D, E	16
G	4	F	5
Н	1	С	4
Ι	5	B, G, N	3
К	2	С	2
L	2	Н	6
М	2	Н	2
N	4	А, D	1
Р	2	F	10
Q	6	-	10
R	3	Q	7

Calculation of the model network diagram parameters

Operation	Initial data for calculating Calculate calculate calculate	ulated o ramo	diagram pa- neters						
identitier	q_k	τ_k	ω'_k	ω_k''	t'_k	t_k''	Δτ	t_k^0	
D	20	1	-	B, N, F	1	2	1	0	
С	20	1	-	В, Н,К	1	5	4	0	
Е	4	2	-	F	2	2	0	0	
А	8	1	-	B, N	1	5	4	0	
F	16	3	D, E	G, P	5	5	0	2	
Н	4	1	С	L, M	2	12	10	1	
Q	10	6	-	R	6	11	5	0	
N	1	4	A, D	Ι	5	9	4	1	
G	5	4	F	Ι	9	9	0	5	
В	13	4	A, C, D	Ι	5	9	4	1	
R	7	3	Q	-	9	14	5	6	
Р	10	2	F	-	7	14	7	5	
М	2	2	Н	-	4	14	10	2	
L	6	2	Н	-	4	14	10	2	
К	2	2	С	-	3	14	11	1	
Ι	3	5	B, G, N	-	14	14	0	9	
Table	Table 2 the following symbols are used: a page								

Table 2 the following symbols are used: q_k – operation intensity; t''_k – late completion date of the current operation; ω'_k – immediately preceding operations; ω'_k – immediately subsequent operations; $\Delta \tau$ – time reserve for the operation. The rest of the designations correspond to those taken in formula (1).

The ranking of operations by importance is performed based on the calculation of the weights of operations according to the principle: the weight of an operation is equal to the number of operations following it plus the weight of each of these operations.

3. The aim and objectives of research

The aim of research is to select a method for the analytical description of the quality of the network diagram as a function of the time characteristics of operations. The presence of such a reasonable description should allow to purposefully select alternative solutions for planning and organizing the project, optimal in terms of resource allocation.

To achieve the aim, the following objectives have been set:

1. Select the output variables that can serve as criteria for the quality of the network, and the criteria for its optimization.

2. Select input variables that significantly affect the network quality.

3. Conduct numerical experiments to build adequate mathematical models describing the effect of time characteristics of operations on the selected criteria for the quality of the network diagram.

4. Find optimal solutions that allow to determine the start times of current operations that significantly affect the quality criteria of the network diagram, in terms of time constraints on the deadlines for performing operations within the framework of the entire project.

4. Research of exiting solutions of the problem

In [18], the author understands the optimization of the network diagram as a sequential adjustment of the network to achieve the most effective results and specified parameters in terms of time and resources. In this context, the problem of optimizing network models in terms of time and resources is being solved. In the first case, it is proposed to use methods for reducing the duration of critical works, dividing critical works and their parallelization, changing the topology by changing the work technology. In the second case, it is proposed to consider as priority, in terms of optimization, the distribution of labor resources and the distribution of material resources. It is concluded that multivariate recalculations of the network diagram during its optimization are expedient to be carried out on an electronic computer (PC). However, in this work, there are no models that allow evaluating the proposed solutions from the point of view of their optimality, and the corresponding mathematical apparatus for computer software, which allows multivariate recalculations of the network diagram when optimizing it. The author limits itself only to staged definitions and gives only the initial and final network diagrams without describing the methods and stages of solving the problem.

In [19], the methods of the set-theoretic approach and the theory of directed diagrams were used to solve the problem of simplifying the structure of complex work-arc network diagrams at the design and optimization stages. A feature of the solutions obtained was the transformation of the network diagram «work – tops» into the network diagram «work – arcs», in order to reduce the number of fictitious jobs. They illustrate the principle of structural transformation, which is an alternative approach to the formation of the principle of choosing alternatives in the context of optimizing the original network diagrams.

In [20], for a generalized stochastic network diagram with a given multidimensional density of distribution of arc travel times, a technique of analytical-simulation modeling is proposed. It was aimed at calculating the probabilities of the realization of all critical paths, the distribution function and the first two moments of the diagram execution time. The technique is based on the construction of systems of inequalities that describe the relationship between the travel times of arcs. As noted in this work, it allows to perform analytical and simulation research in order to obtain the probabilistic parameters of the generalized stochastic network diagram of interest to the user. However, the work did not highlight aspects related to the choice of parameters related to optimization.

The novelty of work [21] the author considers the use of methods of network planning and management for deterministic economic and mathematical modeling of investment design processes. Using the example of the process of creating a restaurant business, it is shown that the field of investment design is one of the areas of application of network planning methods. However, this work does not contain a description of the principles for obtaining solutions, limiting itself to the well-known procedure for constructing network diagrams with minimization in terms of the cost parameter.

In [22], an approach to modeling a network planning system based on the use of information technology (IT) for building a planning system in the Microsoft Project environment is presented. In particular, the issues of building network plans under risk conditions are considered and the results of optimization of the network diagram are given according to the criterion of minimizing the time spent on performing individual stages. The proposed solutions, as noted by the author of this work, allow reducing the costs of implementing the planning system, optimizing its parameters, and choosing a realistic scenario of the firm's behavior under risk conditions. However, this work does not describe the principles of formalizing the problem being solved, highlighting only the IT side of its solution.

In [23], the objective function is the sum of the durations of all critical jobs, which should tend to a minimum. A feature of the results obtained in this work is the revealed pattern associated with the fact that a conflict situation may take place. It manifests itself in the fact that in the process of improving the network diagram, new critical paths should not arise and, at the same time, funds should be redistributed in the best way. From this it is concluded that the output can be such a configuration of costs for each of the processes, in which all the work of the network diagram is critical. Thus, a compromise is reached: the original critical path is preserved, and further reallocation of resources is impossible. The solution found in this way can be considered final and optimal under the given conditions. From this, the author concluded that the most important in the optimization process is the condition, the fulfillment of which ensures the equality of all paths of the network diagram. At the same time, the author noted that the model of minimizing the duration of a project with a fixed budget for its implementation can't always be technically implemented. This is related to the problematic part in relation to the peculiarities of the solution of this problem, as a mathematical programming problem, which may not have solutions under the chosen real constraints. With regard to its presence, it should be noted that this work does not solve the problem. This is due to the fact that it contains only a general description in the formulation of the optimization problem, but does not offer specific examples that allow one to evaluate the quality of the proposed solutions when using the objective function as the sum of the durations of all works lying on the critical path.

The analysis of the above works allows to conclude that the issues of obtaining a mathematical description of the quality of the network diagram, for its subsequent optimization, are not given due attention.

In [24], an algorithm for optimizing the plan for performing a complex of works is proposed, a feature of which is the possibility of using interchangeable resources. The optimization problem is solved in relation to the smallest increase in the total cost of work while reducing the period of their implementation. However, the proposed minimization principles do not contain the solution of the optimization problem based on the preliminary construction of a mathematical model.

In work [25], the concept of integral risk analysis is introduced, which implies optimization of the diagram of an innovative project according to several criteria. Using the basic processes of risk management [26], and noting that the risk management process should be continuous [27], the author proposed several scenarios for diagram optimization:

- without implementing a preliminary risk reduction plan;
 with the implementation of the preliminary plan to reduce risks;
- with synchronous risk justice.

It is emphasized that the choice of the most preferable option depends on the degree of risk acceptance by the project managers, and the maximum permissible level of risk tolerance must be established. However, it should be borne in mind that the above analysis of changes in costs and time for the implementation of the project within the framework of solving the optimization problem should take into account not only the analysis of micro-, but also macro-risks. That is, it is necessary to receive an answer to the question whether it is worth initiating the project at all and continuing to work on it. Analyzing this work, it can be assumed that it would be important to have models that allow one to assess the influence of various potentially significant factors on the optimization results.

Following the results of the analysis of the above works, there is reason to conclude that the issues of obtaining a mathematical description of the quality of the network diagram, for its subsequent optimization, are not given due attention. The presence of such a description in the form of models describing the influence of significant factors on the behavior of the variables being optimized would make it possible to purposefully select measures for transforming network diagrams.

5. Methods of research

The working hypothesis of the study is that the methods of mathematical planning of the experiment can be used to select the method of analytical description of the quality of the network diagram. Based on the fact that the start time of the operation can vary arbitrarily within the allowable ranges of the early and late completion of operations, taking into account their duration, there are grounds for the implementation of designs for a full factorial active experiment. Moreover, these plans can be considered as orthogonal, therefore it is possible to talk about the construction of central orthogonal compositional design, the general view of which is presented in Table 3, and the parameters in Table 4 [28]. In this case, the analytical description of the quality of the network diagram is a regression equation.

Structural location of experime	No of												
	experiment			Design 2	X matrix		2 0		2 0				
the D	CAPETIMON	x ₀	<i>x</i> ₁	x 2		Xn	$x_1^2 - \beta$		$x_n^2 - \beta$	$x_1 x_2$		$\boldsymbol{X}_{n-1}\boldsymbol{X}_n$	
	1	+1	+1	+1		+1	1–β		1–β	+1			
	2	+1	-1	+1		+1	1–β		1–β	-1			
	3	+1	+1	-1		+1	1–β		1–β	-1			
	4	+1	-1	-1		+1	1–β		1–β	+1			
Core	5	+1	+1	+1		+1	1–β		1–β				
	6	+1	-1	+1		+1	1–β		1–β				
	2 ^{<i>n-p</i>}	+1					1–β		1–β				
	2 ^{<i>n</i>-<i>p</i>} +1	+1	$+\alpha$	0		0	$\alpha^2 - \beta$		-β	0		0	
	2 ^{<i>n</i>-<i>p</i>} +2	+1	-α	0		0	$\alpha^2 - \beta$		-β	0		0	
		+1	0	+α		0	-β						
		+1	0	-α		0	-β						
Star point													
		+1	0	0		+α	-β		α²-β	0		0	
	2 ^{<i>n-p</i>} +2 <i>n</i>	+1	0	0		-α	-β		α²-β	0		0	
Center	N=2 ^{n-p} +2n+1	+1	0	0		0	_β		_β	0		0	

t

The matrix F of the central orthogonal compositional design in general form

Table 3

 Table 4

 Central orthogonal composite design parameters

Dimen-	Dimen- Core		~	ß	Dispersion matrix elements				
sion	LUIE	14	u	þ	CO	<i>C</i> ₁	C2	C3	
2	2 ²	9	1	0.6667	0.1111	0.1667	0.5	0.25	
3	2 ³	15	1.215	0.73	0.0667	0.0913	0.2298	0.125	
4	24	25	1.414	0.8	0.04	0.05	0.125	0.0625	
5	25-1	27	1.547	0.77	0.03704	0.0481	0.0871	0.0625	
6	26-1	45	1.722	0.843	0.0222	0.0264	0.0564	0.03125	
7	2 ⁷⁻¹	79	1.885	0.9	0.0127	0.0141	0.0389	0.0156	

The elements of the dispersion matrix given in Table 4 are used to estimate the variances of the coefficients of the regression equation:

$$s_{i}^{2} = \begin{cases} s^{2}c_{0}, i = 0, \\ s^{2}c_{1}, i = 1, \dots, n, \\ s^{2}c_{2}, i = n + 1, \dots, 2n, \\ s^{2}c_{3}, i = 2n + 1, \dots, k, \end{cases}$$
(2)

where s^2 – estimate of the inade quacy variance of the regression equation.

The use of just such an estimate of the variance is explained as follows. A numerical experiment, in which the timing of the completion of operations is varied and the value of the selected quality indicator of the network diagram is determined for each experimental point, does not allow parallel experiments. Therefore, it is not possible to choose a variance estimate of experimental errors as the value s^2 .

The network parameters are determined using the following formulas:

$$t'_{k} = \max(t'_{i} + \tau_{k}), \tag{3}$$

$$t_k'' = \min\left(t_j'' - \tau_j\right),\tag{4}$$

$$\Gamma = \max_{k} t'_{k},\tag{5}$$

$$t_k^0 = t_k' - \tau_k, \tag{6}$$

$$t_k^0(\min) = t_k' - \tau_k,\tag{7}$$

$$t_k^0(\max) = t_k'' - \tau_k. \tag{8}$$

In formulas (3)–(8), the following designations are used: i – index indicating the previous operation; j – index indicating the subsequent operation; τ – operation duration; T – target date of the project; $t_k^0(\min)$ – the lower value of the variation range of the input variable, which is the moment when the current operation starts; $t_k^0(\max)$ – the upper value of the variation range of the input variable, which is the moment when the current operation starts.

The value $t_k^0(\min)$ can be chosen either equal to the early start of the current operation t_k^0 , in the case of constructing a design with a core 2^2 , or a value that provides the possibility of carrying out a numerical experiment at a star point (Tables 3, 4) so that this point corresponds to the value t'_k .

The value $t_k^0(\min)$ can be selected taking into account the time reserve for the corresponding operation. In this case, this value can correspond either to an equal difference $(t_k''-\tau_k)$, in the case of constructing a design with a core 2^2 ,

6. Research results

6.1. Selecting output variables. The output variables in the work are those variables, the value of which should be improved in the process of optimizing the network diagram. Improved means the ability to achieve a given optimization goal:

- goal 1: the possibility of shifting the operation with the maximum intensity in a given time interval – in this case, the output variable is estimated by the number of the operation execution time period; – goal 2: the uniformity of the workload of personnel, assessed by the ratio of the maximum intensity to the minimum intensity $y=q_{\text{max}}/q_{\text{min}}$ for the analyzed version of the network diagram.

Goal 1 is relevant for the case when personnel to perform operations are outsourced or can be employed to perform similar work for another employer.

Goal 2 is relevant for the case when the maximum load in a certain period can cause negative consequences in the organizational part of the project, or when it is difficult to ensure funding for the execution of operations in full for such «overloaded» periods.

6.2. Selection of input variables that significantly affect the quality of the network diagram. As input variables, the value of the times of the beginning of the execution of those operations that satisfy the following requirements is chosen:

- intensity of the operation is maximum;
- number of immediately preceding operations is minimal;
 time reserve for the operation is sufficient for variation
- time reserve for the operation is sufficient for variation.
 Input variables are normalized:

$$x_{inorm} = \frac{2x_i - (x_{i\max} + x_{i\min})}{x_{i\max} - x_{i\min}}, \ i = 1, 2, ..., N, \ j = 1, 2, ..., n,$$
(9)

where x_i – value of the *i*-th input variable in natural form; x_{imax} – the maximum value of the *i*-th input variable in the selected area of variation in natural form; x_{imin} – the minimum value of the *i*-th input variable in the selected range of variation in natural form; x_{inorm} – the value of the *i*-th input variable in normalized form. Formula (9) performs the procedure for normalizing the values of input variables, transforming their absolute values into a dimensionless range [-1; +1].

Table 5 illustrates the principle of selecting input variables based on the data in Table 2. The following color highlights are adopted:

- red and pink colors - alternatives No. 1 and No. 2, respectively, for shifting operations to optimize the network diagram;

- orange – dependent input variables – the corresponding operations are shifted depending on the position of the previous operations selected for evaluating the independent input variables for alternatives No. 1 and No. 2.

Table	5
-------	---

Selection of input variables and variation intervals for the model network diagram

		•									
Work identifier	q_k	τ_k	ω'_k	ω_k''	t'_k	t_k''	Δτ	t_k^0	$t_k^0(\min)$	$t_k^0(\max)$	$t_{k}^{0}\left(0\right)$
D	20	1	-	B, N, F	1	2	1	0	-	-	-
С	20	1	-	В, Н, К	1	5	4	0	0	4	2
Е	4	2	-	F	2	2	0	0	-	-	-
А	8	1	-	B, N	1	5	4	0	-	-	-
F	16	3	D, E	G, P	5	5	0	2	-	-	-
Н	4	1	С	L, M	2	12	10	1	_	-	-
Q	10	6	-	R	6	11	5	0	0	4	2
N	1	4	A, D	Ι	5	9	4	1	-	-	-
G	5	4	F	I	9	9	0	5	-	-	-
В	13	4	A, C, D	Ι	5	9	4	1	_	_	-
R	7	3	Q	-	9	14	5	6	_	-	-
Р	10	2	F	-	7	14	7	5	5	11	8
М	2	2	Н	-	4	14	10	2	-	-	-
L	6	2	Н	-	4	14	10	2	_	-	-
К	2	2	C	-	3	14	11	1	_	_	-
Ι	3	5	B, G, N	-	14	14	0	9	-	-	-

Table 5 shows that there are two alternatives for transforming the network diagram in order to optimize it:

- alternative No. 1: operation C, which has the maximum intensity value $(q_k=20)$, is subject to a time shift; - alternative No. 2: the operations Q and P, having the total maximum intensity value $(q_k=20)$, are subject to time shift.

Therefore, the following ranges of variation are selected: x=(0;4) – for alternative No. 1, $x_1=[0;4]$, $x_2=[5;11]$ – for alternative No. 2.

6.3. Numerical experiments. The construction of the regression equation for alternative No. 1 is carried out on the basis of the application of the D-optimal design [28], which allows calculating the estimates of the coefficients of the equation in such a way as to minimize the determinant of the variance matrix:

$$\mathbf{A} = \frac{1}{3} \mathbf{D} \mathbf{F}^{\mathrm{T}} \mathbf{Y},\tag{10}$$

where ${\bf A}$ – matrix of estimates of the regression equation coefficients; ${\bf D}$ – normalized variance matrix for design F:

$$\mathbf{D} = \begin{pmatrix} 3 & 0 & -3 \\ 0 & 1.5 & 0 \\ -3 & 0 & 4.5 \end{pmatrix},$$

 $\mathbf{F}^{\mathbf{T}}$ – transposed design matrix:

$$\mathbf{F} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{F}^{\mathrm{T}} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix},$$

 \mathbf{Y} – column vector of the values of the output variable, calculated on the basis of the construction of transformed

network diagrams (Fig. 2-6 – for the case when 1 month is taken as a unit of time for performing an operation).



Fig. 2. Distribution of the intensity of operations at point (-1) of the D-optimal design



Fig. 3. Network diagram for point (+1) of the D-optimal design



Time intervals

Fig. 4. Distribution of the intensity of operations at point (+1) of the D-optimal design



Fig. 5. Network diagram for point (0) of the D-optimal design



Fig. 6. Distribution of the intensity of operations at point (0) of the D-optimal design

For the diagrams describing the intensity distribution, the following color designations are adopted: red – the interval with the maximum intensity, gray – the interval with the minimum intensity, green – the value of the output variable.

For the design point (-1), the network diagram has the form shown in Fig. 1, since $t_k^0(\min)=0$ (Table 5), i. e., the early start date of the operation C corresponds to the calculated value t_k^0 of the original network diagram.

From Fig. 2, 4, 6 it can be seen that the matrix \mathbf{Y} has the form:

$$\mathbf{Y} = \begin{pmatrix} 20.667\\ 16.667\\ 15.667 \end{pmatrix}$$

Therefore, the matrix of estimates of the coefficients **A** has the form:

$$A = \begin{pmatrix} 16.667 \\ -2.5 \\ 1.5 \end{pmatrix},$$

and the regression equation has a normalized form:

$$y = 16.67 - 2.5x_{norm} + 1.5x_{norm}^2.$$
⁽¹¹⁾

An analysis of equation (11) shows that the minimum of the output variable is achieved not within the variation range of the input variable, but at its upper boundary, at x=4. This suggests that the use of alternative No. 1 does not give a better result than $y_{\min}=15.667$, i. e., the possibilities of alternative No. 1 in the sense of achieving an optimal solution for goal 2 are limited. However, if goal 1 is pursued and there is a need to shift the maximum intensity from time point 1 to time point 5, then the use of this alternative is effective. It may be interesting to answer the question of what results can be obtained if to accept a different unit of time.

In this regard, numerical modeling was carried out for the case when 1 quarter is taken as a unit of operation time. The results are shown in Fig. 7-11.









ISSN 2664-9969



Fig. 9. Distribution of the intensity of operations at point (O) of the D-optimal design in the case when 1 quarter is taken as a unit of time for performing the operation

Using the D-optimal design, the following regression equations are obtained in natural (12) and normalized (13) forms:

$$y = 16.222 + 0.3333x - 0.2778x^2, \tag{12}$$

$$y = 15.7778 - 1.5556x_{norm} - 1.11111x_{norm}^2.$$
 (13)

To check the quality of the regression equation, experiments are performed at points (-0.5) and (+0.5) of the D-optimal design (Fig. 10, 11, respectively).

The variance of the estimate of the output variable is determined by the formula:

$$s_{u}^{2}(\mathbf{x}) = \mathbf{f}^{\mathrm{T}}(\mathbf{x})(\mathbf{F}^{\mathrm{T}}\mathbf{F})^{-1}\mathbf{f}(\mathbf{x})s^{2}, \qquad (14)$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}, \quad \mathbf{f}^{\mathsf{T}}(\mathbf{x}) = \begin{pmatrix} 1 & x & x^2 \end{pmatrix},$$

x=-0.5 (when calculating s_y^2 at the point x=-0.5) and x=+0.5 (when calculating s_y^2 at the point x=+0.5); s^2 – variance estimate characterizing the error of the regression equation (13), calculated as follows:

$$s^{2} = \frac{\sum_{i=1}^{n} (y_{ic} - y_{ie})^{2}}{\varphi},$$
(15)

where n=5, $\varphi=2$, y_{ic} – value of the output variable calculated by equation (13); y_{ie} – the experimental value of the output variable, determined based on Fig. 10, 11.

Taking these data into account, the following results are obtained: $s^2 = 0.138894$, $s_y^2(-0.5) = s_y^2(+0.5) = 2.15625$.



Fig. 10. Network diagram for point (-0.5) of the D-optimal design



Fig. 11. Network diagram for point (+0.5) of the D-optimal design

Fig. 12 is a graphical description of equation (13) with the results of checking its quality. The red dots correspond to the experimental results at the points (-0.5) and (+0.5).



Fig. 12. Graphical description of equation (13) with the results of checking its quality

The results obtained allow to conclude that the quality of equation (13) allows to deliberately shift the timing of the start of operations, ensuring a decrease in the unevenness of the intensity distribution. However, even in this case, the result obtained can't be considered optimal, since the minimum of the output variable is reached at the upper boundary of the input variable: $y_{min}=13.111$.

Consequently, the possibilities of alternative No. 1 are limited in terms of maximizing the uniformity of efficiency distribution. But it can be useful if it is necessary to shift the intensity peak to a different time interval.

6.4. Finding optimal solutions. The use of alternative No. 1 does not allow obtaining optimal solutions understood in the sense of minimizing $y=q_{\text{max}}/q_{\text{min}}$. Therefore, optimal solutions should be sought using alternative No. 2. Considering that the number of independent variables is equal to two, a two-factor central orthogonal design is used (Table 6), which is simplified in relation to Table 3 view.

 Table 6

 Central orthogonal design for two independent variables

 using alternative No. 2

No. of experiment	<i>x</i> 0	<i>x</i> ₁	x 2	$x_1^2 - \beta$	$x_2^2 - \beta$	<i>x</i> ₁ <i>x</i> ₂	у
1	+1	+1	+1	0.33	0.33	+1	y 1
2	+1	-1	+1	0.33	0.33	-1	<i>y</i> 2
3	+1	+1	-1	0.33	0.33	-1	y 3
4	+1	-1	-1	0.33	0.33	+1	y 4
5	+1	+1	0	0.33	-0.67	0	y 5
6	+1	-1	0	0.33	-0.67	0	y 6
7	+1	0	+1	-0.67	0.33	0	y 7
8	+1	0	-1	-0.67	0.33	0	y 8
9	+1	0	0	-0.67	-0.67	0	y 9

Fig. 13–29 show the results of model experiments to obtain the components of the column vector of the output variable y and fill the last column of the Table 6. The time points corresponding to the maximum intensity are marked in pink on the intensity diagrams.



Fig. 13. Network diagram for point (+1; +1) of the central orthogonal compositional design



Fig. 14. Distribution of the intensity of operations at the point (+1; +1) of the central orthogonal compositional design



Fig. 15. Network diagram for point (-1; +1) of the central orthogonal compositional design





The network graph for the point (-1; -1) of the central orthogonal compositional plan corresponds to the original graph (Fig. 1).



Fig. 17. Network diagram for point (+1; -1) of the central orthogonal compositional design



Fig. 18. Distribution of the intensity of operations at the point (+1; -1) of the central orthogonal compositional design



Fig. 19. Network diagram for point (-1; -1) of the central orthogonal compositional design







Fig. 21. Network diagram for point (+1; 0) of the central orthogonal compositional design

ECONOMICS OF ENTERPRISES: ECONOMIC CYBERNETICS



Fig. 22. Network graph for point (-1; 0) of the central orthogonal compositional design



















Fig. 27. Distribution of the intensity of operations at the point (0; -1) of the central orthogonal compositional design



Fig. 28. Network graph for point (0; 0) of the central orthogonal compositional design



Fig. 29. Distribution of the intensity of operations at the point (0; 0) of the central orthogonal compositional design

Table 7 shows a completed data table for calculating estimates of the coefficients of the regression equation.

Table 7

Data table for calculating estimates of the coefficients of the regression equation

No. of experiment	<i>x</i> 0	<i>x</i> ₁	<i>x</i> 2	$x_1^2 - \beta$	$x_2^2 - \beta$	<i>x</i> ₁ <i>x</i> ₂	у
1	+1	+1	+1	0.33	0.33	+1	17.333
2	+1	-1	+1	0.33	0.33	-1	20.667
3	+1	+1	-1	0.33	0.33	-1	17.333
4	+1	-1	-1	0.33	0.33	+1	20.667
5	+1	+1	0	0.33	-0.67	0	17.333
6	+1	-1	0	0.33	-0.67	0	20.667
7	+1	0	+1	-0.67	0.33	0	17.333
8	+1	0	-1	-0.67	0.33	0	17.333
9	+1	0	0	-0.67	-0.67	0	17.333

According to formulas (16)–(19) [28] based on the data in Table 7, the following matrix of coefficient estimates is obtained:

$$a_i = c_1 \sum_{j=1}^{N} x^j y^j, i = 1, ..., n,$$
(16)

$$a_i = c_2[(x_{i-n}^j)^2 - \beta]y^j, i = n+1, \dots, 2n,$$
(17)

$$a_{i} = c_{3} \sum_{j=1}^{N} x_{\mu}^{j} x_{\lambda}^{j} y^{j}, i = 1, ..., n, \mu \neq \lambda, i = 2n + 1, ..., k,$$
(18)

$$a_0 = \frac{1}{N} \sum_{j=1}^{N} y^j - \beta \sum_{j=1}^{N} a_{n+i}.$$
 (19)

The coefficients included in equations (16)-(19) are given in Table 4.

Therefore, the regression equation is:

$$y = 18.44433 - 1.66733x_{1norm} + + 1.390335x_{1norm}^2 - 0.27667x_{2norm}^2.$$
(20)

Further, using the example of the methodology given in [29], the search for optimal solutions is carried out in a parametric form. The parameter is λ , the region of existence of which is limited by the ranges: $[-\infty; -0.27667]$, [-0.27667; 1.390335], $[1.390335; +\infty]$.

The results are shown in Fig. 30-32.





From Fig. 32 it follows that by shifting operations Q and P it is impossible to significantly reduce the unevenness of the intensity distribution $-y_{min}=18$ (Fig. 30). Therefore, if 1 month is taken as a unit of time, alternative No. 1 turns out to be better. The same conclusion concerns the possibility of shifting the time interval with the maximum intensity value. Further, the possibility of improving the result is checked if the problem of optimizing the network diagram is solved for the case when a quarter is selected per unit of time. The experimental results are shown in Fig. 33–41 and are summarized in Table 8 to calculate the estimates of the coefficients of the regression equation.













ECONOMICS OF ENTERPRISES:

ΠΝΠΜΙ



Fig. 36. Distribution of the intensity of operations at the point (-1; -1) of the central orthogonal compositional design



Fig. 37. Distribution of the intensity of operations at the point (+1; 0) of the central orthogonal compositional design



Fig. 38. Distribution of the intensity of operations at the point (-1; 0) of the central orthogonal compositional design



Fig. 39. Distribution of the intensity of operations at the point (0; +1) of the central orthogonal compositional design



Fig. 40. Distribution of the intensity of operations at the point (0; -1) of the central orthogonal compositional design



Fig. 41. Distribution of the intensity of operations at the point (0; 0) of the central orthogonal compositional design

The matrix of estimates of the coefficients of the regression equation, calculated by formulas (16)-(19), has the form:

	(9.84867)	l
	-4.1555	
_	-3.31316	
<i>L</i> =	-1.99973	ŀ
	-3.46023	
	(1.48775)	

A

Therefore, the regression equation is:

$$y = 9.84867 - 4.1555x_{1norm} - 3.31316x_{2norm} - - 1.99973x_{1norm}^2 - 3.46023x_{2norm}^2 - 0.27667x_{2norm}^2 + + 1.48775x_{1norm}x_{2norm}.$$
 (21)

The estimate of the variance, which makes it possible to judge the quality of the equation, is $s_r^2 = 2.51755$, which reflects the closeness of the calculated and experimental data in accordance with Fig. 42.



Fig. 42. Visualization of the equation quality evaluation (21)

The optimization results are shown in Fig. 43–45 for the regions of existence of the parameter λ : $[-\infty; -3.77241]$, [-3.77241; -1.68755], $[-1.68755; +\infty]$.

The results of the analysis show that in the case of solving the problem of optimizing the network diagram, when 1 quarter is selected per unit of time, it is possible to minimize the unevenness of the intensity. As follows from Fig. 45 (ridge line I), the minimum output variable at $y_{\min}=3$ can be achieved. This solution allows to conclude that for the original network diagram in the case of using alternative No. 2, the optimal solution can be obtained only within the distribution of the intensities of the operations performed on a quarterly basis. But in this case, the maximum intensity all the time falls on the first time interval, i. e., the peak of the maximum intensity can't be shifted beyond the first quarter.





7. SWOT analysis of research results

Strengths. Among the strengths of this research, it is necessary to note the established possibility of a targeted distribution of the intensity of operations. Depending on what priority tasks are at the planning stage, the application of the proposed methods allows for their initial data to obtain a mathematical description of the influence of early terms of operations on the uniformity of the workload of personnel. Based on this description, it is possible to estimate the time interval within which an optimal solution can be obtained in the sense of minimizing the unevenness of the workload of personnel in operations. For such time intervals, it is possible to calculate early dates for the start of operations, ensuring the minimum

uneven workload of personnel. In addition, an assessment can be made of the possibility of shifting the intensity peaks to other intervals of the project implementation time, which is especially important when using third-party labor resources or resources employed at other facilities.

Weaknesses. The weaknesses of this research are related to the fact that all solutions are obtained only for a model network diagram. The variety and infinity of options for the initial data for building network diagrams does not allow applying the obtained solutions to an arbitrary case. That is, there is no way to talk about the universality of the solutions obtained and the possibilities of using the obtained equations as ready-made mathematical models for arbitrary initial data.

Opportunities. Additional opportunities when using the above results in real projects are associated with the fact that, using the above procedures, for an arbitrary project, their own regression equations can be obtained, which make it possible to obtain optimal solutions in the sense of minimizing the unevenness of the workload of personnel. In addition, the application of the proposed principles for evaluating alternatives allows to choose solutions that allow to shift the peaks of the intensity of the workload of personnel to other time ranges.

Threats. Obvious risks when using the results obtained are associated with the fact that errors in estimating the parameters of network diagrams can lead to erroneous decisions, i. e., the factor of subjectivity in the implementation of the proposed procedures is the most significant.

8. Conclusions

1. It is shown that the choice of output variables for optimizing the network diagram depends on the goals that are set at the stage of project planning as priorities. This can be, in particular, the possibility of shifting the peaks of the staff load intensity in a given time interval or minimizing the unevenness of the staff load in the full scope of the project, estimated as the ratio of the maximum staff load intensity to the minimum intensity $y=q_{\rm max}/q_{\rm min}$.

2. The early dates of the start of operations can be used as input variables. However, when choosing those that can be used to optimize networks, special conditions should be used with respect to the dependence of such variables. Based on this, the concept of independent input variables was introduced, in which independence is defined as the ability of each input variable to satisfy the following mandatory set of requirements:

- intensity of the operation for which the input variable is selected is maximum, or, in the general case, as large as possible in absolute value;

 the number of immediately preceding operations should be minimal:

- time reserve for the operation must be sufficient to vary the values of the input variable.

3. To make targeted decisions on the choice of early dates for the start of operations that provide the best results in relation to the selected criteria for optimizing network diagrams, it is effective to build regression equations. Such equations describe the influence of the early start dates of those operations that satisfy the independence requirements on the selected optimization criteria. The quality of these equations can be assessed by a measure of the proximity of the calculated values of the selected optimization criteria to the experimental values. The latter include the data obtained by a model experiment, the essence of which is reduced to the displacement on the network diagram of the selected operations within the ranges of variation of the input variables. In this case, either D-optimal designs on a segment, in the case of one input variable, or central orthogonal compositional designs, in the case of several input variables, can be used. For the considered model case, when calculating the intensities on a quarterly basis, qualitative regression equations were obtained with the following indicators: $s_y^2(x) = 2.15625$ – when using the D-optimal design on a segment, $s_r^2 = 2.51755$ – when using a central orthogonal compositional design.

4. It is shown that optimal solutions in relation to uneven workload of personnel can be found at the boundaries of the planning area, determined by the intervals of variation of the selected independent input variables. The proposed principles for choosing alternatives allow to assess the possibilities of achieving the goals of optimizing the network diagram.

For the considered model case, when calculating the intensities by quarter, the following solutions are obtained:

- the use of alternative No. 1 provided the possibility of reducing the unevenness of the workload by 24.2 % $(y=q_{\max}/q_{\min}=20.667$ for the original network diagram, $y=q_{\max}/q_{\min}=0.667$ for the optimized network diagram); - the use of alternative No. 2 provided the possibility of reducing the unevenness of the workload of personnel by 85.4 % $(y=q_{\max}/q_{\min}=20.667$ for the original network diagram, $y=q_{\max}/q_{\min}=3$ for the optimized network diagram);

alternative No. 1 turned out to be more preferable from the point of view of the shift of the intensity peaks;
obtaining an optimal solution in terms of minimizing the unevenness of the workload of personnel depends on the selected time interval for which the possibility of such minimization is assessed.

References

- Gasanbekov, S. K., Lubenets, N. A. (2014). Setevoe planirovanie kak instrument upravleniia proektami. *Izvestiia MGTU «MAMI»*, 5 (1 (19)), 21–25.
- Butsenko, E. V., Shorikov, A. F. (2015). Setevoe modelirovanie protsessa upravleniia investitsionnym proektirovaniem i ego prilozheniia. Nauchno-tekhnicheskie vedomosti SPbGPU. Ekonomicheskie nauki, 6 (233), 233–244. doi: http://doi.org/10.5862/ je.233.24
- Musatova, T. E. (2016). Primenenie grafoanaliticheskogo metoda v investitsionnom proektirovanii. *Traektoriia nauki*, 4 (9), 7.1–7.12.
- Butsenko, E. V. (2016). Praktika primeneniia setevogo ekonomiko-matematicheskogo modelirovaniia protsessa investitsionnogo proektirovaniia. Vestnik Tomskogo gosudarstvennogo universiteta. Ekonomika, 1 (33), 147–158.
- Popova, I. N., Anikina, D. L. (2018). Mekhanizm upravleniia i optimizatsii investitsionnogo proekta s ispolzovaniem metoda setevogo modelirovaniia. *Diskussiia*, 91, 6–16.
- Akhtiamov, R. G., Elizarev, A. N., Vdovina, I. V., Planida, Iu. M., Khaertdinova, E. S. (2012). Primenenie setevykh modelei pri planirovanii avariino-spasatelnykh i drugikh neotlozhnykh rabot. Nauchnye i obrazovatelnye problemy grazhdanskoi zaschity, 2, 29–34.
- Demina, A. V., Aleksentseva, O. N. (2017). Ispolzovanie setevogo planirovaniia dlia organizatsii uchebnogo protsessa. Vestnik Saratovskogo gosudarstvennogo sotsialno-ekonomicheskogo universiteta, 1 (65), 16–18.
- Bashtannik, N. A., Lobeiko, V. I., Poliakov, S. V. (2010). Optimizatsiia putei obmena informatsiei mezhdu elementami ASU setevym metodom. *Izvestiia VolgGTU*, 6 (66), 48–51.

- 9. Wang, Y., Chen, J., Ning, W., Yu, H., Lin, S., Wang, Z. et. al. (2020). A time-sensitive network scheduling algorithm based on improved ant colony optimization. *Alexandria Engineering Journal*. doi: http://doi.org/10.1016/j.aej.2020.06.013
- Akberdina, V. V., Smirnova, O. P. (2017). Kontseptsiia setevykh sopriazhennykh proizvodstv v kontekste chetvertoi promyshlennoi revoliutsii. *Vestnik ZabGU*, 23 (7), 104–113.
- Galper, J. (2001). Three Business Models for the Stock Exchange Industry. *The Journal of Investing*, 10 (1), 70–78. doi: http:// doi.org/10.3905/joi.2001.319454
- Judith, G., Mark, G. (2003). The US Wine Industry and the Internet: An Analysis of Success factors for Online Business models. *Electronic Markets*, 13 (1), 59–66. doi: http://doi.org/10.1080/ 1019678032000039877
- Bobrova, T. V., Dubenkov, A. A. (2017). Kalendarno-setevoe planirovanie stroitelstva lineinykh obektov v srede MSProject. *Vestnik SibADIB*, 4-5 (56-57), 68-77.
- 14. Chavada, R., Dawood, N., Kassem, M. (2012). Construction workspace management: the development and application of a novel nD planning approach and tool. *Journal of Information Technology in Construction*, 17, 213–236. Available at: http:// www.itcon.org/2012/13
- Ting, W., Ying, Y. K., Xiao, L. P. (2013). The Impact of BIM Application to the Project Organizational Process. *Applied Mechanics and Materials*, 357-360, 2524–2528. doi: http://doi.org/ 10.4028/www.scientific.net/amm.357-360.2524
- Moon, H., Dawood, N., Kang, L. (2014). Development of workspace conflict visualization system using 4D object of work schedule. Advanced Engineering Informatics, 28 (1), 50–65. doi: http://doi.org/10.1016/j.aei.2013.12.001
- Korshunov, Iu. M. (1980). Matematicheskie osnovy kibernetiki. Moscow: Energiia, 424.
- Karenov, R. S. (2013). Metodika analiza i optimizatsii setevogo grafika. Vestnik Karagandinskogo universiteta. Seriia «Matematika», 3 (71), 53–65.
- Postovalova, I. P. (2014). Effektivnii sintez setevoi modeli «rabotydugi» s minimalnym chislom fiktivnykh rabot. Upravlenie bolshimi sistemami, 52, 118–132.
- Ivanov, N. N. (2015). Analitiko-imitatsionnoe modelirovanie obobschennykh stokhasticheskikh setevykh grafikov. Upravlenie bolshimi sistemami, 53, 27–44.
- Shorikov, A. F., Butsenko, E. V. (2015). Metodika optimizatsii investitsionnog proektirovaniia na osnove setevogo modelirovaniia i ee prilozheniia. Vestnik Permskogo universiteta. Ekonomika, 4 (27), 62–70.
- 22. Rygalov, G. M. (2010). modelirovanie sistemy setevogo planirovaniia v microsoft project. *Transportnoe delo Rossii*, 58–64. Available at: http://cyberleninka.ru/article/n/modelirovanie-sistemysetevogo-planirovaniya-v-microsoft-project
- 23. Kushner, M. A. (2010). Model minimizatsii srokov vypolneniia proekta v ramkakh setevykh tekhnologii pri fiksirovannom biudzhete. Vestnik AGTU. Seriia Ekonomika, 2, 124–129.
- 24. Tikhobaev, V. M., Tolokonnikova, L. A., SHatokhina, A. G. (2010). Optimizatsiia plana kompleksa rabot pri vzaimozameniaemykh resursakh s ispolzovaniem setevogo grafika. *Izvestiia Tulskogo gosudarstvennogo universiteta. Ekonomicheskie i iuridicheskie nauki, 2-2,* 143–150.
- Shmat, V. V., Iuva, D. S. (2017). Razrabotka metodiki riskoptimalnogo planirovaniia dlia innovatsionnogo proekta v neftegazovom sektore. *Innovatsii, 6 (224),* 113–121.
- Epstein, M. (2002). Risk Management of Innovative R&D Project. Helsinki: School of Economics, 273.
- A guide to the project management body of knowledge (PMBOK Guide). (2016). Newtown Square: Project Management Institute, 615. Available at: http://www.pmi.org/PMBOK-GuideandStandards/pmbok-guide.aspx
- Hartman, K., Leckiy, E., Shefer, V. et. al. (1977). Planirovanie ehksperimenta v issledovanii tekhnologicheskih processov. Moscow: Mir, 552.
- 29. Demin, D. (2017). Synthesis of optimal control of technological processes based on a multialternative parametric description of the final state. *Eastern-European Journal of Enterprise Technologies*, 3 (4 (87)), 51–63. doi: http://doi.org/10.15587/1729-4061.2017.105294

Domina Olena, Member of the Board «Scientific Route» OÜ, Tallinn, Estonia, ORCID: http://orcid.org/0000-0001-8202-4054