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## ANALYSIS OF SEPARATE CHANNELS IN A MULTI-CONNECTED CONTROL SYSTEM

The object of research is the control system of an autonomous mobile robot equipped with an anthropomorphic manipulator with four degrees of mobility. When a mobile robot of variable configuration moves along a given route, the control system must ensure a minimum deviation of the center of mass of the platform from a given trajectory. In this case, the control moments are directed along the axes of the coordinate system associated with the platform of the autonomous mobile robot. With relative movements of the manipulator, the tensor of inertia of the system of bodies in the coordinate system associated with the platform becomes off-diagonal and non-stationary, which determines the interconnection of control channels. The number of control actions: when moving the trajectory – four (for each wheel), when the manipulator is working – four (for each generalized coordinate). Thus, the control system is multidimensional, the connection between control channels is carried out due to the physical properties of the control object.

The paper presents the results of the first stages of the development of a multi-connected control system: study of separate control channels, synthesis of a separate controller, adjustments of the controller and analysis of the quality of the created control system. The research is carried out using matrix transfer functions; Besekersky formulas are applied to determine the parameters of the desired transfer function. During the analysis, logarithmic and amplitude frequency characteristics were constructed for each separate channel, a block diagram was chosen, and a transfer function was compiled. The quality assessment of the synthesized separate channel is carried out according to the following criteria: accuracy, speed, oscillation, transient time, overshoot, amplitude and phase distortions, stability margins. The properties of the synthesized separate channel in terms of accuracy, speed and oscillation correspond to the conditions.

Further research and synthesis of multi-connected control systems of a mobile robot with a manipulator will increase its survivability and efficiency in autonomous operation. Since a mobile robot with a manipulator is an example of an «autonomous mobile robot of variable configuration» object class, the results obtained can be applied to all objects of this class.

**Keywords:** autonomous mobile robot of variable configuration, anthropomorphic manipulator, four degrees of mobility, separate channel.

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### 1. Introduction

The expansion of the scope and complication of tasks for modern autonomous mobile robots (AMR) determines the presence of structural elements that are movable relative to the work platform. An example of such structural elements can be a manipulator, executive bodies of specialized equipment or movable attachments: probe, drill, bucket, etc. [1]. During AMR operation, these elements move relative to the platform, changing the geometry of the system masses.

AMR with a manipulator is a complex mechanical system that can be considered a manipulator on a movable basis. The link of the manipulator – the bodies were transferred, the platform – the carrying body [2].

When moving the AMR of a variable configuration along a given route, the control system must ensure the minimum deviation of the center of mass of the platform from the given trajectory. In this case, the control moments are directed along the axes of the coordinate system associated with the AMR platform [3].

With the relative motion of the structural elements of the AMR with a variable configuration, its main central axes of inertia are not directed with the axes of the coordinate system associated with the AMR platform. In [4], the analysis of the tensor of inertia of AMR with a manipulator was carried out with different configurations of their mutual position with the mass of the manipulator with a load up to 20 % of the mass of the AMR platform. The research results given in [4] show that the value of the centrifugal moments of inertia of the system with the relative motion of the manipulator is comparable to the value of the axial moments of inertia of the system, even if the mass of the moving structural elements is less than 10 % of the mass of the platform. Therefore, in the general case, the tensor of inertia of such a system should be taken off-diagonal and non-stationary. The results of mathematical modeling and analysis of the dynamics of AMR of a variable configuration, presented in [5, 6], show the relationship of control channels due to the off-diagonal and nonstationarity of the inertia tensor. Thus, the analysis

of separate AMR control channels of variable configuration is an urgent scientific and applied problem.

The object of research is the control system of an autonomous mobile robot equipped with an anthropomorphic manipulator with four degrees of mobility.

The aim of research is to ensure effective control of a multi-connected and multifunctional system.

**2. Methods of research**

**2.1. Problem statement.** A diagram of the AMR design with a manipulator, similar to that proposed in [7], is shown in Fig. 1. The structure consists of an all-wheel drive 4-wheeled platform AMR and an anthropomorphic manipulator made up of a ring rotating around a vertical axis and rod links – an arm connected by rotational kinematic pairs of the fifth class.

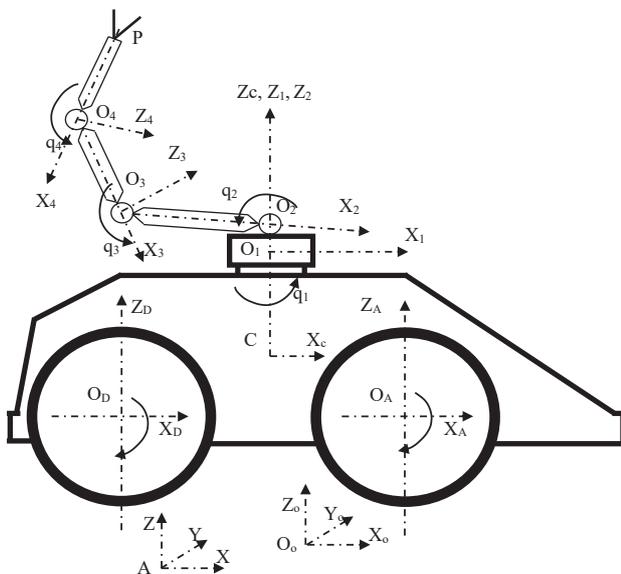


Fig. 1. Scheme of coordinate systems for an autonomous mobile robot with a manipulator

The AMR work cycle can be divided into several stages: – movement of the AMP platform from the starting point *S* to the finishing point *F* with a stationary manipulator in compliance with the requirements of optimality (speed, performance or economy, etc.); – performance of technological operations with a manipulator, attachments or information equipment with a stationary platform; – return of the AMP platform to the starting point *S* or movement to a predetermined end point *K* with a stationary manipulator.

At the first and third stages of work, AMR is a trolley with an adaptive control system that moves in a predetermined environment. To ensure the stable movement of AMR from the start point *S* to the finish point *F* with a stationary manipulator in compliance with the requirements of optimality, it is necessary to perform mathematical and simulation modeling. The dynamics and controllability of a four-wheel all-wheel drive autonomous mobile robot were studied in detail in [7]. Control actions – moments applied to each wheel, cause the manipulator to deviate from the initial position as a result of the non-diagonality of the inertia tensor. At the second stage of work, AMR can

be viewed as an anthropomorphic manipulator on a mobile basis. The number of control actions: when moving the trajectory – four (for each wheel), when the manipulator is working – four (for each generalized coordinate).

**2.2. Method of analysis of separate control channels.**

The block diagram of a multiply connected control system (MCS) is shown in Fig. 2. Features of the dynamics, methods of analysis and synthesis of such a control system are due to the presence of cross-links between regulation channels [8].

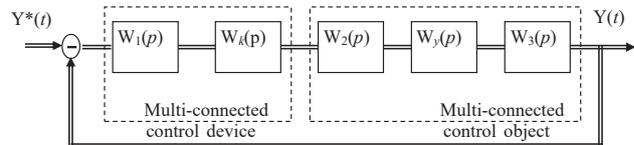


Fig. 2. Block diagram of a multi-connected control system

Transfer matrices (TM) of linear links in a four-dimensional MCS are as follows:

$$W_1(p) = \text{diag}\{W_{p1}(p); W_{p2}(p); W_{p3}(p); W_{p4}(p)\};$$

$$W_2(p) = \text{diag}\{W_{01}(p); W_{02}(p); W_{03}(p); W_{04}(p)\};$$

$$W_3(p) = \text{diag}\{1/p; 1/p; 1/p; 1/p\},$$

where  $W_{pi}(p)$  – transfer functions of the *i*-th separate controller;  $i=1, \dots, 4$ ; transfer functions of the unchanged part of the *i*-th separate channel:

$$W_{0i}(p) = \frac{K_i}{(1 + pT_{ai})(1 + pT_{bi})};$$

$W_k(p) = [E + W_x(p)]$  – compensator TM with direct cross-links, where:

$$W_x(p) = \begin{bmatrix} 0 & 0 & 0 & W_{x4}(p) \\ 0 & 0 & W_{x3}(p) & 0 \\ 0 & W_{x2}(p) & 0 & 0 \\ W_{x1}(p) & 0 & 0 & 0 \end{bmatrix},$$

and  $W_{x1}(p)$ ,  $W_{x2}(p)$ ,  $W_{x3}(p)$  and  $W_{x4}(p)$  – transfer functions of forward (or reverse) cross-links as part of a serial compensator that provides the property of autonomy of control channels;

$$W_y(p) = \begin{bmatrix} 1 & 1 & 1 & W_{y4}(p) \\ 1 & 1 & W_{y3}(p) & 1 \\ 1 & W_{y2}(p) & 1 & 1 \\ W_{y1}(p) & 1 & 1 & 1 \end{bmatrix},$$

where the transfer functions of cross-links in the control object:

$$W_{yi}(p) = \frac{K_{yi}}{1 + pT_{yi}}.$$

At the first stage of MCS development for AMR, it is advisable to consider separate channels, that is, control

channels without taking into account all cross-links in the facility and in the regulator. Under such conditions, the MCS is a set of four unconnected separate channels.

According to [8], using the formulas of V. Besekersky, a typical «symmetric» logarithmic amplitude-frequency characteristic for each separate channel, let's choose the type and parameters of the transfer function of the controller  $W_{pi}(p)$ , provided that the specified quality indicators are ensured. For AMR of variable configuration, the critical indicators are accuracy (quality factor  $K=K_i$ ), speed ( $\omega_{av}$ ) and oscillation (M).

According to the technique described in [9], the synthesis of separate controllers in the MCS, the structural diagram of which is shown in Fig. 2 is preceded by an analysis of each of the separate channels. Thus, for the synthesis of separate control channels, it is necessary to:

- 1) develop a block diagram for each separate control channel;
- 2) determine the transfer functions of the separate channels;
- 3) construct the logarithmic and frequency response of each separate channel;
- 4) investigate the properties of a separate channel in terms of accuracy, speed and oscillation;
- 5) determine the transfer functions of the separate controllers;
- 6) correct the transfer functions of the separate regulators in the conditions of ensuring the autonomy of the control channels;
- 7) investigate the properties of the synthesized separate MCS channels.

For definiteness, as an example, consider the MCS with the initial data given in Table 1. It is necessary to adjust the transfer functions of the separate regulators to achieve the «desired» quality indicators (Table 1) and ensure the autonomy of the channels.

$$W_2(p) = \begin{bmatrix} \frac{125}{(1+0.07p)(1+0.01p)} & 0 & 0 & 0 \\ 0 & \frac{135}{(1+0.11p)(1+0.009p)} & 0 & 0 \\ 0 & 0 & \frac{110}{(1+0.06p)(1+0.035p)} & 0 \\ 0 & 0 & 0 & \frac{120}{(1+0.15p)(1+0.032p)} \end{bmatrix}$$

**Table 1**

Initial data

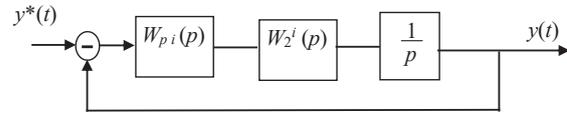
Channel No.	Coefficients of the unchanged part of the $i$ -th separate channel			«Desired» quality indicators	
	$K_i, s^{-1}$	$T_{air}, s$	$T_{bir}, s$	$\omega_{avi}$	$M_i$
1	125	0.07	0.01	145	1.05
2	135	0.11	0.009	29	1.35
3	110	0.06	0.035	26	2.8
4	120	0.15	0.032	23	2.8

All calculations will be performed using the Mathcad software package.

### 3. Research results and discussion

**3.1. Development of a structural diagram of each separate control channel.** In the absence of cross-links in the multidimensional control device and in the multidimensional control object, the MCS structural diagram turns

into a set of autonomous separate channels. The block diagram of the fifth separate channel is shown in Fig. 3.



**Fig. 3.** Block diagram of the fifth separate channel

1. Let's write down the transfer matrices with the numerical values of parameters given in Table 1:

$$W_1(p) = \begin{bmatrix} W_{p1}(p) & 0 & 0 & 0 \\ 0 & W_{p2}(p) & 0 & 0 \\ 0 & 0 & W_{x3}(p) & 0 \\ 0 & 0 & 0 & W_{x4}(p) \end{bmatrix};$$

$$W_3(p) = \begin{bmatrix} \frac{1}{p} & 0 & 0 & 0 \\ 0 & \frac{1}{p} & 0 & 0 \\ 0 & 0 & \frac{1}{p} & 0 \\ 0 & 0 & 0 & \frac{1}{p} \end{bmatrix};$$

- Let's write down the transfer function of the  $i$ -th open-loop separate channel at  $W_{pi}(p)=1$ :

$$W_2^1(p) = \frac{125}{p(1+0.07p)(1+0.01p)};$$

$$W_2^2(p) = \frac{135}{p(1+0.11p)(1+0.009p)};$$

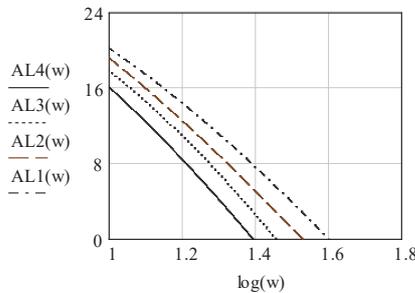
$$W_2^3(p) = \frac{110}{p(1+0.06p)(1+0.035p)};$$

$$W_2^4(p) = \frac{120}{p(1+0.15p)(1+0.032p)}.$$

Using the Mathcad software package, using the known open-loop transfer function, let's construct the logarithmic amplitude-frequency characteristics (LAFC) for each separate channel (Fig. 4).

By tracing the LAFC graphs, let's determine the actual communication frequencies and compare the obtained values

with the «desirable» ones (Table 2). Based on the results of the study, let's conclude that the actual values of the communication frequency of the third and fourth channels are close to the desired values. So, it is advisable to make adjustments for the first and second separate channels.



**Fig. 4.** Logarithmic frequency characteristics for each separate channel

**Table 2**

Communication frequencies

Channel No. <i>i</i>	Actual values of the communication frequency $\omega_{ai}, s^{-1}$	Desired values of the communication frequency $\omega_{avi}, s^{-1}$
1	39.35	145
2	33.51	29
3	26.26	26
4	22.88	23

Let's define the closed transfer function of each channel by the formula:

$$W_{ci}(p) = \frac{W(p)}{1 + W(p)}$$

The amplitude-frequency characteristics is equal to:

$$A(\omega) = |W(j\omega)|,$$

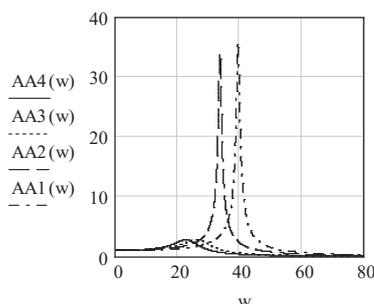
where  $W(j\omega)$  – frequency transfer function. If to write:

$$W(j\omega) = U(\omega) + jV(\omega),$$

where  $U(\omega)$  – real part;  $V(\omega)$  – imaginary part; then  $A(\omega)$  can be defined by the formula:

$$A(\omega) = \sqrt{U^2(\omega) + V^2(\omega)}.$$

Let's construct the amplitude-frequency characteristics for each closed separate channel (Fig. 5).



**Fig. 5.** Amplitude-frequency characteristics of closed separate channels

2. Determine whether the «desired» properties of the separate channel are provided. Determine the actual vibrational index  $M$  using the formula:

$$M_a^i = \frac{AAi_{max}}{AAi(0)}, \tag{1}$$

where  $AAi_{max}$  – maximum of the amplitude-frequency characteristics of the  $i$ -th closed channel, taking into account:

$$M_a^1 = \frac{35.84}{1} = 35.84; \quad M_d^1 = 1.05;$$

$$M_a^2 = \frac{33.94}{1} = 33.94; \quad M_d^2 = 1.35;$$

$$M_a^3 = \frac{2.83}{1} = 2.83; \quad M_d^3 = 2.8;$$

$$M_a^4 = \frac{2.82}{1} = 2.82; \quad M_d^4 = 2.8,$$

conclude that the properties of the first and second separate channels do not meet the desired vibration requirements.

3. Adjusted transfer function of the separate regulator will look like:

$$W_d^i = \frac{K_i(1 + T_2^i p)}{p(1 + T_1^i p)(1 + T_3^i p)(1 + T_4^i p)}.$$

To estimate the parameters of the desired transfer function, let's use the Besekersky formulas [10]:

$$T_2^i \geq \frac{M}{\omega_{ci}(M-1)}; \quad T_3^i \leq \frac{M}{\omega_{ci}(M+1)}; \quad T_3^i + T_4^i \leq \frac{M}{\omega_{ci}(M-1)}.$$

Let's construct the desired LAFC, determine the time constant  $T_1$  from the graph and calculate the coefficients of the separate controller. The transfer functions of the regulators have been adjusted to ensure the specified quality indicators in the first and second separate channels, have the form:

$$W_{p1}(p) = \frac{(1 + 0.017p)(1 + 0.07p)}{(1 + 0.0065p)(1 + 0.000473p)}$$

$$W_{p2}(p) = \frac{(1 + 0.11p)(1 + 0.11p)}{(1 + 0.01p)(1 + 0.447p)}$$

and the transfer functions of the open separate channels:

$$W_p^I(p) = \frac{125(1 + 0.017p)}{(1 + 0.0065p)(1 + 0.000473p)(1 + 0.01p)p}$$

$$W_p^{II}(p) = \frac{135(1 + 0.11p)}{(1 + 0.447p)(1 + 0.0009p)(1 + 0.01p)p}$$

Let's construct the amplitude-frequency characteristic of the first and second closed separate channels (Fig. 6) and, using formula (1), determine the oscillation index of the corrected separate channel:

$$M^1 = \frac{1.04}{1} = 1.04; \quad M^2 = \frac{1.3}{1} = 1.3.$$

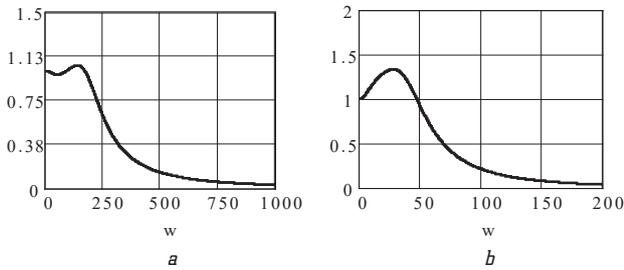


Fig. 6. Amplitude-frequency characteristic of a closed channel with a corrected controller: *a* – for the first separate channel; *b* – for the second separate channel

**3.2. Investigation of the properties of synthesized separate MCS channels.** Indicators of the transient process quality. To determine the direct indicators of the transient process quality, let's obtain the transient characteristics of the first and second separate channels using the Mathcad software package.

The amplitude-frequency characteristic for the first and second separate channels is shown in Fig. 7.

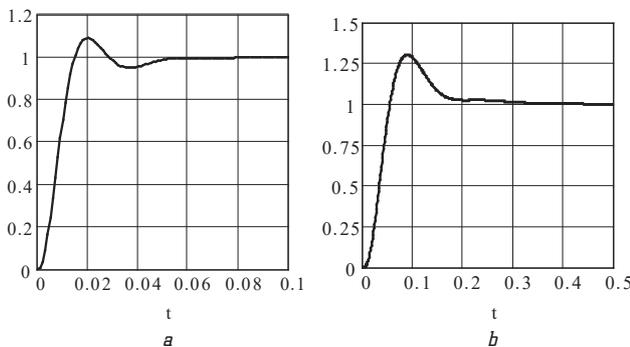


Fig. 7. Transient response: *a* – for the first separate channel; *b* – for the second separate channel

Using the graph of the transient function, let's determine the time of the transient process  $t_{tp}^1 = 0.6$  s,  $t_{tp}^2 = 0.4$  s. Let's define overshoot as the ratio of the maximum gain of the controlled value to its steady-state value:

$$\sigma = \frac{h_{\max} - h_{st}}{h_{st}} \cdot 100\%;$$

$$\sigma^1 = \frac{1.089 - 1}{1.089} \cdot 100\% = 8.2\%;$$

$$\sigma^2 = \frac{1.303 - 1}{1.303} \cdot 100\% = 23.2\%.$$

**Accuracy indicators.** Let's determine the accuracy indicators in the form of amplitude ( $\delta_A$ ) and phase ( $\delta_\varphi$ ) distortions at a frequency  $\omega_1 = 0.3\omega_{av2}$ .

The cutoff frequency of the second separate channel is determined by the terms of reference:

$$W_{av2} = 29.25 \text{ rad/s,}$$

$$\omega_1 = 0.3 \cdot 29.25 = 8.775 \text{ rad/s.}$$

Let's find the amplitude-phase distortions for each separate channel by the formulas:

$$\delta_A = \frac{|\Phi(\omega^i) - 1|}{1} \cdot 100\%;$$

$$\delta_\varphi = |\arg(\Phi(\omega^i))|,$$

where  $\Phi(\omega)$  – frequency transfer function of the closed separate channel.

Let's write down the frequency transfer functions of the first and second closed channels:

$$\Phi^I(\omega) = \frac{125(1 + 0.017j\omega)}{125(1 + 0.017j\omega) + (1 + 0.0065j\omega)(1 + 0.000473j\omega)(1 + 0.01j\omega)j\omega};$$

$$\Phi^{II}(\omega) = \frac{135(1 + 0.11j\omega)}{135(1 + 0.11j\omega) + (1 + 0.447j\omega)(1 + 0.0009j\omega)(1 + 0.01j\omega)j\omega}.$$

Let's define amplitude and phase distortions:

$$\delta_A^I = \frac{|\Phi^I(\omega^2) - 1|}{1} \cdot 100\% = \frac{|0.998 - 1|}{1} \cdot 100\% = 0.2\%;$$

$$\delta_\varphi = |\arg(\Phi^I(\omega^2))| = 0.07 \text{ rad.}$$

**Stability margins.** Let's determine the stability margins of the separate channels using the Nyquist criterion for the LAFC. LAFC plots of corrected open separate channels are shown in Fig. 8.

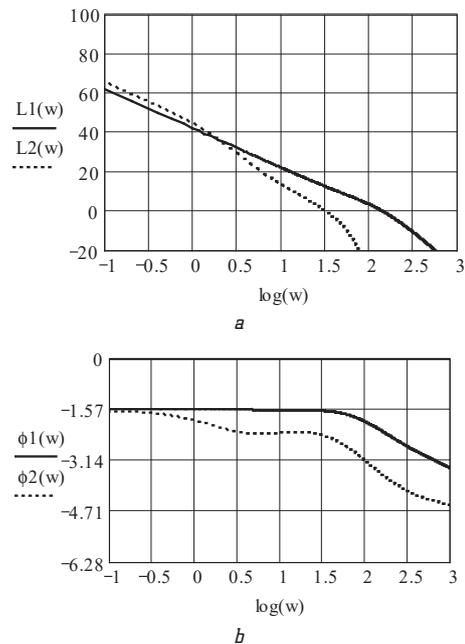


Fig. 8. Graphs of the logarithmic characteristics of the corrected open-loop separate channels: *a* – amplitude characteristics; *b* – frequency characteristics

Using the graphs, let's determine the stability margins:  $L_m^1 = 23.09$  dB,  $L_m^2 = 0.83$  rad,  $L_m^2 = 33.52$  dB,  $j_m^1 = 0.79$  rad.

#### 4. Conclusions

The paper presents the results of the first stages of the development of a multi-connected control system: study of separate control channels, synthesis of a separate controller, adjustments of the controller and analysis of the quality of the created control system. The research is carried out using matrix transfer functions; to determine the parameters of the desired transfer function, the Besekersky formulas are applied.

During the analysis, logarithmic and amplitude frequency characteristics were constructed for each separate channel, a block diagram was chosen, and a transfer function was compiled.

The quality assessment of the synthesized separate channel is carried out according to the following criteria: accuracy, speed, oscillation, transient time, overshoot, amplitude and phase distortions, stability margins. Quality indicators for the first and second synthesized separate channel are: oscillation  $M^1=1.04$ ,  $M^2=1.3$ ; the time of the transient process  $t_{ip}^1=0.6$  s,  $t_{ip}^2=0.4$  s; overshoot  $\sigma^1=8.2$  %,  $\sigma^2=8.2$  %; amplitude and phase distortion:  $d_A^1=0.2$  %;  $\delta_\phi=0.07$  rad; stability margins:  $L_m=23.09$  dB,  $L_m^1=0.83$  rad,  $L_m^2=33.52$  dB,  $j_m^1=0.79$  rad.

Since a mobile robot with a manipulator is an example of an «autonomous mobile robot of variable configuration» object class, the results obtained can be applied to all objects of this class.

#### References

1. Iurevich, E. I. (2017). *Osnovy robototekhniki*. Saint Petersburg: BKHV-Peterburg, 304.
2. Vorobev, E. I., Popov, S. A., Sheveleva, G. I.; Frolova, K. V., Vorobev, E. I. (Eds.) (1988). *Mekhanika promyshlennykh robotov. Kn. 1. Kinematika i dinamika*. Kyiv: Vischa shkola, 304.
3. Zenkevich, S. L., Iuschenko, A. S. (2004). *Osnovy upravleniia manipuliatsionnymi robotami*. Moscow: Izd-vo MGTU im. N. E. Baumana, 480.
4. Ashhepkova, N. S., Zbrutskiy, O. V., Bytsenko, O. V. (2020). Analiz nediahonalnosti i nestatsionarnosti tenzoru inertsii avtonomnoho mobilnoho robota. *Mekhanika hiroskopichnykh sistem*, 39, 24–34.
5. Ashhepkova, N. S., Zbrutskiy, A. V. (2020). Modeliuvannia dynamiky avtonomnoho mobilnoho robota z manipulatorom. *Vestnyk NTU «KhPY»*. *Seryia: Informatyka i modelyrovanye*, 31, 4–14.
6. Goidin, O. P., Krusanov, V. S., Kuraev, E. V., Solovev, V. P., Timofeev, A. V. (2013). Pat. No. 142363U1 RU. *Mobilnii robot*. Available at: <https://patents.google.com/patent/RU142363U1/>
7. Martynov, Iu. G. (2005). Upravlenie dvizheniem mobilnykh kolesnykh robotov. *Fundamentalnaia i prikladnaia matematika*, 11 (8), 29–80.
8. Zyrianov, G. V. (2010). *Sistemy upravleniia mnogosviaznyimi obektami*. Cheliabinsk: Izdatelskii tsentr IUUrGU, 112.
9. Kim, D. P. (2007). *Teoriia avtomaticheskogo upravleniia mnogosviaznyimi obektami. Vol. 2: Mnogomernye, nelineinye, optimalnye i adaptivne sistemy*. Moscow: Fizmatlit, 440.
10. Besekerskii, V. A., Popov, V. A. (1972). *Teoriia sistem avtomaticheskogo regulirovaniia*. Moscow: Nauka, 768.

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