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DEVELOPMENT OF A METHOD FOR SYNTHESIS THE FIR FILTERS WITH A CASCADE STRUCTURE BASED ON GENETIC ALGORITHM

The object of research is the process of digital signal processing. The subject of research is methods of synthesis of digital filters with a finite impulse response based on a genetic algorithm. Digital filtering is one of the tasks of digital signal processing. FIR filters are always stable and provide a constant group delay. There are various methods for synthesizing digital filters, but they are all aimed at synthesizing filters with a direct structure.

One of the most problematic areas of a digital filter with a direct structure in digital processing is the high sensitivity of the filter characteristics to inaccuracies in setting the filter coefficients. Genetic algorithm-based filter synthesis methods use an ideal filter as the approximated filter. This approach has a number of disadvantages: it complicates the search for an optimal solution; computation time increases.

The study used random search method, which is the basis of genetic algorithm (used for solving optimization problems); theory of digital filtering in filter analysis; numerical methods for modeling in a Python program.

Prepared synthesis method FIR filter with the cascade structure, which is less sensitive to the effect of finite bit width. Computation time was reduced. This is due to the fact that the proposed method searches for the most suitable filter coefficients based on a genetic algorithm and has a number of features, in particular, it is proposed to use a piecewise-linear function as an approximated amplitude-frequency response.

This makes it possible to reduce the number of populations of the genetic algorithm when searching for a solution. The synthesis of an FIR filter with a cascade structure based on a genetic algorithm showed that for a 24-order filter it took about 30–40 generations to get the filter parameters close to the optimal values. In comparison with classical methods of filter synthesis, the following advantages are provided: calculations of the coefficients of a filter with a cascade structure directly, the possibility of optimizing coefficients with limited bit depth.

Keywords: *genetic algorithm, FIR filter, cascade structure of digital filter, standard deviation, piecewise-linear function.*

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1. Introduction

Digital signal processing (DSP) is directed at solving the problems of receiving, processing and transmitting information in real time. Its application allows to obtain results that were unattainable when using analog methods of information processing. DSP methods are of great interest to scientists and engineers working in various fields of science and technology: communication systems, image processing, process control systems, medical diagnostic equipment, acoustics, measuring technology and instrumentation [1].

DSP is used in the fields, where it is necessary to perform such tasks as filtering, compressing, recovering, controlling, measurement of signal parameters: audio, video processing, or any signal coming from any source.

Filtering is the most common digital processing task. Its implementation is based on using digital filters: filters with a finite impulse response (FIR filters), filters with infinite impulse response (IIR filters). In many applications with digital signal processing, the use of FIR filters is preferable because it has the following advantages:

- filter group delay constant;
- FIR filters are always stable.

Algorithms based on intelligent data processing algorithms are now widely used. One of such algorithms is genetic [2, 3].

A genetic algorithm is a heuristic method, which is a variation of evolutionary algorithms. It solves optimization problems using natural evolution methods similar to natural selection in real world.

Digital filter design consists of the following steps [4]:

1. Determining filter requirements;
2. Calculation of the digital filter coefficient;
3. Representation of a digital filter by a suitable computational structure (algorithm);

4. Analysis of the effect of the final bit depth on the filter operation;

5. Implementation of the digital filter at the software and/or hardware level.

At the first stage of filter design, need to set: sampling rate; technical requirements for amplitude-frequency response (AFR), phase-frequency response or impulse response; choose the type of filter (FIR filter, IIR filter); hardware platform requirements (bit depth, processor/microcontroller clock speed, memory size, etc.), etc.

At the second stage, the synthesis method is selected and the filter coefficients are calculated. The obtained coefficients are intended for the implementation of a digital filter using a direct structure. However, such a structure has a number of disadvantages and may not meet the technical requirements; therefore, proceed to the third stage – the choice of the filter structure.

Implementing a digital filter using a direct structure leads to a number of problems, especially when the filter order is large. They are due to the high sensitivity of the filter characteristics to inaccuracies in setting the digital filter coefficients. Factor setting errors can lead to unwanted frequency response deviations and cause filter instability (IIR filter only). Therefore, for the implementation of digital filters, as a rule, other structures are used [4, 5], and in particular a cascade structure based on the use of elementary links of the first and second orders.

However, calculating the coefficients of a cascade filter from the coefficients of a straight filter introduces additional error. This problem can be eliminated if the coefficients of the filter with a cascading structure are calculated directly for a given structure. The use of a genetic algorithm makes it possible to solve this problem.

At the fourth stage, it is necessary to take into account the effect of the limited bit depth of the chosen platform on the filter characteristics. Due to the fact that the characteristics of the filter with a cascade structure are less sensitive to deviations of the filter coefficients than with a direct structure, it means that their bitness may be less and, therefore, the requirements for the hardware platform will be lower.

To minimize the effect of finite bit depth, mathematical methods of operations research are used [6]. However, the genetic algorithm will allow not only to calculate the coefficients of the digital filter, but also to optimize the filter coefficients taking into account the effect of quantization. Considering the above, the synthesis of FIR filters with a cascade structure based on a genetic algorithm is an actual task.

2. The object of research and its technological audit

The object of research is the process of digital signal processing, the subject of research is methods of synthesis of digital filters with a finite impulse response based on a genetic algorithm.

Digital processing in general is performed in accordance with the structural diagram (Fig. 1):

Fig. 1. General structure of the DSP system:
ADC – analog-to-digital converter; DSP – digital signal processor;
DAC – digital-to-analog converter

Digital filtering is the most common digital processing task. FIR filter belongs to the class of linear discrete systems. The relationship between input $x(n)$ and output $y(n)$ digital signals is determined by the following difference equation (1) [5]:

$$y(n)=\sum_{i=0}^{N-1} b_i \cdot x(n-i), \tag{1}$$

where $y(n)$, $x(n)$ – output and input digital signals, respectively; b_i – FIR filter coefficients; N – number of filter coefficients.

In accordance with the difference equation (1), the direct form of the FIR filter is implemented, shown in Fig. 2.

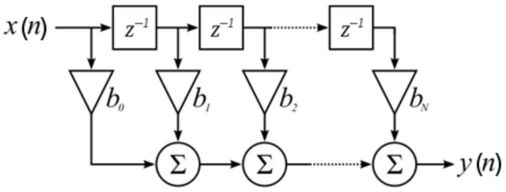


Fig. 2. FIR filter with a direct form

Another form of FIR filter is cascading, which is given by the following expression (2):

$$y(n)=\prod_{k=0}^{K-1}\left(b_{0k}\cdot v_k(n)+b_{1k}\cdot v_k(n-1)+b_{2k}\cdot v_k(n-2)\right), \tag{2}$$

where

$$v_{k+1}(n)=b_{0k}\cdot v_k(n)+b_{1k}\cdot v_k(n-1)+b_{2k}\cdot v_k(n-2),$$

$$v_0(n)=x(n),\quad v_0(n-1)=x(n-1),\quad v_0(n-2)=x(n-2),$$

K – number of stages.

In accordance with equation (2), the cascade form of the FIR filter is implemented, shown in Fig. 3.

Fig. 3. FIR filter cascade form

Each cascade is represented mainly in the form of second-order filters in accordance with Fig. 4 (one of the cascades can be of the first order if the filter order is even).

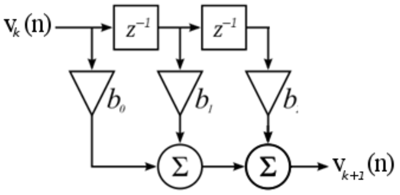


Fig. 4. Second order cascade structure

A digital filter with a direct structure is simple to implement, but it has a high sensitivity of the filter characteristics to inaccuracies in setting the filter coefficients. Calculating the coefficients of a cascaded filter based on the coefficients of a direct filter also has its drawbacks.

3. The aim and objectives of research

The aim of research is to analyze methods for the synthesis of digital filters with a finite impulse response, to develop a method for the synthesis of FIR filters with a cascade implementation structure based on a genetic algorithm.

This will make it possible to improve the accuracy of the digital filter with a cascade structure.

To achieve the aim, the following objectives have been set:

1. Explore the existing methods for the synthesis of digital filters with a finite impulse response.
2. Develop a method for the synthesis of FIR filters with a cascade implementation structure based on a genetic algorithm.
3. Carry out the synthesis of an FIR filter with a cascade implementation structure.

4. Research of existing solutions of the problem

In the scientific researches, digital filters and methods for their design have been deeply analyzed [4, 5]. Among the most widely used classical methods for calculating FIR filters are window method, frequency sampling method, least squares method, best uniform approximation method. The first two are not optimization methods, but are simple to use. The third method requires the use of numerical methods, and, therefore, the calculation accuracy will depend on the accuracy of those methods. The fourth method allows obtaining the best results, but, as a rule, it is impossible to determine analytically the function of the best uniform approximation. One of the most effective iterative methods for determining the best uniform approximation is the Remez exchange algorithm (modified Remez algorithm).

The use of modern heuristic algorithms for data processing [3] makes possible to get some advantages: to increase the efficiency of searching an optimal solution; perform multi-criteria optimization.

In [7, 8], the FIR filter synthesis using a genetic algorithm is considered. The work presents a diagram (Fig. 5), which explains the principle of calculating the coefficients.

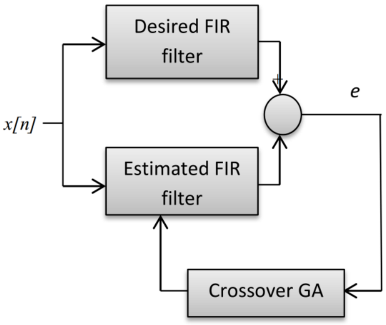


Fig. 5. Diagram for estimating FIR filter coefficients using a genetic algorithm

When designing a digital filter, two main synthesis methods can be distinguished: synthesis of digital filters in the frequency domain; synthesis of digital filters in the time domain.

In accordance with Fig. 5 to optimize the coefficients of the FIR filter the following objective function (3) is proposed in [7, 8]:

$$MSE=\sum_{n=0}^{T}\left(y(n)-\hat{y}(n)\right)^{2}, \qquad (3)$$

where $y(n)$, $\hat{y}(n)$ – output digital signals of the approximated and approximating filters respectively; T – the number of signal samples.

The nature of the input signal $x(n)$ is not clear from the work, so it is not clear in which domain the filter is synthesized: frequency or time? It can be easily shown that if the input signal $x(n)$ is harmonic, then the optimization of the synthesized filter will occur in the frequency domain and only at one point of the amplitude-frequency response (AFR).

In works [7–9], an ideal filter is used as an approximated filter (Fig. 6), which has a piecewise-constant AFR (equal to a constant value in the passband and zero in the stopband). The ideal filter is unrealizable, therefore the real (approximating) filter deviates from the ideal filter. In addition, a real filter has a transition band in addition to the pass and attenuation bands. In the transition band, the greatest deviation of the ideal and real filters is observed, which causes additional difficulties in filter design.

In [7, 8], the filter design is performed in accordance with the objective function (3) and, therefore, does not take into account the indicated problem.

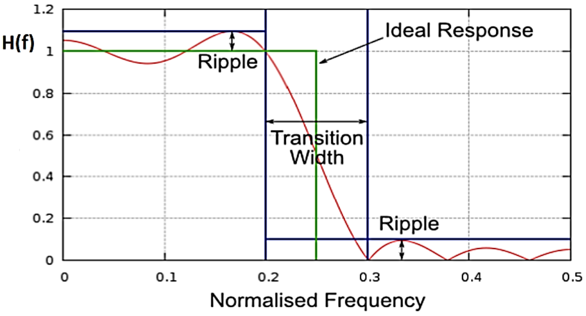


Fig. 6. AFR of the approximated and approximating low-pass filters

In [9], this problem is solved by using two objective functions to minimize the deviation: MSE (mean square error) in the passband and stopband; MAE (mean absolute error) in the transition band. Although this approach eliminates the problem, it complicates the implementation of the method.

It follows from the above analysis that the works devoted to the synthesis of FIR filters are focused on the implementation of filters with a direct structure. If it is necessary to implement filters with a cascade structure, it is proposed to recalculate the coefficients. Therefore, it is necessary to find a solution that would allow directly calculating the coefficients of the FIR filter with a cascade structure.

5. Methods of research

The following scientific methods were used in the study:

- random search method, which is the basis of genetic algorithm (used for solving optimization problems);
- theory of digital filtering in filter analysis;
- numerical methods for modeling in a Python program.

Methods of mathematical analysis, the theory of linear discrete systems, the theory of digital filtering, operational calculus, numerical methods, mathematical methods of operations research, elements of the theory of computational algorithms and programming were used for the development.

6. Research results

Let's perform the synthesis of the FIR filter in the frequency domain, so it is necessary to set the requirements for the AFR of the filter. As an approximated filter, not an ideal filter will be used, but a filter with the appearance shown in Fig. 7.

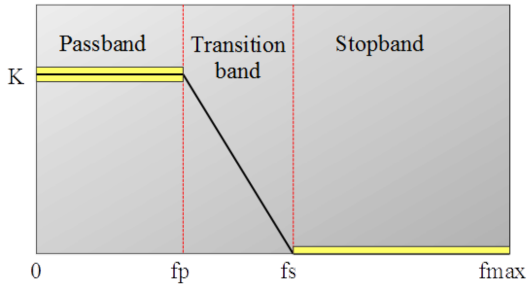


Fig. 7. AFR of the approximated low-pass filter

As an approximating filter, a filter will be used, the AFR of which is given by expression (4).

$$H(\omega)=\sum_{n=0}^{N-1} h(n)\cdot e^{-j\omega n}, \tag{4}$$

where $h(n)$ – the final impulse response of the FIR filter; ω – the angular frequency.

However, as already mentioned, linear phase FIR filters are widely used. This requires the impulse response to be symmetric or antisymmetric [5]. In this case, four types of FIR filters are possible (Table 1).

Table 1

Linear phase filter types

Filter type	Impulse response	Number of impulse response coefficients	Amplitude-frequency response
I	symmetrical	odd	$H(\omega)=\sum_{n=0}^{(N-1)/2} a(n)\cdot\cos(\omega n)$
II	symmetrical	even	$H(\omega)=\sum_{n=1}^{N/2} b(n)\cdot\cos(\omega(n-1/2))$
III	antisymmetric	odd	$H(\omega)=\sum_{n=1}^{(N-1)/2} c(n)\cdot\sin(\omega n)$
IV	antisymmetric	even	$H(\omega)=\sum_{n=1}^{N/2} d(n)\cdot\sin(\omega(n-1/2))$

To optimize filter coefficients, an objective function (fitness function) is required. As an objective function, let's use the standard deviation between the approximated and approximating AFR of the FIR filter (5).

$$e=\sqrt{\int_0^{\pi} W(\omega)\left(H(\omega)-\widehat{H}(\omega)\right)^2 d\omega}\to\min, \tag{5}$$

where $W(\omega)$ – the weight function.

When solving a problem with a genetic algorithm, it is necessary to identify the phenotype that determines the real object. In our case, the filter coefficients will be a phenotype, which for our task will form a chromosome.

As an implementation of the genetic algorithm, a chromosome is a Python class that stores a list of real filter coefficients and other necessary parameters:

```
class Fir1T:
  “”” Non-recursive digital filter of the first type “””

  def init(self, fmax, a):
      self._N=len(a)      # number of FIR coefficients
      self._fmax=fmax     # sampling rate/2 (maximum)
      self._a=a           # list of coefficients
```

The general view of the proposed method for the synthesis of an FIR filter with a cascade structure based on a genetic algorithm is as follows:

1. Creation an initial population;
2. Calculation of the chromosomes fitness;
3. Selection of initial chromosomes (solutions) with the best fitness values for creating a new population;
4. Performing the operation of the chromosomal crossover;
5. Performing the mutation operation;
6. If the stop condition is triggered, let's return the chromosome with the best fitness value, otherwise go to step 2 to process the new population.

To test the efficiency of the method can be used a package of applied programs for solving technical computing problems – Matlab [10, 11]. However, later it was decided to replace it with the Python programming language in mind: not the expressiveness of the built-in Matlab programming language; the high cost of the package. Let's implement modeling for the first type filter (Table 1). The approximated frequency response function will have the form shown in Fig. 8.

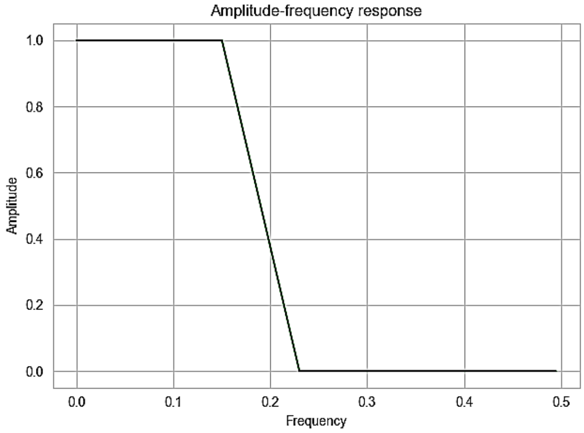


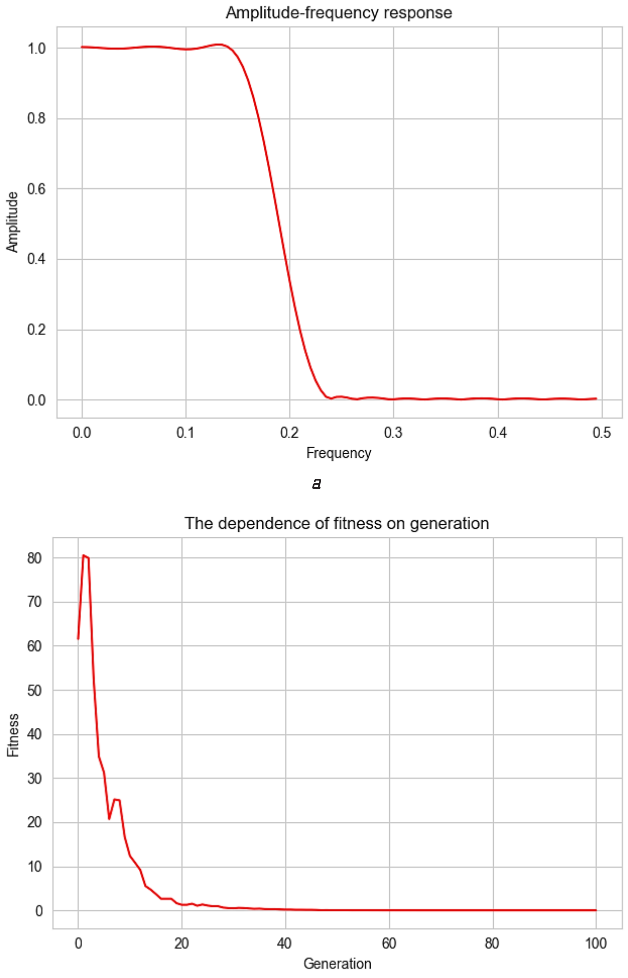
Fig. 8. AFR of the desired FIR filter

To run the genetic algorithm, it is necessary to setup some hyperparameters. In our case, it will look like this:

```
ORDER=24              # filter order
POPULATION=150        # number of individuals
                        in the population
SURVIVOR=0.25         # survival probability
MUTATION=0.05         # probability of an
                        individual mutating
GENERATIONS=150       # maximum number of
                        generations
```

The simulation results are shown in the next two figures: Fig. 9, *a* – synthesized FIR filter of the 24th order; Fig. 9, *b* shows the dependence of fitness on the chromosomes population.

Calculated FIR filter coefficients using a genetic algorithm (Table 2).



a

b

Fig. 9. The results of the proposed method:
a – AFR of the synthesized FIR filter; *b* – the fitness change when searching a solution

Table 2

Coefficients of each stage of the FIR filter

k	b_{0k}	b_{1k}	b_{2k}
0	0.00500739	–0.0155937	0.01283741
1	1	–2.28907715	2.29340521
2	1	0.95592767	1
3	1	1.98590483	1
4	1	1.65039403	1
5	1	0.1683833	1
6	1	0.59197151	1
7	1	1.87621986	1
8	1	1.32148575	1
9	1	–0.1459856	1
10	1	–0.99811282	0.43603285
11	1	–1.21470794	0.39006209

From the given example, it can be seen that it took about 30–40 generations for the result of the synthesis of a filter with a cascade structure based on the genetic algorithm to be close to the optimal parameters.

7. SWOT analysis of research results

Strengths. The studies have shown that the use of the genetic algorithm makes it possible to calculate the coefficients of the FIR filter directly for the cascade structure. This calculation can be performed taking into account the effect of quantizing the coefficients, and not using an additional mathematical apparatus to minimize the error when quantizing the filter coefficients.

Weaknesses. The disadvantages of the method of synthesis of an FIR filter with a cascade structure based on a genetic algorithm can be attributed to a relatively long computation time compared to classical methods of synthesis of filters with a direct structure. The disadvantage can be partially eliminated by using compiled programming languages.

Opportunities. In the future, it is necessary to pay attention to the formation of the initial population, as well as the operations of the genetic algorithm – crossing and mutation, as well as their choice.

Threats. The method for the synthesis of an FIR filter with a cascade structure based on a genetic algorithm, proposed in this work, belongs to evolutionary algorithms, therefore, special attention should be paid to tuning hyperparameters. The proposed version of hyperparameters in most cases is suitable for solving the problem, but in some cases it may be necessary to adjust them.

8. Conclusions

1. The analysis of the existing methods for the synthesis of FIR filters showed that the calculation of the filter coefficients is carried out for the direct form of implementation. If it is necessary to implement filters with a cascade form, it is proposed to recalculate the coefficients.

2. The analysis showed that the use of an ideal filter as an approximated frequency response has a number of disadvantages when using the genetic algorithm: it complicates the search for an optimal solution; computation time increases. To eliminate the disadvantage, it was proposed to use a piecewise-linear function.

3. The result of the synthesis of an FIR filter with a cascade structure based on a genetic algorithm showed that for a 24-order filter it took about 30–40 generations to get the filter parameters close to optimal values.

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