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DEVELOPMENT OF A MATHEMATICAL MODEL OF THE OPERATION OF THE SWAB GENERATOR VALVE

The object of research in the work is a mathematical model of the operation of a hydrodynamic well generator of the spring-valve type. The existing problem lies in the fact that vibrating wave swabbing differs from known methods of vibrating wave influence on the near-breakout zone of the formation in its ease of execution and, as a result, a much smaller range of equipment. At the same time, its effectiveness should not be inferior to known methods because the treatment of the formation is carried out by elastic low-frequency pressure fluctuations against the background of depression on the formation. Since the swab generator valve is, in fact, a hydrodynamic well generator (HWG) of the spring-valve type, the processes of pressure fluctuations in the considered devices will be similar. However, because of the review of HWG designs, the presence of a mathematical model describing the operation of rotor-type devices, and not valve type, was revealed. Despite the fact that the mathematical model takes into account all the main factors that determine the process of the HWG piston movement, this model cannot be used to describe the operation process of the swab generator valve. Due to the absence of a pulsating flow created at the wellhead by pumping units. Taking this fact into account, as well as taking into account the compressibility of the liquid passing through the valve, a new mathematical model was developed. It describes both the process of operation of a spring-valve type HWG operating in a well on pump-compressor pipes, and the operation of a swab generator valve that is lowered into a well on a rope. Thanks to this, it is possible to obtain the parameters of the oscillating system: geometric dimensions, inertial mass of moving parts, stiffness of the spring, coefficient of viscous friction between the piston and the valve body, as well as parameters of the working fluid: pressure, flow and its rheological properties. In comparison with the similarly known technology of vibration wave impact on a formation with known HWG, it provides such advantages as a higher speed of movement and frequency of oscillations of the moving end of the spring from the identified essential parameters of the oscillating system and the working fluid of the spring (100–1000 N/mm), consumption of working liquid (10–130 m^3/h), valve opening pressure (1–7 MPa). This will make it possible to calculate a helical cylindrical compression spring made of round steel for its operation in the valve of the swab generator without collision of turns.

Keywords: *hydrodynamic well generator, mathematical model, spring deformation, valve opening pressure, spring stiffness.*

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1. Introduction

Vibratory wave swabbing differs from the known methods of vibrowave impact on the bottomhole formation zone (BFZ) by the ease of implementation and, as a result, by a much smaller range of equipment. At the same time, its efficiency should not be inferior to known methods, because the treatment of the formation is carried out by elastic low-frequency pressure fluctuations against the background of depression on the formation.

In this regard, the researchers tried to compile a mathematical model for the operation of a hydrodynamic downhole generator of a spring-valve type [1], as well as to substantiate its optimal parameters [2]. In the developed mathematical model [3], the movement of the HWG piston is determined by the parameters of the oscillatory system. Namely, the mass of the piston, the stiffness of the spring, the coefficient of viscous friction between the piston and the generator housing. Also, the impact parameters are determined by pressure, speed, frequency of the pulsating flow and its rheological properties.

An analysis of the mathematical model of the operation of a swab generator valve is necessary, first of all, in order to identify the parameters of the oscillatory system and the working fluid. They have a significant impact on the operating parameters of the valve spring, which must be known for the correct selection or design of the spring, during which there is no collision of its adjacent coils, and, therefore, the specified reliability of the valve operation is ensured.

The object of research in the work is a mathematical model of the operation of a hydrodynamic downhole generator of a spring-valve type.

The aim of research is to substantiate the optimal parameters of the downhole generator.

2. Research methodology

Changes in these parameters depending on the parameters of the oscillatory system and the parameters of the working fluid are considered. In this case, the parameters of the oscillatory system will be understood as the inertial mass of the piston and parts of the spring (m) and the stiffness of the spring (c).

Under the parameters of the working fluid, we mean the flow rate of the working fluid (Q) and the pressure of the working fluid at which the valve windows open (p_2). As mentioned earlier, the system of differential equations (1) describes three stages in the operation of the HWG valve [4].

In the system of differential equations (1), the first equation describes the process of piston movement at the first stage – the separation of the piston from the valve seat before opening the windows. The second equation describes the process of piston movement at the second stage – from the moment the valve windows open and the piston moves down, as well as its closing and the piston moves up until the windows close. The third equation describes the process of piston movement at the third stage – from closing the valve windows to top dead center [5].

For a true description of the process, the dependence of the coefficients of equations (1) on variables is taken into account. Consequently, differential equations are nonlinear and numerous integration methods are used to solve them [6]:

$$\begin{cases} z=\dfrac{\dfrac{Q\cdot t}{V_w+Q\cdot t}\cdot\dfrac{1}{\beta_p}\cdot S_p,}{c}-s_1, \\ A_1\cdot\dfrac{d^2z}{dt^2}+B_1\dfrac{dz}{dt}+C_1\cdot z-D_1\cdot z^2-E_1=0, \\ A_2\cdot\dfrac{d^2z}{dt^2}+B_2\dfrac{dz}{dt}+C_2\cdot z+D_2=0, \end{cases} \quad (1)$$

where z – the coordinate of the upper part of the piston; Q – the flow rate of the working fluid; t – time; V_w – the volume of the working fluid; β_p – the coefficient of relative volumetric bending of the working fluid at a given hydrostatic pressure of the working fluid; S_p – the cross-sectional area of the piston head; S_1 – the area of the side surface of the piston, along which the working fluid layer is displaced; A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, E_1 – coefficients of transformations into a simplified form; dz/dt – the piston speed towards the z axis; d^2z/dt^2 – the acceleration of the piston towards the z axis.

The system of equations (1) was solved in the mathematical package MathCAD using the 4th order Runge-Kutta method [7] with a constant integration time step.

3. Research results and discussion

The HWG valve operation process is divided into three stages, each of which is described by its own equation. In this case, the fitting method [8, 9] was used, the essence of which is that the values of the variables obtained at the end of the previous stage are taken as the initial ones for the next stage. The joint solution of the system of equations within each stage excludes the occurrence of numerical instability of the solution and thereby eliminates the perversion of the process [10].

The graphical solution of the system of differential equations (1) with the values of the variables (Table 1) that determine the value of the coefficients of the system is shown in Fig. 1.

During the time period $T1$, the spring is compressed as a result of the pressure exerted over the piston area of the valve and the activity on the piston. In the time period $T2$, the valve opens and pressure is released above the piston area of the valve. During the time period $T3$, the piston fluid is compressed under the action of the spring force and the piston returns to its original position.

Table 1

The value of the variables of the system of equations (1)

Value	Parameters
Valve opening pressure (p_2), MPa	2.0
Valve flow rate (μ_v)	0.62
Valve window width (b), mm	24
Distance between valve and piston (δ), mm	0.2
Piston head diameter (d_p), mm	61.6
Piston head height (h_p), mm	61.6
Piston tail diameter (d_{tp}), mm	38
Piston tail height (h_{tp}), mm	15
Piston material density (ρ_p), kg/m^3	7850
Piston mass (m_p), kg	1.575
Spring outer diameter (D_1), mm	60
Spring wire diameter (d), mm	12
Number of spring turns (n_1), pcs.	21.5
Spring mass (m_s), kg	2.861
Spring stiffness (s), N/m	91900
Working fluid barrel diameter (d_{wf}), mm	62
Working fluid barrel length (L_{wf}), mm	1000
Working fluid flow rate (Q), m^3/h	50
Working fluid density (ρ_{wf}), kg/m^3	1000
Coefficient of relative volumetric bending of the working fluid (β_{wf}), 1/Pa	$48\cdot10^{-10}$
Dynamic viscosity of the working fluid (μ), Pa·s	$893.7\cdot10^{-10}$
Static displacement stress of the working fluid (τ_0), Pa	0

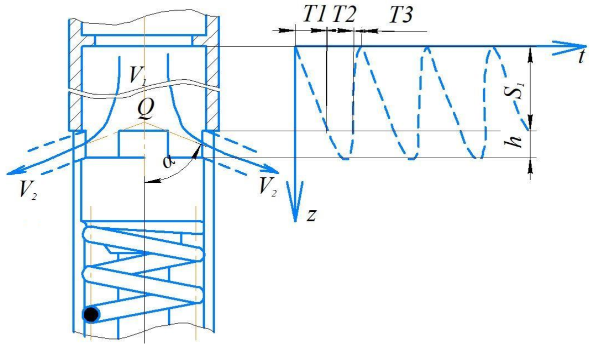

Fig. 1. Schematic representation of the piston movement: Q – flow rate of the liquid; z – piston displacement; t – time; s_1 – preliminary deformation of the spring; h – working stroke of the spring; $T1$ – period of spring compression by the pressure of the working fluid; $T2$ – valve opening period; $T3$ – period of compression of the working fluid by the spring; V_1 – initial volume of the working fluid; V_2 – final volume of the working fluid, α_v – angle of inclination of the flow flowing out of the valve window

As a result of the research, it was found:

– that the spring preload (s_1) is influenced by the spring constant (c) and the valve opening pressure (p_2). As p_2 increases by a factor of 4, s_1 decreases by a factor of 4. As p_2 increases by 4, s_1 increases by a factor of 4. Thus, the spring rate and the valve opening pressure have a significant effect on the pre-strain of the spring, however, the valve opening pressure has a direct effect, and the spring rate has the opposite effect;

– that the working stroke of the spring (h) is affected by the spring constant (c), the inertial mass (m) and the flow rate of the working fluid (Q). The effect of the valve opening pressure (p_2) is negligible. When c increases by 4 times, h decreases by 6.5 times. When m is increased by 4 times, it increases by 2 times. With an increase in Q by 4 times, it increases by 4.1 times. Thus, the greatest influence on the working stroke of the spring is exerted by the stiffness of the spring and the flow of the working fluid;

– that the maximum speed of movement of the moving end of the spring (v_{max}) is affected by the stiffness of the spring (c) and the flow rate of the working fluid (Q). When c increases by 4 times, v_{max} decreases by 4 times. As Q quadruples, v_{max} quadruples. Thus, the greatest speed of movement of the movable end of the spring has a significant, but opposite effect of the spring stiffness and the flow of the working fluid;

– that the oscillation frequency of the moving end of the spring (ν) is affected by the stiffness of the spring (c), the flow rate of the working fluid (Q), the opening pressure of the valve (p_2). With an increase in c by 4 times, it increases by 1.8 times. With an increase in Q by 4 times, v_{max} increases by 1.7 times. With an increase in p_2 by 4 times, it decreases by 1.7 times.

Thus, the stiffness of the spring has the greatest influence on the frequency of oscillation of the moving end of the spring.

As a result of the study of the mathematical model of the operation of the swab generator valve, the essential parameters of the oscillatory system and the working fluid were found, and a relationship was established with the operating parameters of the valve spring in the form of formulas. Spring preload is most significantly affected by spring stiffness and valve opening pressure. Fig. 2 shows the dependence of the pre-strain of the spring on the specified parameters. These dependencies are described by the formula:

$$s_1 = 0.00298 \cdot p_2 \cdot c^{-1}. \quad (2)$$

The working stroke of the spring is most significantly affected by the stiffness of the spring and the flow rate of the working fluid. The inertial mass of the piston and the movable part of the spring, due to small practical variations, is recognized as having no significant effect, as well as the opening pressure of the valve.

Fig. 3 shows the dependence of the working stroke of the spring on the identified significant parameters. These dependencies are described with sufficient accuracy by the formula:

$$h = 12 \cdot Q \cdot c^{-1}. \quad (3)$$

The error of the values obtained by formula (3) does not exceed 6.8 %.

The maximum speed of movement of the moving end of the spring is most significantly affected by the stiffness of the spring and the flow rate of the working fluid. Fig. 4 shows the dependence of the maximum speed of movement of the moving end of the spring on the identified significant parameters. These dependencies are described with sufficient accuracy by the formula:

$$v_{max} = 0.572 \cdot Q \cdot c^{-1}. \quad (4)$$

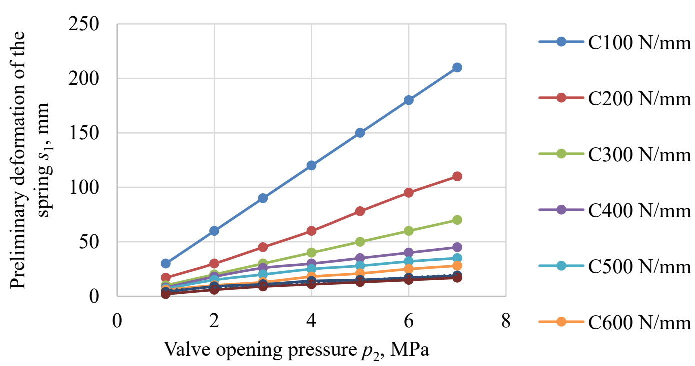

Fig. 2. Dependence of the preliminary deformation of the spring on the opening pressure of the valve at different spring stiffness

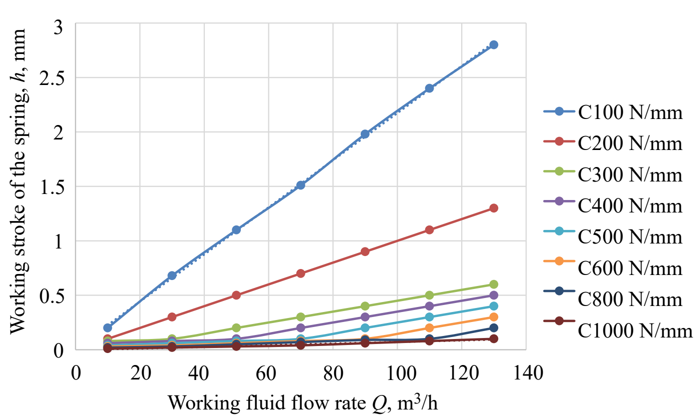

Fig. 3. Dependences of the working stroke of the spring on the working fluid flow rate for different spring stiffness

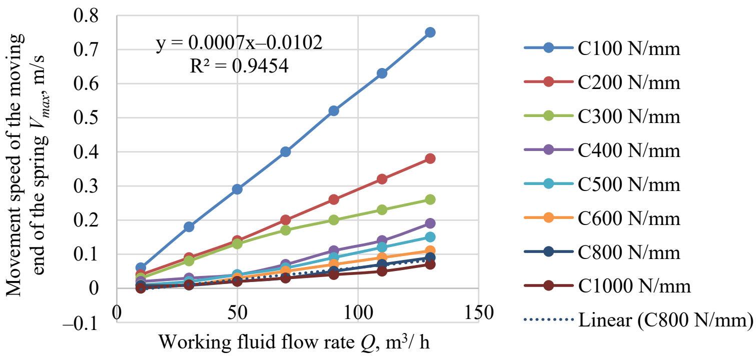

Fig. 4. Dependences of the maximum speed of movement of the moving end of the spring on the working fluid flow rate for different spring stiffness

structive and technological point of view. From a constructive point of view, their advantages lie in the simplicity of design and manufacturability. From a technological point of view, the advantage of a valve-type HWG lies in the possibility of creating high-amplitude and, at the same time, low-frequency pressure fluctuations, which is unattainable in most known rotary-type HWG designs.

Due to the absence of the need to create high costs of the working fluid, valve-type HWGs have relatively low hydraulic losses; therefore, the efficiency is greater than that of the rotary HWG.

The error of the values obtained by formula (4) does not exceed 4.6 %.

The nature of these dependencies is shown in Figs. 5–7.

Fig. 5 shows that even at the maximum flow rate of the working fluid (up to 130 m^3/h) and the maximum stiffness of the valve spring (900 N/mm), the frequency of elastic pressure oscillations will not exceed 10 Hz. This is well within the range recommended for vibrowave action frequencies (up to 20 Hz).

However, from the point of view of increasing the efficiency of stimulation of the formation, it is necessary to strive to reduce the frequency of elastic oscillations, as well as increase their amplitude, which means opening the swab generator valve under the action of as much pressure as possible that is, the valve opening pressure.

Fig. 6 clearly shows that the valve opening pressure reduces the frequency, which also increases the efficiency of the vibration wave assimilation.

In further studies, it is necessary to take into account the fact that the inertial mass of the moving parts of the valve does not affect the preliminary deformation of the spring, practically does not affect the highest speed of the moving end of the spring and the frequency of oscillation of the moving end of the spring. The inertial mass only affects the degree of opening of the valve, such as the stroke of the spring. Also, the opening pressure of the valve has little effect on the working stroke of the spring and the highest speed of movement of the moving end of the spring, but it has a significant effect on all other parameters of the spring. The working stroke of the spring is most significantly affected by the stiffness of the spring and the flow rate of the working fluid. The inertial mass of the piston and the movable part of the spring, due to small practical variations, is recognized as having no significant effect, as well as the opening pressure of the valve.

As a result of the research, it was found that the designs of valve-type hydrodynamic downhole generators are the most promising for joint use with a swab, both from a con-

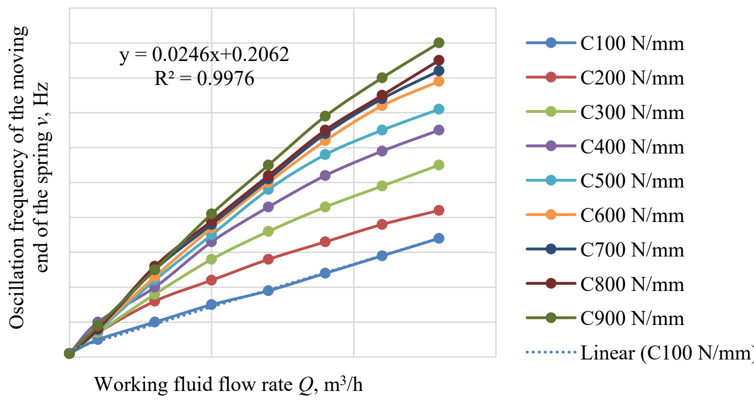

Fig. 5. Dependences of the oscillation frequency of the moving end of the spring on the working fluid flow rate for different spring stiffness

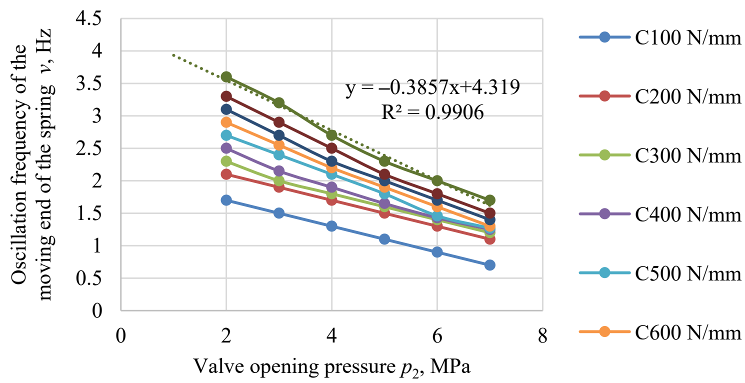

Fig. 6. Dependences of the oscillation frequency of the moving end of the spring on the valve opening pressure for different spring stiffness

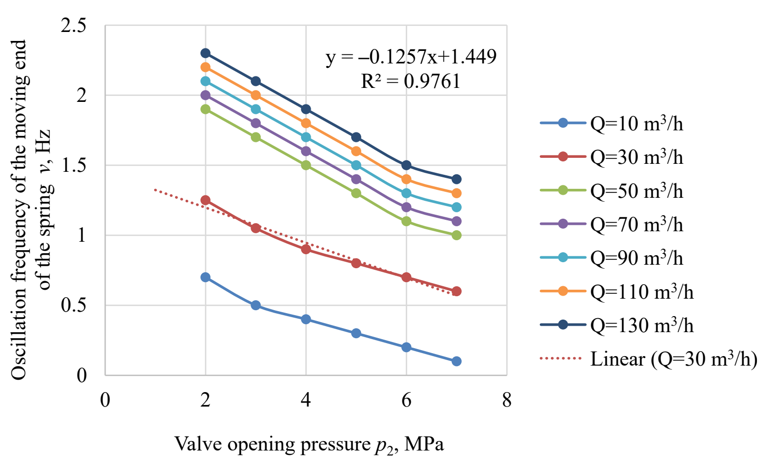

Fig. 7. Dependences of the oscillation frequency of the moving end of the spring on the valve opening pressure at different flow rates of the working fluid

In the future, it is possible to conduct a comparative analysis of rotary and valve type devices, justify which of them will be the best inducing inflow to the well, and conduct their technical and economic assessment during implementation.

4. Conclusions

As a result of the study of the mathematical model of the operation of the spool generator valve, it was found that the oscillation frequency of the movable end of the valve spring depends to the greatest extent on the stiffness of the spring, the flow rate of the working fluid passing through the valve, and the opening of the valve pressure.

The parameters of the oscillatory system and the working fluid that most affect the working parameters of the spring are determined. The nature of the dependencies between the specified parameters has been established, which allow designing springs taking into account the provision of the necessary inertial clearance, which will make it possible to exclude the collision of adjacent coils of the spring. This, in turn, will lead to an increase in the reliability of the operation of hydrodynamic generators of the borehole-valve type.

It has been established that even at the maximum flow rate of the working fluid (up to 130 m^3/h) and the maximum stiffness of the valve spring (900 N/mm), the frequency of elastic pressure oscillations will not exceed 10 Hz, which is well within the frequency range of vibrowave action (up to 20 Hz).

Conflict of interests

The authors declare that there is no conflict of interest regarding this study, including financial, personal nature, authorship or other nature that could affect the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

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