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PROPERTY ANALYSIS OF MULTIVARIATE CONDITIONAL LINEAR RANDOM PROCESSES IN THE PROBLEMS OF MATHEMATICAL MODELLING OF SIGNALS

The object of research is the process of mathematical modelling of a multidimensional random signal, which in the structure of its generation is the sum of a large number of random impulses that occur at random times. Examples of stochastic signals of this type can be, in particular, electroencephalographic and cardiographic signals, photoplethysmography signals, resource consumption processes (electricity, gas, water consumption), radar signals, vibrations of bearings of electric machines and others.

A common mathematical model (especially in the multidimensional case) of this type of signal is a linear random process that allows the signal to be represented as the sum of a large number of stochastically independent random impulses that occur at Poisson moments. If the impulses are stochastically dependent (or the moments of time of their appearance are not Poisson), then the mathematical model is a conditional linear random process. The definition and analysis of the probabilistic properties of such processes for the multidimensional case have not been conducted.

The paper defines a multidimensional conditional linear random process, each component of which is represented as a stochastic integral of a random kernel driven by a process with independent increments. Expressions for the characteristic function and moment functions of the specified process are obtained. The approach used was to use the mathematical apparatus of conditional characteristic functions, as well as the known representation of an infinitely divisible characteristic function of a linear random process as a functional of a process with independent increments.

The obtained results provide a possibility for theoretical analysis of probabilistic properties of multichannel stochastic signals, the mathematical model of which is a multidimensional conditional linear random process. Justification of their properties of stationarity or cyclostationarity, which are the consequence of corresponding properties of the kernel and process with independent increments, can be carried out.

Keywords: *multivariate signal, conditional linear random process, stochastic integral, characteristic function, mathematical expectation, covariance function.*

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1. Introduction

Justification of mathematical and computer simulation models of random information signals is one of the most important stages in the applied problems of information systems development for signal statistical processing for technical or medical diagnostics, forecasting, use in control systems, identification, etc. Important classes of mathematical models that have become widespread are linear [1, 2] and conditional linear random processes (CLRP) [3]. Such models are used for the mathematical representation of signals, the generation mechanism of which allows representing them as the sum of a large number of random impulses that occur at random moments of time. Such signals include, in particular, electroencephalographic signals, photoplethysmography signals, cardio signals, resource consumption processes (electricity, gas, water consumption), vibration signals of electric machines, etc. [2–4].

Analysis of the literature sources shows that for the first time the concept of «conditionally linear random process» has been proposed by the author of [5] in the context of his research conducted at RAND (Research and Development) Corporation on the problem of modelling radar signals and interference. The physically reasonable signal model in [5] is presented as a random process:

$$\xi(t)=\sum_{k=-\infty}^{\infty}\varphi_k(t-\tau_k),$$

which is the sum of large number of random impulses occurring at random moments of time $\ldots<\tau_{k-1}<\tau_k<\tau_{k+1}<\ldots$.

Process $\xi(t)$ is called in [5] linear if $\varphi_k(t)$ are stochastically independent impulses and time moments $\ldots<\tau_{k-1}<\tau_k<\tau_{k+1}<\ldots$ form a Poisson flow. If $\varphi_k(t)$ are stochastically dependent or/and time moments $\ldots<\tau_{k-1}<\tau_k<\tau_{k+1}<\ldots$ are not Poisson times, then $\xi(t)$ is a conditional linear random process. The theory of such random processes and their application for modelling and processing of stochastic signals in technical and medical systems have recently been developed, in particular, by the authors of [3, 4, 6].

However, the multidimensional CLRP and their properties have not been studied yet, and it is an important task for applied mathematical modelling and computer simulation of multichannel stochastic signals.

Thus, the *object of research* is the process of multidimensional random signal mathematical modelling, which in the structure of its generation is the sum of a large number of random impulses that occur at random times.

The aim of research is to obtain expressions for the characteristic function and moment functions of the first and second order of a multidimensional conditional linear random process.

2. Research methodology

Conditional linear random process (one-dimensional) $\xi(\omega,t)$, $\omega\in\Omega$, $t\in(-\infty,\infty)$, given on some probability space $\{\Omega,\mathrm{F},\mathrm{P}\}$, is a stochastic integral of the form [3]:

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)\mathrm{d}\eta(\omega,\tau), \qquad (1)$$

where $\varphi(\omega,\tau,t),\tau,t\in(-\infty,\infty)$ is a real *random* function (kernel); $\eta(\omega,\tau)$, $\tau\in(-\infty,\infty)$, $\mathrm{P}(\eta(\omega,0)=0)=1$ is a real Hilbert stochastically continuous random process with independent increments (generating process); $\mathrm{M}\eta(\omega,\tau)=a(\tau)<\infty$ and $\mathrm{D}\eta(\omega,\tau)=b(\tau)<\infty$, $\forall\tau$; random functions $\varphi(\omega,\tau,t)$ and $\eta(\omega,\tau)$ are *stochastically independent*.

The results of the study of the probabilistic characteristics of process (1) within the framework of correlation theory have been represented in [4], and the analysis of one-dimensional and multidimensional distributions by the method of characteristic functions has been carried out in [3]. Based on the results [3], it is possible to propose the following approach (technique) to finding and analyzing the probabilistic characteristics of a multidimensional conditional linear random process, using the mathematical apparatus of conditional characteristic functions [7–9]. Namely:

– finding the conditional (with respect to σ-subalgebra $\mathrm{F}_\varphi\subset\mathrm{F}$, generated by random function $\varphi(\omega,\tau,t)$) characteristic function of multivariate CLRP using the infinitely divisible representation of the characterisitic function of linear random process [1, 2];

– finding the unconditional characteristic function of multivariate CLRP;

– finding the moment functions of multivariate CLRP using the representation of its multivariate characteristic function.

Further, to simplify the notation, let's omit the symbol ω everywhere.

3. Research results and discussion

Let $\eta(\tau),\tau\in(-\infty,\infty)$ be the Hilbert random process with independent increments and $\Xi(t)=(\xi_1(t),\xi_2(t),\ldots,\xi_n(t))'$ be the n-dimensional conditional linear random process (here and further $(\cdot)'$ denotes a vector, which is transposed to the vector $(\cdot)$) with the elements of the following form:

$$\xi_k(t)=\int_{-\infty}^{\infty}\varphi_k(\tau,t)\mathrm{d}\eta(\tau),\ t\in(-\infty,\infty),\ k=\overline{1,n}, \qquad (2)$$

where $\varphi_k(\tau,t)$, τ, $t\in(-\infty,\infty)$ is a random kernel of the k-th element of the vector (2), $k=\overline{1,n}$.

The diagram representation of the formation of a three-dimensional conditional linear random process with components (2) has been given in the Fig. 1.

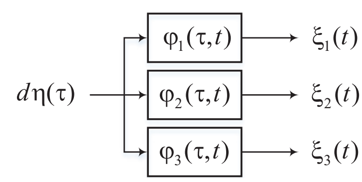

Fig. 1. The diagram representation of a three-dimensional conditional linear random process formation with components (2)

Thus, the components of the vector $\Xi(t)$ are the conditional linear random processes (2) driven by the same process with independent increments but with the different kernels.

The multidimensional CLRP defined above can be used in mathematical modelling problems of multichannel stochastic information signals, which from a physical point of view can be interpreted as driven by a single process, but the channels themselves do not interact.

Let's now consider another case of constructing a multidimensional conditional linear random process that can be used for mathematical modelling of a multichannel signal with cross-channel interaction, but the components of such a signal are driven by separate independent processes.

Let's consider n-dimensional random process $\Xi(t)=(\xi_1(t),\xi_2(t),\ldots,\xi_n(t))'$ with the components of the following form:

$$\xi_k(t)=\sum_{m=1}^{n}\int_{-\infty}^{\infty}\varphi_{km}(\tau,t)\mathrm{d}\eta_m(\tau),\ t\in(-\infty,\infty),\ k=\overline{1,n}, \qquad (3)$$

where $\varphi_{km}(\tau,t)$, $\tau,t\in(-\infty,\infty)$, $k,m=\overline{1,n}$ are stochastically independent real random functions, such that $\int_{-\infty}^{\infty}|\varphi_{km}(\tau,t)|\mathrm{d}\tau<\infty$, $\forall t$ with probability 1; $\eta_m(\tau)$, $\tau\in(-\infty,\infty)$, $\mathrm{P}(\eta_m(0)=0)=1$, $m=\overline{1,n}$ are real Hilbert stochastically continuous random processes with independent increments; processes $\eta_m(\tau)$, $m=\overline{1,n}$ are mutually independent with the following characteristics:

$$K_m(x;\tau),\ \mathrm{M}\eta_m(\tau)=a_m(\tau),$$

$$\mathrm{D}\eta_m(\tau)=b_m(\tau),\ m=\overline{1,n},$$

where $K_m(x;\tau)$, $x\in(-\infty,\infty)$ is a real non-decreasing function with bounded variation (Poisson jump spectrum in Kolmogorov's form) such that:

$$K_m(-\infty;\tau)=0,\ K_m(\infty;\tau)=\int_{-\infty}^{\infty}d_xK_m(x;\tau)=\mathrm{D}\eta(\tau),\ \forall\tau.$$

The diagram representation of the components (3) formation of multivariate conditional linear random process (where $n=3$) has been given in the Fig. 2.

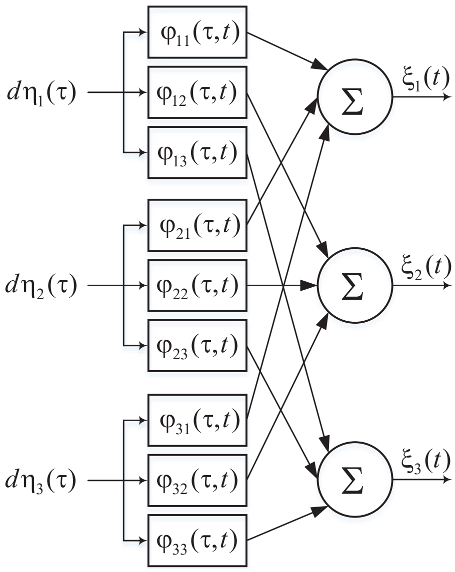

Fig. 2. The diagram representation of the components formation of three-dimensional conditional linear random process (3)

In formula (3) each sum element is a conditional linear random process, and the sum elements are stochastically independent. The characteristic function of the sum of independent random variables is equal to the product of their characteristic functions. Therefore, taking into account the results [3], let's obtain for the characteristic function of n-dimensional CLRP $\Xi(t)$ with the components (3) the following expression:

$$f_{\xi}(u_1,u_2,...,u_n;t_1,t_2,...,t_n)=\mathrm{M}e^{i\sum_{k=1}^{n}u_k\xi_k(t_k)}=$$
$$=\mathrm{M}\left[\mathrm{M}\left(e^{i\sum_{k=1}^{m}u_k\xi_k(t_k)}\middle|\mathrm{F}_{\varphi}\right)\right]=$$
$$=\mathrm{M}\left\{\exp\left[i\sum_{m=1}^{n}\sum_{k=1}^{n}u_k\int_{-\infty}^{\infty}\varphi_{km}(\tau,t_k)da_m(\tau)+\right.\right.$$
$$\left.\left.+\sum_{m=1}^{n}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\begin{array}{c}e^{ix\sum_{k=1}^{n}u_k\varphi_{km}(\tau,t_k)}-1-\\ -ix\sum_{k=1}^{n}u_k\varphi_{km}(\tau,t_k)\end{array}\right)\frac{d_x d_{\tau}K_m(x;\tau)}{x^2}\right]\right\},$$
$$u_k,\ t_k\in(-\infty,\infty),\ k=\overline{1,n}. \qquad (4)$$

Mathematical expectation of each component of multivariate CLRP (3) is equal to:

$$\mathrm{M}\xi_k(t)=\sum_{m=1}^{n}\int_{-\infty}^{\infty}\mathrm{M}\varphi_{km}(\tau,t)\mathrm{d}a_m(\tau),\quad k=\overline{1,n},$$

cross-covariance function of the components $\xi_k(t)$ and $\xi_l(t)$ is equal to:

$$R_{kl}(t_1,t_2)=\mathrm{M}\big((\xi_k(t_1)-\mathrm{M}\xi_k(t_1))(\xi_l(t_2)-\mathrm{M}\xi_l(t_2))\big)=$$
$$=\sum_{m=1}^{n}\sum_{j=1}^{n}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{M}\big(\varphi_{km}^{(0)}(\tau_1,t_1)\varphi_{lj}^{(0)}(\tau_2,t_2)\big)\mathrm{d}a_m(\tau_1)\mathrm{d}a_j(\tau_2)+$$
$$+\sum_{m=1}^{n}\int_{-\infty}^{\infty}\mathrm{M}\big(\varphi_{km}(\tau,t_1)\varphi_{lm}(\tau,t_2)\big)\mathrm{d}b_m(\tau),\ k,l=\overline{1,n}, \qquad (5)$$

where $\varphi_{km}^{(0)}(\tau,t)=\varphi_{km}(\tau,t)-\mathrm{M}\varphi_{km}(\tau,t)$ is a centered kernel of CLRP.

The obtained results make it possible to conduct a theoretical analysis of the probabilistic properties of multichannel stochastic signals, the mathematical model of which is a multidimensional conditional linear random process. In particular, the conditions under which multidimensional CLRP is a wide-sense stationary or strict-sense stationary, periodically correlated, cyclostationary [10], or ergodic, can be obtained on the basis of the above relations for any specific application, taking into account the specifics of the kernel and process with independent increments. An example of establishing such conditions for one-dimensional CLRP has been represented, for example, in [3]. Based on the relevant theoretical analysis, a decision is also made on the strategy and choice of methods of statistical processing and computer simulation of the investigated signals, as well as the identification of informative features.

Obviously, the condition for applying the above results to specific practical problems is the necessity to study and taking into account the physical (biophysical, economic, etc.) nature of the generation of the investigated multichannel signal when building its mathematical model. In this case, it is possible, for example, that the times flow of occurrence of random impulses that form a signal is not Poisson, then the obtained expression for the characteristic function does not hold.

The methodology of the practical application of the model of multidimensional CLRP in the direction of methods development of its characteristics statistical estimation, informative features detection, etc. should be further developed.

Table 1 illustrates the main differences in the structure of multidimensional CLRP (defined in this paper) and other known models of random signals allowing representation in the form of a stochastic integral driven by a process with independent increments. It can be seen that the model of multidimensional CLRP (expressions for the main probabilistic characteristics of which are obtained in this paper) is the most general among those represented in the table.

Table 1

Comparison of the random signal models allowing representation as a stochastic integral driven by a process with independent increments

Model	Model components		Range of a process, R^m
	Kernel	Generating process	
Linear random process [1, 2, 11]	Non-random function	Process with independent increments	$m\geq 1$
Conditionally linear random process [5]	Random function	Levy process	$m=1$
Filtered stochastic point process [6]	Non-random function with random parameters	Poisson process, general counting process	$m\geq 1$
Volatility modulated Levy-driven Volterra process [12]	Product of non-random function and nonnegative stochastic process	Levy process	$m\geq 1$
Conditional linear random process [3, 4]	Random function	Process with independent increments	$m=1$
Multivariate conditional linear random process	Random function	Process with independent increments	$m>1$

Note: $\mathbf{R}=(-\infty,\infty)$; Levy process is a stochastically continuous random process with independent stationary increments

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The obtained results can be used, for example, in the problems of mathematical modelling and processing of multichannel electroencephalographic (EEG) signals for the purpose of medical diagnostics. In [4] the justification of the mathematical model of such a one-channel signal in the form of one-dimensional CLRP has been given. On the basis of the signal model, the methods of its statistical processing with the use of the random coefficients autoregressive model are further developed. However, complex medical diagnostics based on EEG signals is usually based on the analysis of electrical activity generated simultaneously by many different parts of the cerebral cortex (up to 20 electrodes can be used simultaneously), which interact with each other. Therefore, the development of results [4] in the direction of mathematical modelling of multichannel EEG signals based on multidimensional CLRP is a prospective task.

4. Conclusions

The definition of a multidimensional conditional linear random process with the components represented in the form of stochastic integrals from random kernels driven by processes with independent increments has been given in the paper. If the generating process is an inhomogeneous compound Poisson process, then multidimensional CLRP can be used for mathematical modelling of a multichannel stochastic signal, which can be represented as the sum of a large number of stochastically dependent impulses that occur at Poisson times.

Expressions for the characteristic and moment functions of multidimensional CLRP have been obtained, which makes it possible to carry out a theoretical analysis of the investigated signal probabilistic characteristics, and to justify the methods for their statistical processing.

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