UDC 537.8:539.184:535.14:535.33:621.373
DOI: 10.15587/2706-5448.2023.277346

**Khadidja Khelil,
Azzeddine Dekhane,
Aissa Benselhoub,
Stefano Bellucci**

HIGHER ORDER DISPERSIONS EFFECT ON HIGH-ORDER SOLITON INTERACTIONS

The object of the research is deleting the interaction of the higher order soliton interaction by introducing the third and fourth order dispersions inside an optical fiber. The results are obtained by the simulation of the nonlinear Schrödinger equation, which models the propagation of solitons in the optical fiber using the method of Fast Fourier Transform.

The interaction of two higher order solitons due to the attraction of their electric field can lead to losing the solitons' properties. Hence, this can prevent the use of solitons in high-bit-rate optical fiber communication systems because it increases the bit error rate, significantly limiting the potential of the communication system. To resolve this problem, we should diminish the bit rate error by avoiding the interaction of the co-propagative solitons when they are too close.

It is well known that, during higher order soliton propagation in the presence of the third order dispersion, the irregular shape of the higher order soliton disappears, and a splitting towards its fundamental constituents occurs after a considerable propagation. As for the fourth order, dispersion gives rise to two dispersive wave sidebands on the red or blue side. Our results reveal that bringing two higher order solitons together in the presence of the fourth order dispersion, a series of interactions between the components generated after their fission is obtained. In the third-order distribution, besides the fourth-order diffusion, the rare form and the supercontinuum generated by the fission of the higher-order solitons disappear, and we get two fundamental solitons propagating in parallel with a temporal shift and some inconsiderable dispersive waves. The most important aspect is that both higher-order dispersions are able to suppress the interactions of higher-order solitons thanks to the time shift induced by the third-order distribution and the intermittent compression caused by the fourth-order scattering. These results can be obtained in practice inside the dispersion-engineered photonic crystal waveguide (PhC-wg), which allows for manipulating the high order dispersion.

Keywords: *higher order solitons, soliton fission, dispersions, nonlinearity, optical fiber, supercontinuum.*

Received date: 19.02.2023
Accepted date: 14.04.2023
Published date: 28.04.2023

How to cite
Khelil, K., Dekhane, A., Benselhoub, A., Bellucci, S. (2023). Higher order dispersions effect on high-order soliton interactions. Technology Audit and Production Reserves, 2 (1 (70)), 24–29. doi: https://doi.org/10.15587/2706-5448.2023.277346

1. Introduction

A higher-order soliton is a soliton pulse, the energy of which is higher than that of a fundamental soliton by a factor which is the square of an integer number (i. e. 4, 9, 16, etc.). The temporal shape of such a pulse is not constant but somewhat varies periodically during propagation. The period of their evolution is the so-called soliton period.

Higher-order solitons take a physically interpreted periodic pattern through the higher cheerful chirp produced by Self Phase Modulation (SPM) than the negative chirp produced by Groupe Velocity Dispersion (GVD). Consequently, they need help to fully counterbalance each other. Contrarily to the fundamental solitons, the higher order solitons are destructible in the presence of all kinds of perturbations: Hamiltonian (birefringence, dispersion) or non-Hamiltonian (attenuation, Raman Effect, self-stiffening) [1–3]. The impressive phenomenon concerning higher order solitons is the fission of higher order solitons into N fundamental solitons where N is the order of the soliton. This phenomenon is nicknamed «higher order soliton fission» and has led to extensive applications, the most famous of which is supercontinuum generation [4–6].

Nevertheless, an additional phenomenon provokes the fission of the higher order solitons. This is the interaction of higher order solitons creating ultrashort solitons with a tunable wavelength [7]. Interaction fission appears due to electric field attraction that depends on the temporal separation between the pulses and the difference in the relative phases of the vibrations.

The fission of the higher-order solitons caused by the interaction brings two solitons closer together. This creates a mutually exchanged perturbation between two solitons that fission after a distance L_s [8] defined as the fission distance:

$$L_S = \frac{L_D}{N} = \sqrt{\frac{\tau_0}{|\beta_2 \gamma P_0|}}, \quad (1)$$

where L_D – dispersive length; N – soliton order; τ_0 – time width of the pulse; β_2 – second order dispersion; γ – nonlinearity coefficient; P_0 – pulse power.

The fundamental solitons created after the interaction do not have the same intensity or undergo the same time shift during propagation.

The intensity of the fundamental solitons generated is:

$$P_j = P_0 (2N - 2j - 1)^2. \qquad (2)$$

Such that N is the order of the soliton; j is the number of fundamental solitons generated; P_0 is the initial power of the pulse.

The fission of the solitons by bringing the adjacent solitons closer together induces a great attraction that causes their disintegration into N fundamental solitons. No energy is exchanged between the fundamental solitons created by fission since their intensity is constant during propagation. The length of the fission depends on the order of the soliton.

The fundamental solitons do not appear simultaneously if the phases are opposite. They appear one by one. While solitons are in-phase, they are born at the same point, and the fundamental solitons are therefore generated together. The most energetic soliton ($k=1$) with the shortest temporal width is the Raman soliton, primarily responsible for non-solitonic radiation (NSR). As for the secular shift, if they are in phase, the time shift of the fundamental solitons differs from the offset when they are out of phase because in the first case, the dress is constant, while in the second case, it is done with a variable speed.

Since this study focuses on the influence of the effect of the second, third and fourth order dispersion on the interactions of the higher order solitons [9–12], it is possible to review their influence on the propagation of the fundamental solitons and their propagation before introducing it on the higher order solitons. The Third Order Dispersion (TOD) has two significant applications in transmission chains: In the literature, [13–16] it is mentioned that the shift of a soliton under the effect of the dispersion of the third order is accompanied by a creation of a wave nicknamed «the dispersive wave» [17–20]. This occurs a temporal shift more than the main soliton under the effect of the TOD, and the energy transfer from the soliton to the dispersive wave is also proportional to the TOD. This effect cannot be overlooked for solitons with a temporal width of less than 5 ps [21–23]. When two solitons propagate together, their interactions are avoidable thanks to the term of the TOD. Let's note that the periodic attraction disappears, and the two solitons remain separate if the TOD is with or without GVD. If the TOD and the GVD act together, each fundamental soliton shifts in time and expands according to the sign of the TOD without any interaction induced by the Kerr effect.

Nevertheless, when the GVD is zero, the TOD weakens the interaction induced by the Kerr effect but does not entirely suppress it because there is a slight attraction after propagation of about $27L_D$ [24–26]. It is clear that the appeal is not symmetrical: the leading soliton is attracted to the trailing soliton if the TOD is negative, and vice versa if the TOD is negative. Except for this point of attraction, the two solitons remain separate and keep their initial characteristics; there is only a time lag that depends on the sign of the TOD, as explained above.

As for the fourth-order dispersion [27, 28], properly designed microstructured fibers may possess this particular dispersion characteristic. It induces a periodic pulse compression without any temporal shift or fission. This period is inversely proportional to the fourth-order dispersion. Let's also note that the intensity at the points with maximum reduction is higher when the fourth-order distribution is more robust.

The mixture of the GVD and Fourth order dispersion and the proper value of TOD may be a solution to avoid the interaction of the fundamental solitons [29]. Still, this solution has never been used to prevent the interactions of the higher order solitons to the best of our knowledge.

For two co-propagative solitons subject to Fourth Order Dispersion (FOD) and GVD, the addition of the TOD may create a separation between them because of the shift in time that it induces. The direction of the temporal growth depends on the sign of the TOD, and its strength is proportional to the value of the TOD. However, the leading and trailing pulses are not affected by the TOD in the same way, except for when gathering the GVD, TOD and FOD is precisely balanced by the Kerr nonlinearity, here they stay in parallel. Otherwise, if gathering the three distribution orders is not counterbalanced with the nonlinearity, the two pulses interact after considerable propagation. The interaction can happen because the trailing pulse catches the leading pulse or vice versa, according to the sign. On the other hand, the TOD may weaken the intermittent compression induced by the FOD.

2. Materials and Methods

The propagation of solitons through an optical fiber can be modeled by the famous Non-Linear Schrodinger (NLS) equation. For optical pulses in the picosecond regime, vibrations can be described using the NLS model. But, in the femtosecond regime, the higher-order effects such as the third-order dispersion, fourth-order dispersion, self-steepening, and delayed non-linear response should be considered. In this case, the propagation is described.

By the Higher-Order Nonlinear Schrodinger (HNLS) equation [12, 13], what is typical between higher-order effects is that they cause a shift of the fundamental soliton in the time spectrum by leading to a variation in the travelling speed of the fundamental soliton. The temporal change is transcribed in the frequency spectrum as the creation of a dispersive wave, a result confirmed in several experimental works [5, 15]. Indeed, the NLS equation is a partial differential linear equation, and it is challenging to solve analytically:

$$\frac{\partial A(Z,T)}{\partial Z} + \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} - \\ -\frac{i}{6}\beta_3 \frac{\partial^3 A}{\partial T^3} - \frac{i}{24}\beta_4 \frac{\partial^4 A}{\partial T^4} = +i\gamma\left[|A|^2 A\right]. \qquad (3)$$

Or the normalized form:

$$i\frac{\partial u}{\partial \xi} + |u|^2 u + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} - i\delta_3 \frac{\partial^3 u}{\partial^3 \tau} - i\delta_4 \frac{\partial^4 u}{\partial^4 \tau} = 0. \qquad (4)$$

The solution to the equation above is:

$$A = \left[3\cdot(b^2)\cdot A_0(sech\cdot(T/T_0)^2))\right]\cdot e^{i(8/5)\cdot(dz/2)\cdot b^2}, \qquad (5)$$

where

$$\delta_4 = \frac{\beta_4}{24\cdot|\beta_2|\cdot(T_0)^2},$$

ISSN 2664-9969

as

$$b = \frac{1}{(40 \cdot \delta_4)^2}.$$

The nonlinear Schrodinger equation for higher order solitons is written as:

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + N^2 |u|^2 u = 0°. \quad (6)$$

The soliton propagates in the form of a higher-order soliton in the case where N takes values more significant than 1:2, 3, 4 and 5; as it depends on Z, its expression is given by:

$$A = N^2 \cdot A_0 \cdot sech\left(\frac{T}{T_0}\right) \cdot exp\left(i \cdot \frac{\pi}{4} \cdot \frac{Z}{Z_0}\right), \quad (7)$$

or

$$u(z,\tau) = N \cdot sech(\tau) \cdot e^{iz/2}. \quad (8)$$

The soliton period is:

$$Z_0 = \frac{\pi}{2} \cdot \frac{T_0^2}{|\beta_2|}. \quad (9)$$

The soliton order is:

$$N^2 = \gamma \cdot P_0 \frac{T_0^2}{|\beta_2|}. \quad (10)$$

Dispersive length is:

$$L_D = \frac{T_0^2}{|\beta_2|}. \quad (11)$$

Nonlinear length is:

$$L_{NL} = \frac{1}{\gamma P_0}. \quad (12)$$

3. Results and Discussion

Table 1 illustrates the value used for the simulation of the equations (5), (13) and (14).

Table 1

Simulation values

Parameter	Values	Units
Non-linear parameter γ	1.0	1/W·km
Dispersion of the second order β_2	–25	ps^2/km
Fibre length L	80	Km
Width of the impulse T_0	5.0	Ps

3.1. The propagation of one higher-order soliton with the second and the fourth-order dispersion. Fig. 1 shows the propagation of the 2nd, 3rd and 4th order soliton with the second and the fourth order dispersion.

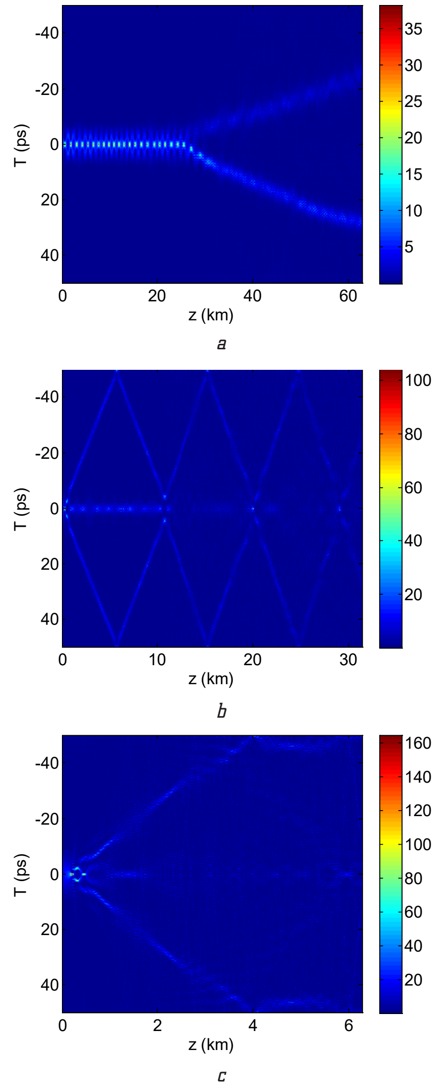

Fig. 1. The effect of FOD on the propagation of one higher-order soliton: *a* – the propagation of the 2nd-order soliton with the second and the fourth-order dispersion $\beta_4 = 1000$ ps^4/km; *b* – the propagation of the 3rd-order soliton with the second and the fourth-order dispersion $\beta_4 = 1000$ ps^4/km; *c* – the propagation of the 4th order soliton with the second and the fourth-order dispersion $\beta_4 = 1000$ ps^4/km

The propagation of the higher order solitons subjected to the FOD leads to the fission into three components: the main pulse and two dispersive waves at the blue and the red side succeeded by an attraction of the three components. This phenomenon is repeated in a periodic may, but the period is not constant because it gets smaller with the propagation. This scenario depends on two parameters: the order of the soliton and the value of the FOD coefficient. Notice that by keeping the same order of the soliton (Fig. 1) and changing the value of the FOD coefficient, the fission length and the speed of the attraction and repulsion get clearly affected. Those proprieties also change if to keep the same FOD value and change the soliton order.

3.2. The propagation of two higher-order solitons with the second and the fourth-order dispersion $A = [3 \cdot (b^2) \cdot A_0(sech \times \times (T/T_0 - 5)^2)) + 3 \cdot (b^2) \cdot (A_0(sech \cdot (T/T_0 + 5)^2))] \cdot e^{i(8/5) \cdot (dz/2) \cdot b^2}$. Fig. 2 shows the propagation of two 2nd, 3rd and 4th order soliton with the second and the fourth order dispersion.

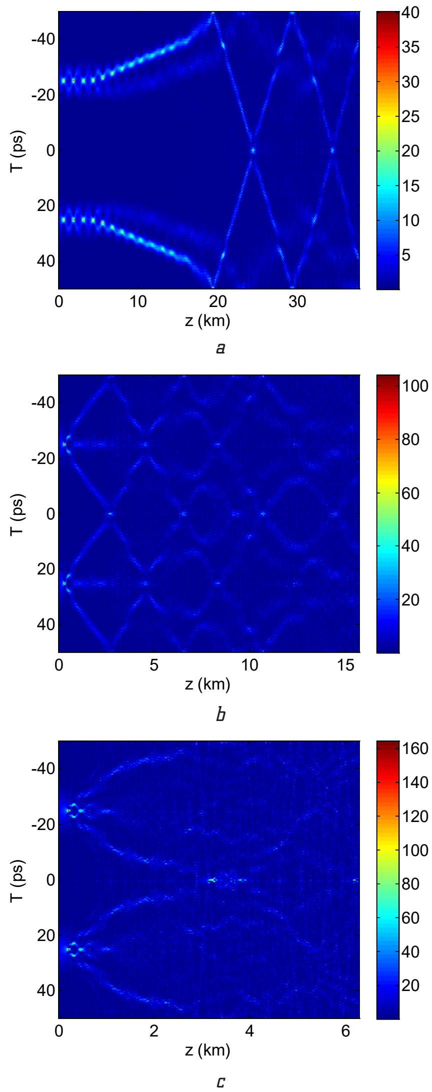

Fig. 2. The effect of FOD on the propagation of two higher-order soliton: *a* – the propagation of two 2nd order soliton with the second and the fourth-order dispersion $\beta_4=1000$ ps^4/km; *b* – shows the propagation of two 3rd order soliton with the second and the fourth-order dispersion $\beta_4=1000$ ps^4/km; *c* – the propagation of two 4th order soliton with the second and the fourth-order dispersion $\beta_4=1000$ ps^4/km

The propagation of two higher order solitons subjected to the FOD leads to several interactions besides their fission. The fission is the same as described above for the two solitons except for a variation in the fission length that comes significantly earlier because of the Kerr-induced interaction. After the fission, the fundamental solitons born get periodically attracted between them.

3.3. The propagation of two higher-order soliton with the second, third and fourth order dispersion $A=[3\cdot(h^2)\cdot A_0\cdot(\mathrm{sech}\times(T/T_0-6)^2)]+3\cdot(h^2)\cdot A_0\cdot(\mathrm{sech}\cdot(T/T_0+6)^2))]\cdot e^{i(8/5)\cdot(dz/2)\cdot h^2}$. Fig. 3 shows the propagation of two 2nd, 3rd and 4th order soliton with the second, third and fourth order dispersion.

By adding the TOD, the shape of the higher order, soliton changes completely and loses its periodicity. It becomes precisely like the fundamental soliton under the effect of TOD: the main soliton and the dispersive wave. As for the interactions, they disappear thanks to the balance between Kerr-induced nonlinearity and the three dispersions. Conversely, the temporal shift exists because of TOD, and the periodic self-compression occurs because of the FOD.

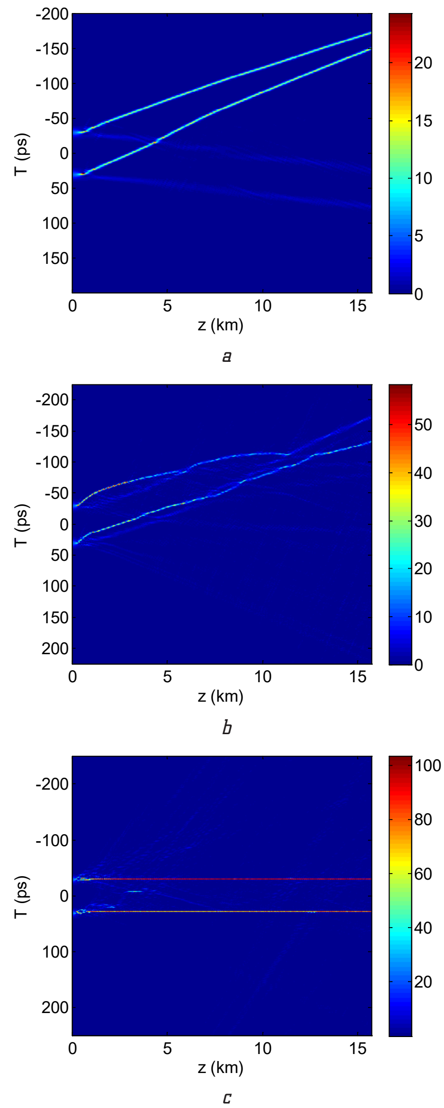

Fig. 3. The effect of FOD in the presence of GVD and TOD on two higher-order solitons: *a* – the propagation of two 2nd order soliton with the second, third and fourth order dispersion: $\beta_3=20$ ps^3/km and $\beta_4=500$ ps^4/km; *b* – the propagation of two 3rd order soliton with the second, third and fourth order dispersion: $\beta_3=20$ ps^3/km and $\beta_4=500$ ps^4/km; *c* – the propagation of two 4th order soliton with the second, third and fourth order dispersion: $\beta_3=20$ ps^3/km and $\beta_4=500$ ps^4/km

3.4. The limits and perspectives of the study. Those results apply to optical fibers with high order dispersion and high order nonlinearity. The two conditions mentioned above are available only in microstructured fibers. In addition, the high nonlinearity and the high order dispersion should be precisely balanced. Thing that is quietly difficult because of the difficulty of manufacturing a fiber with that accuracy. Hence, the limit of manufacturing a classical optical fiber is its susceptibility during the construction or the installation. This defect is duplicated with the microstructured fibers. So the balance of the nonlinearity with the dispersion can't be always attended because of fabrication issues.

One must refrain from speaking about the value of the dispersion or the nonlinearity because it depends on the

 ISSN 2664-9969

design of the optical fiber, seeing that the nonlinearity is related to the effective area of the fiber. Seeing that the effective area of the microstructured fiber is smaller than that of the classical fiber, getting the right value requires more expensive materials to be made and specialist team to be installed.

The convenience of this kind of propagation is the dispersive waves generated that waste the energy and can slightly diminish the amplitude of the soliton. As perspective, let's intend to delete the dispersive waves, which are considered parasites because they disturb the propagation and lose some of the pulse's power. The solution of the dopping fibers seems to be exact solution for this kind dispersion. It is intended to develop this study with a good dopping.

4. Conclusions

In this paper, we could discuss the possibility of deleting both the fission and the interaction of the higher order solitons by simultaneously introducing the third and the fourth order dispersion. Our study confirms that thanks to the temporal shift and the intermittent compression of the third and fourth order dispersion, we can defeat the solid electric attraction of the higher order solitons induced by their gigantic nonlinearity Kerr. We benefit from this fit a better and strict propagation of the higher order solitons by decreasing the bit rate error. However, they lose their irregular shape and get shifted in time. Those results are available only if the values of the high order dispersion are suitable to compensate for the nonlinearity-Kerr according to the design of the optical fiber.

Acknowledgements

The authors would like to thank all colleagues participating in the performance of this research work.

Conflict of interest

The authors declare that they have no conflict of interest concerning this research, whether financial, personal, authorship or otherwise, that could affect the study and its results presented in this paper.

Financing

The study was performed without financial support.

Data availability

The manuscript has associated data in a data repository.

References

1. Khan, K. R., Mahmood, M. F., Biswas, A. (2014). Coherent Super Continuum Generation in Photonic Crystal Fibers at Visible and Near Infrared Wavelengths. *IEEE Journal of Selected Topics in Quantum Electronics, 20 (5),* 573–581. doi: https://doi.org/10.1109/jstqe.2014.2302353
2. Hernandez-Garcia, J. C., Estudillo-Ayala, J. M., Mata-Chavez, R. I., Pottiez, O., Rojas-Laguna, R., Alvarado-Mendez, E. (2013). Experimental study on a broad and flat supercontinuum spectrum generated through a system of two PCFs. *Laser Physics Letters, 10 (7),* 075101. doi: https://doi.org/10.1088/1612-2011/10/7/075101
3. Dudley, J. M., Genty, G., Coen, S. (2006). Supercontinuum generation in photonic crystal fiber. *Reviews of Modern Physics, 78 (4),* 1135–1184. doi: https://doi.org/10.1103/revmodphys.78.1135
4. Smirnov, S. V., Ania-Castanon, J. D., Ellingham, T. J., Kobtsev, S. M., Kukarin, S., Turitsyn, S. K. (2006). Optical spectral broadening and supercontinuum generation in telecom applications. *Optical Fiber Technology, 12 (2),* 122–147. doi: https://doi.org/10.1016/j.yofte.2005.07.004
5. Lü, X., Zhu, H.-W., Meng, X.-H., Yang, Z.-C., Tian, B. (2007). Soliton solutions and a Bäcklund transformation for a generalized nonlinear Schrödinger equation with variable coefficients from optical fiber communications. *Journal of Mathematical Analysis and Applications, 336 (2),* 1305–1315. doi: https://doi.org/10.1016/j.jmaa.2007.03.017
6. Meng, X.-H., Zhang, C.-Y., Li, J., Xu, T., Zhu, H.-W., Tian, B. (2007). Analytic Multi-Solitonic Solutions of Variable-Coefficient Higher-Order Nonlinear Schrödinger Models by Modified Bilinear Method with Symbolic Computation. *Zeitschrift Für Naturforschung A, 62 (1-2),* 13–20. doi: https://doi.org/10.1515/zna-2007-1-203
7. Yu, T., Golovchenko, E. A., Pilipetskii, A. N., Menyuk, C. R. (1997). Dispersion-managed soliton interactions in optical fibers. *Optics Letters, 22 (11),* 793–795. doi: https://doi.org/10.1364/ol.22.000793
8. Herrmann, J., Griebner, U., Zhavoronkov, N., Husakou, A., Nickel, D., Knight, J. C. et al. (2002). Experimental Evidence for Supercontinuum Generation by Fission of Higher-Order Solitons in Photonic Fibers. *Physical Review Letters, 88 (17).* doi: https://doi.org/10.1103/physrevlett.88.173901
9. Wai, P. K. A., Chen, H. H., Lee, Y. C. (1990). Radiations by «solitons» at the zero group-dispersion wavelength of single-mode optical fibers. *Physical Review A, 41 (1),* 426–439. doi: https://doi.org/10.1103/physreva.41.426
10. Elgin, J. N. (1993). Perturbations of optical solitons. *Physical Review A, 47 (5),* 4331–4341. doi: https://doi.org/10.1103/physreva.47.4331
11. Karpman, V. I. (1993). Radiation by solitons due to higher-order dispersion. *Physical Review E, 47 (3),* 2073–2082. doi: https://doi.org/10.1103/physreve.47.2073
12. Akhmediev, N., Karlsson, M. (1995). Cherenkov radiation emitted by solitons in optical fibers. *Physical Review A, 51 (3),* 2602–2607. doi: https://doi.org/10.1103/physreva.51.2602
13. Roy, S., Bhadra, S. K., Agrawal, G. P. (2009). Dispersive waves emitted by solitons perturbed by third-order dispersion inside optical fibers. *Physical Review A, 79 (2).* doi: https://doi.org/10.1103/physreva.79.023824
14. Wai, P. K. A., Menyuk, C. R., Lee, Y. C., Chen, H. H. (1986). Nonlinear pulse propagation in the neighborhood of the zero-dispersion wavelength of monomode optical fibers. *Optics Letters, 11 (7),* 464. doi: https://doi.org/10.1364/ol.11.000464
15. Biswas, A. (2004). Stochastic perturbation of optical solitons in Schrödinger-Hirota equation. *Optics Communications, 239 (4-6),* 461–466. doi: https://doi.org/10.1016/j.optcom.2004.06.047
16. Biswas, A., Jawad, A. J. M., Manrakhan, W. N., Sarma, A. K., Khan, K. R. (2012). Optical solitons and complexitons of the Schrödinger-Hirota equation. *Optics & Laser Technology, 44 (7),* 2265–2269. doi: https://doi.org/10.1016/j.optlastec.2012.02.028
17. Kohl, R., Milovic, D., Zerrad, E., Biswas, A. (2009). Soliton perturbation theory for dispersion-managed optical fibers. *Journal of Nonlinear Optical Physics & Materials, 18 (2),* 227–270. doi: https://doi.org/10.1142/s0218863509004592
18. Topkara, E., Milovic, D., Sarma, A. K., Zerrad, E., Biswas, A. (2010). Optical soliton perturbation with full nonlinearity in non-Kerr law media. *Journal of Optical and Fiber Communications Research, 7 (1-4),* 43–59. doi: https://doi.org/10.1007/s10297-010-9007-3
19. Biswas, A., Topkara, E., Johnson, S., Zerrad, E., Konar, S. (2011). Quasi-stationary optical solitons in non-kerr law media with full nonlinearity. *Journal of nonlinear optical physics & materials, 20 (3),* 309–325. doi: https://doi.org/10.1142/s0218863511006108
20. Biswas, A., Milovic, D., Savescu, M., Mahmood, M. F., Khan, K. R., Kohl, R. (2012). Optical soliton perturbation in nanofibers with improved nonlinear Schrodinger's equation by semi-inverse variational principle. *Journal of Nonlinear Optical Physics & Materials, 12 (4).* doi: https://doi.org/10.1142/s0218863512500543

21. Biswas, A., Milovic, D., Girgis, L. (2013). Quasi-stationary optical Gaussons. *Optik – International Journal for Light and Electron Optics, 124 (17),* 2959–2962. doi: https://doi.org/10.1016/j.ijleo.2012.09.055
22. Xu, Y., Jovanoski, Z., Bouasla, A., Triki, H., Moraru, L., Biswas, A. (2013). Optical solitons in multi-dimensions with spatio-temporal dispersion and non-kerr law nonlinearity. *Journal of Nonlinear Optical Physics & Materials, 22 (3),* 1350035. doi: https://doi.org/10.1142/s0218863513500355
23. Biswas, A., Khan, K., Rahman, A., Yildirim, A., Hayat, T., Aldossary, O. M. (2012). Bright and dark optical solitons in birefringent fibers with Hamiltonian perturbations and Kerr law nonlinearity. *The Journal of Optoelectronics and Advanced Materials, 14 (7-8),* 571–576.
24. Bhrawy, A. H., Alshaery, A. A., Hilal, E. M., Manrakhan, W. N., Savescu, M., Biswas, A. (2014). Dispersive optical solitons with Schrödinger-Hirota equation. *Journal of Nonlinear Optical Physics & Materials, 23 (1),* 1450014. doi: https://doi.org/10.1142/s0218863514500143
25. Wang, J., Wang, S., Chu, X., Sun, M. (2013). Numerical Study on Optical Solitons Transmission System with 40 Gbit/s in the Photonic Crystal Fiber. *Optics and Photonics Journal, 3 (2),* 141–146. doi: https://doi.org/10.4236/opj.2013.32023
26. Elgin, J. N., Brabec, T., Kelly, S. M. J. (1995). A perturbative theory of soliton propagation in the presence of third order dispersion. *Optics Communications, 114 (3-4),* 321–328. doi: https://doi.org/10.1016/0030-4018(94)00602-q
27. Biswas, A., Milovic, D. (2009). Optical Solitons with Fourth Order Dispersion and Dual-power Law Nonlinearity. *International Journal of Nonlinear Science, 7 (4),* 443–447.
28. Elshater, M. E. M., Zayed, E. M. E., Al-Nowehy, A-G. (2016). Solitons and other solutions to nonlinear Schrödinger equation with fourth-order dispersion and dual power law nonlinearity using several different techniques. *Optik, International Journal for Light and Electron Optics.*
29. Khelil, K., Saouchi, K., Bahloul, D. (2020). Effect of fourth order dispersion on solitonic interactions. *Ukranian Journal of Physics, 4.*

Khadidja Khelil, *PhD, Department of Physics, Badji Mokhtar University, Annaba, Algeria, ORCID: https://orcid.org/0009-0003-1168-7231*

Azzeddine Dekhane, *Lecturer, LTSE Laboratory, Ecole Nationale Supérieure de Technologie et d'Ingénierie, Annaba, Algeria, ORCID: https://orcid.org/0000-0003-0328-8452*

✉***Aissa Benselhoub,*** *PhD, Associate Researcher, Environmental Research Center (C.R.E), Annaba, Algeria, ORCID: https://orcid.org/0000-0001-5891-2860, e-mail: aissabenselhoub@cre.dz*

Stefano Bellucci, *Senior Researcher, INFN Frascati National Laboratories, Frascati, Italy, ORCID: https://orcid.org/0000-0003-0326-6368*

✉*Corresponding author*