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RESEARCH OF ORTHOTROPIC COMPOSITES FAILURE TAKING INTO ACCOUNT THEIR STRUCTURAL STOCHASTICITY

The object of the study is the construction of the reliability assessing algorithm for the orthotropic composite plate, taking into account the stochasticity of its structure under the conditions of plane deformation. The plate consists of a matrix and reinforcement elements. The main orthotropic directions of the material coincide with the directions of the loading. The conducted studies are based on the failure criteria expressed through the components of macro stresses. The hypothesis of the weakest link is used, which for the case of the statistical theory of strength sounds like this: the ultimate (failure) loading for an orthotropic composite plate is equal to the ultimate loading for its weakest element. Defects-cracks are characterized by independent random variables – the half-length and the orientation angle between the defect line and the axis of orthotropy with a higher modulus of elasticity. The proposed model of orthotropic composite material corresponds to known experimental studies epoxy phenolic fiberglass on the cord glass fiber. The distribution probabilities density of defect orientation takes into account their predominant orientation in the direction of reinforcement. On the basis of the obtained composite failure loading integral probability distribution function, the construction and study of the dependence of the plate failure probability diagrams for different number of cracks, structural inhomogeneity and type of applied loading was carried out.

Complex application of the composite materials fracture mechanics deterministic solution and the methods of probability theory and mathematical statistics allows for a more adequate assessment of the composite materials reliability, taking into account the stochasticity of their structure.

The main content of this work is the construction and analysis of dependence of stochastically defective reinforced composite materials failure probability diagrams.

The obtained results make it possible to evaluate the reliability of orthotropic stochastically defective materials under conditions of plane deformation.

The algorithm of a compatible combination of defectiveness and randomness of the orthotropic composite material structure makes it possible to qualitatively investigate its failure under various types of applied loading.

Keywords: *orthotropic composite material, reliability, structure stochasticity, failure loading, distribution function, failure probability, plate.*

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1. Introduction

Composite materials, in particular orthotropic, are an important component in the design of structural materials. When assessing their strength, an important factor is taking into account the randomness and stochasticity (certain probability distribution) of the structure. Therefore, for a more adequate display of their strength properties and reliability, there is a need for the complex application of known deterministic solutions of the composite materials fracture mechanics and probabilistic statistical methods. Such complex application can be traced in the articles of a number of authors.

In the article [1], the composite laminated plates failure under random loading conditions was investigated using the layer-wise plate theory and a number of analytical solutions based on the Kirchhoff-Lave plate theory were written. In paper [2], an analysis of the probability of failure of the composite material microstructure is carried out, taking into account the strength of the components and the random nature of the elastic properties. A conclusion was drawn on the study of the dependence of the microscopic probability of failure of the composite material on the microscopic random variable structure. The work [3] considered the failure of laminated composite plates under the conditions of uniaxial and biaxial loadings. The stochastic method of finite elements is used in the study of random failure, which is based on the complex application of the perturbation method and the theory of shear deformation. In [4], the types of uncertain parameters, properties and geometry of the composite plate material are considered as random variables.

An analysis of plate reliability was carried out using Monte Carlo simulation. In [5], the expediency of taking into account the effects of the sequence of loadings when assessing the reliability of structure orthotropic composite elements under fatigue conditions is substantiated. The effect of initial damage on the reliability of such elements was investigated. A study of the change in the stress state and the failure of a symmetrical laminated plate were carried out [6]. The plate contains elliptical cutouts and is under conditions of flat tensile loading.

A complex combination of probabilistic statistical methods and deterministic solutions of composite materials fracture mechanics is an urgent task. Thus, *the object of the research* is construction of an algorithm for assessing the reliability of a stochastically defective orthotropic composite plate under different conditions of the applied loading (conditions of plane deformation).

The aim of this study is to extension of the methodology for assessing the reliability of stochastically defective materials to orthotropic composite materials with the predominant orientation of defects-cracks in the direction of reinforcement under complex stress conditions. The study is based on the deterministic failure criterion, expressed through macro stress components. An analysis of the effect of heterogeneity, size (number of cracks) and type of applied loading on the probability of failure (reliability) of the composite plate was carried out. This will make it possible to more adequately evaluate the mechanism of composite materials failure, taking into account the randomness and defectiveness of their structure.

2. Materials and Methods

Consider a plate made of composite material, which consists of a matrix and reinforcing elements (Fig. 1). Such a material will be considered orthotropic in terms of elastic properties. Under the action of a uniformly distributed loading, the plate is in plane deformation conditions. The main orthotropic directions of the material coincide with the directions of the action loading P and $Q=\eta P$.

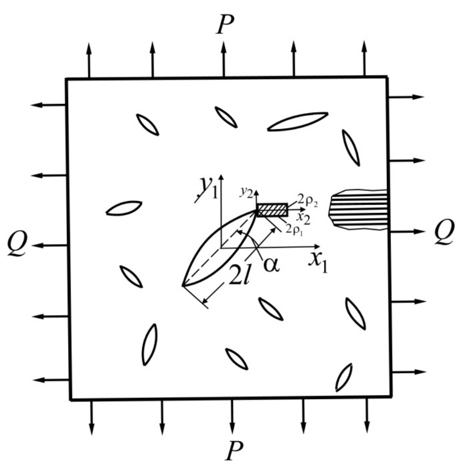

Fig. 1. Model of a stochastically defective reinforced plate

The structure of the material is characterized by defectiveness of various scales, that is, uniformly distributed N defects-cracks that do not interact with each other. The parameters of the defects are considered the half-length and the angle between their line and the axis of orthotropy ox_1. These parameters are statistically independent random variables. For physical reasons, let's consider the most likely cracks orientation in the direction of the axis ox_1 with a higher Young's modulus E_1 of the material, that is, in the direction of reinforcement.

For such a case of crack orientation, let's choose the distribution probability density according to the results of [7]:

$$f(\alpha)=\frac{\lambda^{3/2}}{\pi(\lambda^3\sin^2\alpha+\cos^2\alpha)},\ \lambda=\frac{E_1}{E_2}>1. \tag{1}$$

The proposed model of the orthotropic composite material corresponds to experimental studies of epoxy phenolic fiberglass on the cord glass fiber carried out [8], for which the ratio of Young's moduli $\lambda=3.2$.

Let's assume that the random half-length of cracks can take arbitrary values $(0\le l<\infty)$. The assumption that the maximum crack size is unlimited simplifies material modeling and mathematical calculations, but it makes it possible to obtain quite simply a number of results that adequately reflect reality. This assumption is accepted in a number of works, as indicated in [9].

Let's enter a random variable $L=\rho_1/l$ (ρ_1 is fixed, the size of the structural element), Fig. 1 [10]. The value ρ_1 is small compared to the crack half-length.

In paper [7], the probability distribution density of a random variable L was chosen in the form of a power distribution:

$$f(L)=\frac{(s-1)a^{s-1}}{(L+a)^s},\ s>1,\ a>0. \tag{2}$$

Distribution (2) is a two-parameter statistical model for random variables that vary in the range from zero to infinity.

In this study, let's choose the distribution probability density of a random variable L in the form of a beta distribution [11]. The beta distribution is a statistical model for random variables whose values are bounded by a finite interval. The density of this distribution:

$$f(x)=\frac{1}{d-d_0}\frac{\Gamma(j+\beta)}{\Gamma(j)\Gamma(\beta)}\left(\frac{x-d_0}{d-d_0}\right)^{j-1}\left(1-\frac{x-d_0}{d-d_0}\right)^{\beta-1},$$

$$d_0\le x\le d,\ j>0,\ \beta>0, \tag{3}$$

where $\Gamma(y)$ is gamma function, $\Gamma(y)=\int_0^\infty t^{y-1}e^{-t}\mathrm{d}t$.

With $j=1$ and $\beta=r+1$ $(r\ge 0)$ from expression (3), let's obtain a partial case – a distribution with a constant or monotonically decreasing probability density:

$$f(x)=\frac{r+1}{d-d_0}\left(1-\frac{x-d_0}{d-d_0}\right)^{r},\ d_0\le x\le d,\ r\ge 0. \tag{4}$$

According to the expression (4), the integral probability distributions function:

$$F(x)=1-\left(1-\frac{x-d_0}{d-d_0}\right)^{r+1},\ d_0\le x\le d,\ r\ge 0. \tag{5}$$

The mean value and dispersion of a random variable with distribution (4) have the form:

$$\langle x\rangle=\frac{d_0+d}{r+2},\ D(x)=\frac{(d-d_0)^2(r+1)}{(r+2)^2(r+3)}. \tag{6}$$

The nature of law (4) is reflected in the parameter r. With $r>0$, a decreasing beta distribution describes random variables whose probability of meeting decreases as it increases. At larger value $r>0$, the lower the probability of meeting random values that is close to the largest value d and the greater the probability of meeting small values of the random value (Fig. 2). Formula (4) is confirmed by experimental data [12].

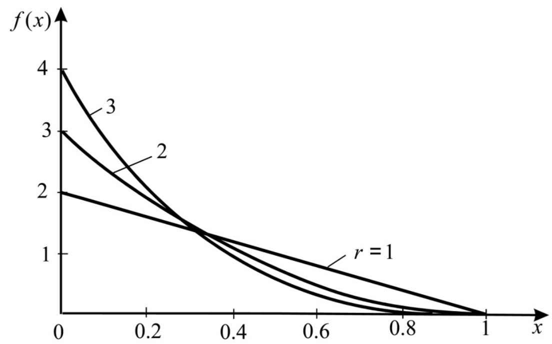

Fig. 2. Beta distribution with a constant or monotonically decreasing probability density

The distribution probability density of the random variable L, as the inverse of l is chosen in the form of a decreasing beta distribution (4):

$$f(L)=(r+1)(1-L)^r,\ 0\le L\le 1,\ r\ge 0. \tag{7}$$

According to (5), the integral probability distributions function of a random variable L:

$$F(L)=1-(1-L)^{r+1},\ 0\le L\le 1,\ r\ge 0. \tag{8}$$

The mean value and dispersion according to (6) will be written as follows:

$$\langle L\rangle=\frac{1}{r+2},\quad D(L)=\frac{r+1}{(r+2)^2(r+3)}.$$

Joint probability distribution density of statistically independent random variables α and L according to (1), (7) has the form:

$$f(\alpha,L)=\frac{(r+1)(1-L)^r\lambda^{3/2}}{\pi(\lambda^3\sin^2\alpha+\cos^2\alpha)}. \tag{9}$$

Graphs of the joint probability distribution density $f(\alpha, L)$ for $L=0.1$, $\lambda=3.2$ (9) are shown in Fig. 3.

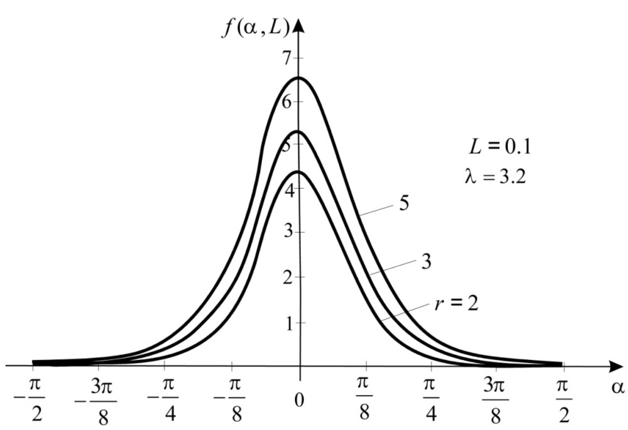

Fig. 3. Joint probability distribution density $f(\alpha, L)$ for different parameter values r

The joint probability distribution density $f(\alpha, L)$ curves are symmetric with respect to the ordinate axis. When the value r changes, the shape of the curve of joint probability distribution density does not undergo significant changes.

3. Results and Discussion

3.1. The failure loading integral probability distribution function. Let's use the hypothesis of the weakest link, which for the case of the statistical theory of strength sounds like this: the limited (failure) loading for an orthotropic composite plate is equal to the limited loading for its weakest element. Let's consider a plate that contains N defects as a random sample of the volume N from the general population of the material primary elements.

In accordance with the proposed in [9] algorithm, the failure loading integral probability distribution function for a plate element with one crack is defined as follows:

$$F_1(P,\eta)=\int_{-\pi/2}^{\pi/2} f(\alpha)F(L)\,\mathrm{d}\alpha. \tag{10}$$

By substituting expressions (1) and (8) into formula (10), let's obtain:

$$F_1(P,\eta)=\frac{\lambda^{3/2}}{\pi}\int_{-\pi/2}^{\pi/2}\frac{1-(1-L(P,\eta,\alpha))^{r+1}}{\lambda^3\sin^2\alpha+\cos^2\alpha}\mathrm{d}\alpha,\ 0\le P\le P_{\max}, \tag{11}$$

where the function $L(P,\eta,\alpha)$ is determined from the failure criterion expressed in terms of macro stress components [7]:

$$[\sigma_{11}]\sin^2\alpha+[\sigma_{22}]\cos^2\alpha+[\sigma_{12}]\sin 2\alpha=\sigma_{cr}, \tag{12}$$

where σ_{cr} is the strength of the composite, macroscopic stresses $[\sigma_{ij}]$ were obtained in paper [10] and, in particular, recorded in [7].

3.2. Probability of failure of an orthotropic composite plate. Under a fixed loading P and Q in accordance with (11), (12), the probability of failure of an orthotropic composite plate under biaxial tension-compression according to the methodology [9] is determined by the following expression:

$$P_f=1-\left(1-\frac{\lambda^{3/2}}{\pi}\int_{-\pi/2}^{\pi/2}\frac{1-(1-L(P,\eta,\alpha))^{r+1}}{\lambda^3\sin^2\alpha+\cos^2\alpha}d\alpha\right)^N. \tag{13}$$

Formula (13) makes it possible to calculate the dependence of the probability of failure P_f of the investigated plate on different types of loading (parameter η), for different number of cracks N (different sizes of the plate) and different structural inhomogeneity of the material (parameter r). Fig. 4–6 show the corresponding diagrams (solid for $\eta=0$, dashed for $\eta=1$, dotted dashed for $\eta=-1$).

In Fig. 4, diagrams of the dependence of the probability of failure on the dimensionless loading P/σ_{cr} for different types of loading cases are constructed. Curves are plotted for different number of cracks.

Fig. 5 shows the dependence of the plate probability of failure at a given loading $P/\sigma_{cr}=0.4$ on the number of cracks and different structural inhomogeneity of the material.

Fig. 6 shows the effect on the probability of failure of the structural material inhomogeneity and the type of applied loading at fixed dimensions of the plate ($N=80$).

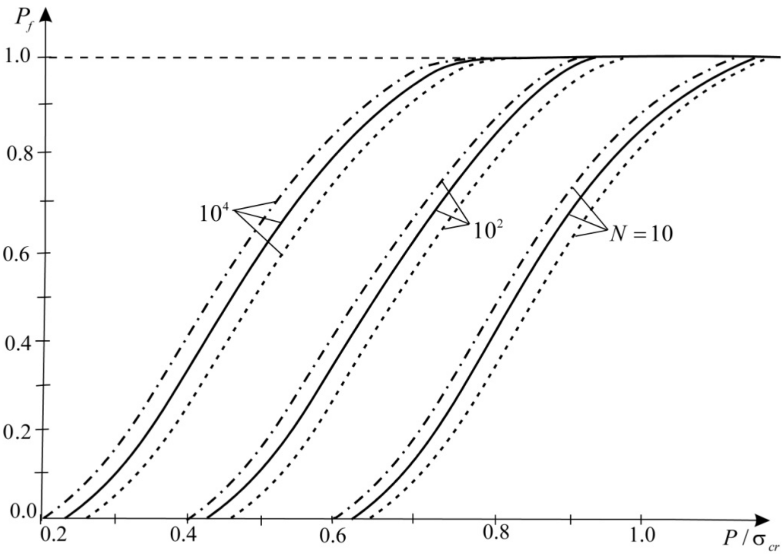

Fig. 4. Probability of failure for various types of stress state

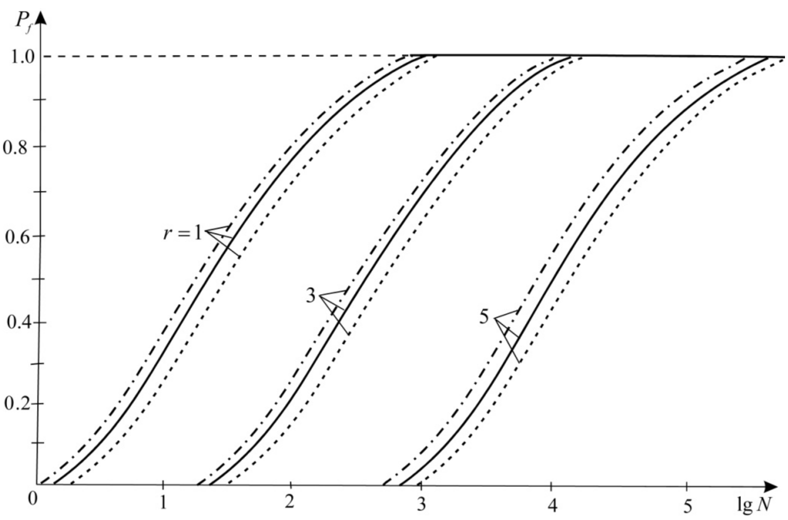

Fig. 5. Probability of failure for the given loading

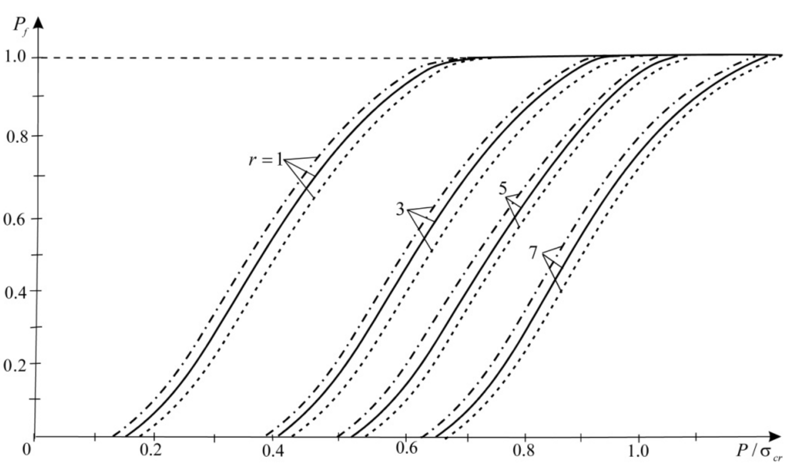

Fig. 6. Probability of failure for different material structural inhomogeneity

In Fig. 4, let's observe a certain loading range, which corresponds to a low probability of failure. Under a fixed loading, an increase in the number of defects leads to an increase in the probability of failure, which depends on the type of stress state (from η).

According to Fig. 5, at each loading level and material structural inhomogeneity corresponds to a range of composite sizes, at which the probability of its failure increases (a certain scale threshold of the probability of failure).

For a fixed loading with an increase in the parameter r (the structure of the material goes to the homogeneous) let's obtain the pattern of decreasing the probability of failure (Fig. 6). This pattern depends on the type of stressed state.

The proposed algorithm for assessing the reliability of orthotropic composite plates is of practical importance in the use of stochastic modeling and statistical description of random variables affecting the process of structural materials failure.

A limitation of this study is the flat model of the orthotropic composite material. Therefore, the construction of a spatial model for assessing the reliability of composite materials opens up new possibilities in the combination of deterministic solutions of the failure materials mechanics with research using probabilistic and statistical methods.

Conducting research under martial law was affected by frequent power outages and, accordingly, the absence of the Internet.

4. Conclusions

The obtained distribution function of failure loading $F_1(P, \eta)$ under a fixed loading determines the orthotropic composite materials probability of failure.

The composite probability of failure P_f depends on the type of the number of defects in plate (dimensions of the composite), the structural inhomogeneity of the material and type of applied loading. There are certain intervals of the composite dimensions and applied loading, in which let's observe a significant increase in the probability of failure P_f.

The highest reliability of the investigated material is observed for equal biaxial tension, the smallest for tension-compression. This regularity can be explained by the influence of the orthotropy of the plate material. In the study, it is proposed to consider the predominance of the defects orientation of in the direction of reinforcement in accordance with the physical meaning of law (1). Therefore, with equal biaxial tension, the action of the loading Q causes the closing of cracks, that is, an increase in the strength of the material and, accordingly, its reliability.

Similar statistical patterns were observed in [7] when choosing a probability distribution density of a random variable L in the power distribution form.

Conflict of interest

The author declares that he has no conflict of interest in relation to this research, whether financial, personal,

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authorship or otherwise, that could affect the research and its results presented in this paper.

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The study was performed without financial support.

Data availability

The manuscript has no associated data.

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