ISSN 2664-9969

UDC 621.311
DOI: 10.15587/2706-5448.2023.293005

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FORCE EFFECT OF A CIRCULAR ROTATING MAGNETIC FIELD OF A CYLINDRICAL ELECTRIC INDUCTOR ON A FERROMAGNETIC PARTICLE IN PROCESS REACTORS

The object of research is the force effect of a circular magnetic field of cylindrical inductors with alternating current windings on the actuator element of technological reactors – a ferromagnetic particle. Technologies using a rotating magnetic field and ferromagnetic particles (RMF and FP) are increasingly used in industry, in devices for fine and ultra-fine grinding, mixing and activation, in the construction and chemical industries, in energy-saving and environmental systems.

In previous studies, the authors proposed a method for calculating the force effect on ferromagnetic particles (FP) of an elliptical rotating magnetic field (RMF) of an external cylindrical inductor with a symmetrical alternating current winding. In this work, based on this technique, formulas for the force effect on the FP of the fundamental harmonic of RMF of cylindrical inductors with different numbers of pole pairs are derived and analyzed.

It is shown that for a hard magnetic and saturated magnetically soft (soft-magnetic) particle in a circular field of a cylindrical inductor with the number of pole pairs greater than one, the magnitude of the magnetic displacement force does not depend on the orientation of the magnetic moment of the ferromagnetic particle, and the direction of action of this force is determined by the angle between the circular field induction vector and the magnetic moment of the particle. While maintaining the similarity of the inductors and the equality of the amplitude of the magnetic induction on the surface of the inductor bore, the magnetic displacement force does not retain the similarity, in particular, while maintaining the values of the magnetic moment of the particle, this force is inversely proportional to the radius of the bore of the cylindrical inductor.

Examples are given of the use of formulas for calculating the ratio of displacement forces to the weight of a particle and the calculation of forces for an unsaturated soft-magnetic particle, where, due to the dependence of the magnetic moment on the field strength, the calculation formulas are modified and take on a slightly different form than the formulas for a particle with a constant modulus magnetic moment.

The research results will be useful for engineers and researchers involved in the research, development, design and operation of reactors with RMF and FP technologies.

Keywords: *electromagnetic mills, circular field, cylindrical magnetic field inductor, ferromagnetic particles, magnetization, magnetic forces.*

Received date: 29.10.2023
Accepted date: 12.12.2023
Published date: 14.12.2023

How to cite

Polshchikov, H., Zhukov, P. (2023). Force effect of a circular rotating magnetic field of a cylindrical electric inductor on a ferromagnetic particle in process reactors. Technology Audit and Production Reserves, 6 (1 (74)), 34–40. doi: https://doi.org/10.15587/2706-5448.2023.293005

1. Introduction

Modern technologies involving the force action of a rotating magnetic field (RMF) on ferromagnetic particles (FP) find application in various spheres of human activity [1].

Electromagnetic mills, activators and separators (hereinafter referred to as «reactors») based on this principle are currently used in mechanical engineering, construction, fuel and chemical industries, as well as in energy-saving and environmental systems. In this case, the source of the RMF can be permanent magnets driven by motors, but more often these are three-phase electrical inductors of various designs: external, internal or combined [2–9]. All magnetic fields in the working area of such reactors belong to the RVF family. However, depending on the design and characteristics of the source, these RMFs have completely different structures. Accordingly, the spectrum of force influences and the behavior of both an individual FP and the entire ensemble of FPs as a whole will be different in different RMFs.

The most common reactor design at present is the design with an external three-phase cylindrical inductor [2–8], Fig. 1.

In these reactors, the structure of the RMF and the FP behavior strongly depend on the number of pole pairs of the inductor. As an example, let's take the photograph in Fig. 2 from [2], which shows the distribution of many moving elongated steel cylinders in the working area of a reactor with a two-pole (Fig. 2, *a*) and four-pole (Fig. 2, *b*) design of an external three-phase inductor.

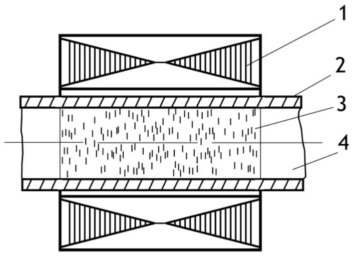
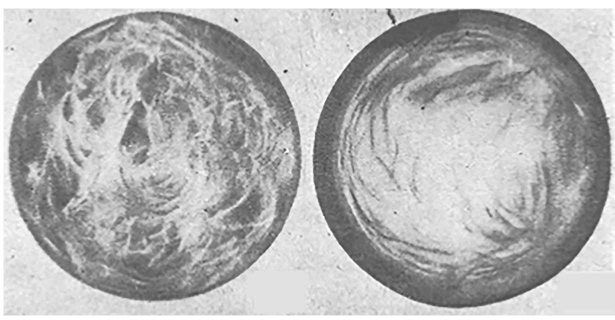


Fig. 1. Schematic representation of a reactor with ferromagnetic particles and an external three-phase inductor: 1 – three-phase inductor generating RMF in the working area of the reactor; 2 – non-magnetic cylindrical working chamber of the reactor; 3 – ferromagnetic particles moving under the influence of the RMF of the inductor in the working area of the reactor; 4 – process medium (gas; liquid; bulk material; sometimes workpiece). The dash-dot line indicates the common axial axis *z* of the reactor and inductor



a *b*

Fig. 2. Distribution of an ensemble of cylindrical particles: *a* – uniform distribution of an ensemble of cylindrical steel particles in a reactor with an external two-pole three-phase inductor; *b* – «bagel» distribution of an ensemble of the same cylindrical steel particles in a reactor with an external four-pole three-phase inductor (particles are distributed near the inner surface of the reactor working chamber, the central part of the reactor working zone is empty) [2]

One of the most important characteristics of the behavior of a FP ensemble in technological reactors is the power density developed by this ensemble. Predicting the power density of a specific ensemble of FPs and a number of other reactor parameters associated with this indicator is a technically important and not yet solved problem. Scientists are trying to find a solution to this problem experimentally and using computer modeling methods and tools [9–12].

Valuable initial information for solving this problem is provided by an analytical study of magnetic force effects on the FP under reasonable simplifying conditions. A general analytical description of the force effect of an elliptical RMF of cylindrical external inductors with a symmetrical alternating current winding on a separate FP was made in [13], in which the total RMF of the inductor is considered as a superposition of circular RMF s of individual field harmonics. In this work, let's highlight as a separate topic and consider in detail the force effect of the fundamental harmonic of the field on various types of FP, presenting it as the only harmonic excited in the bore of an idealized inductor.

A quasi-stationary plane-parallel vector field in the working area of an external electric inductor with a different number of pole pairs is described with varying degrees of accuracy in [13–17]. In the transition to the description of the circular field of an idealized inductor, all descriptions essentially coincide.

The force effect of the RMF of an external inductor on a separate FP was considered in [11–13, 16, 17]. Wherein:

– in papers [16, 17], the obtained formulas contradict the laws of physics, since they show the presence of a displacement force in a uniform field;

– in paper [13] a general description is given of the elliptical rotating magnetic field of an external three-phase inductor and its force effect on the FP, however, this work does not highlight a very important practical analysis of the force effect on various types of FP of an individual field harmonic as a separate topic;

– in papers [11, 12], the force effect is studied by computer modeling methods in isolation from the analytical description of the RMF of the specifically studied inductor.

Summing up the review of previous works, it is possible to state that a detailed analysis of the force effect of the fundamental harmonic of the magnetic field of external inductors with a different number of pole pairs on various types of FPs, most interesting for engineers involved in the development and operation of the reactors in question, is currently in the scientific and technical literature essentially absent.

In this publication let's partially fill this gap.

The aim of this research is to derive and analyze general formulas for the force action on working ferromagnetic particles of reactors of the RMF fundamental harmonic external cylindrical inductors of alternating current with a different number of pole pairs. This can be very useful for the work of specialists involved in the research, development, design and operation of devices using RMF and FP technologies.

2. Materials and Methods

The idealized inductor is presented in the form of an infinite unsaturated ferromagnetic tube, on the inner surface of which, according to the harmonic law, a thin current layer of varying spatial periodicity is specified.

The formulas for the quasi-stationary plane-parallel vector field of induction of a circular RMF of an idealized inductor in the working area of the reactor were taken into consideration.

To calculate magnetic forces, the dipole approximation technique described in [13] was used.

The cases of hard-magnetic, saturated and unsaturated soft-magnetic FP are considered.

3. Results and Discussion

The rotating magnetic field in an idealized cylindrical inductor is defined by the linear current density, which in polar coordinates (r, α) is described as follows:

$$J = J_m \sin(\omega t - p\alpha), \tag{1}$$

which, as follows from the law of total current, is equivalent to the following boundary conditions specified on the bore surface for the magnetic field induction in the bore volume:

$$B_{0\alpha} = B_0 \sin(\omega t - p\alpha), \tag{2}$$

$$B_0 = \mu_0 J_m, \tag{3}$$

ISSN 2664-9969

where J_m – the amplitude value of the linear current density; ω – angular frequency, 1/s ($\omega=2\pi f$, where f – supply voltage frequency, Hz); t – time, s; p – the number of pairs of magnetic field poles; $B_{0\alpha}$ – tangential component of magnetic induction on the surface of the inductor bore; B_0 – amplitude value of the tangential component of magnetic induction on the surface of the inductor bore (T); $\mu_0=4\pi\cdot10^{-7}$ H/m – magnetic constant.

The coordinate system is chosen in such a way that the polar coordinate pole is located on the inductor axis (in the center of the inductor bore), and the polar axis intersects the inductor bore surface at the point where $J=0$ at $t=0$.

From the papers [13–15] for this case, the following solution follows for the magnetic induction vector $\vec{B}$ of a plane-parallel quasi-stationary field (rot $\vec{B}=0$) in the inductor bore:

$$B_r = B_0\left(\frac{r}{r_0}\right)^{p-1}\cos(\omega t - p\alpha), \tag{4}$$

$$B_\alpha = B_0\left(\frac{r}{r_0}\right)^{p-1}\sin(\omega t - p\alpha), \tag{5}$$

where B_r – radial component $\vec{B}$; B_α – tangential component $\vec{B}$; $B_0(r/r_0)^{p-1}$ – vector modulus $\vec{B}$.

Expressions (4), (5) constitute parametric equations of a circle with radius $B_0(r/r_0)^{p-1}$, that is, the hodograph of the magnetic field induction vector at each point of the field is a circle, which defines these expressions as those that describe a circular field.

In the case of $p=1$, the induction vector at each moment of time is the same at all points of the field, that is, the rotating circular field of the inductor is uniform.

The angular velocity of rotation of the induction vector in time at a fixed point of the inductor boring does not depend on the number of pole pairs p:

$$\frac{\partial}{\partial t}(\omega t - p\alpha) = \omega. \tag{6}$$

The angular velocity of rotation $d\alpha/dt$ of a quasi-stationary magnetic field (field poles, field line patterns, magnetic field points) around the axis of the inductor is determined from the condition $d/dt(\omega t - p\alpha)=0$:

$$\frac{d\alpha}{dt} = \frac{\omega}{p}. \tag{7}$$

The effect of the magnetic field under consideration on a small magnetic particle, which has its own or acquired and changing magnetic moment $\vec{M}$, can be quantitatively assessed in the dipole approximation using the well-known formulas [18, 19]:

$$\vec{T} = \vec{M}\times\vec{B}, \tag{8}$$

$$\vec{F} = \left(\vec{M}grad\right)\vec{B} = (\vec{M}\nabla)\vec{B}, \tag{9}$$

where $\vec{T}$ – the torque acting on the magnetic particle from the field (N·m); $\vec{F}$ – displacement force acting on a magnetic particle from the field $\vec{B}$ (N).

In accordance with the provisions of theoretical electrical engineering in formulas (8), (9), the vector $\vec{B}$ is the vector of the external magnetic field into which a particle with a magnetic moment $\vec{M}$ is introduced.

Formulas (8), (9) when applied to the circular field of the inductor take the form:

$$\vec{T} = \left(MB_0\left(\frac{r}{r_0}\right)^{p-1}\sin\psi\right)\vec{i}_z, \tag{10}$$

$$\vec{F}_r = F_r\vec{i}_r = \left(MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos\psi\right)\vec{i}_r, \tag{11}$$

$$\vec{F}_\alpha = F_\alpha\vec{i}_\alpha = \left(MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)sin\psi\right)\vec{i}_\alpha, \tag{12}$$

where $\vec{F}_r$, $\vec{F}_\alpha$ – the radial and tangential components of the vector $\vec{F}$; F_r and F_α – their modules; M – vector $\vec{M}$ module; ψ – the angle between $\vec{M}$ and $\vec{B}$, is taken positive if the vector $\vec{M}$ is considered to be out of phase with the vector $\vec{B}$; $\vec{i}_\alpha$, $\vec{i}_r$ – unit vectors of polar coordinates; $\vec{i}_z$ – axial unit vector.

It is useful to note that according to (10)–(12): $\vec{T}$, $\vec{F}_\alpha$ are proportional to the $B_\perp$-normal component $\vec{M}$ of the vector $\vec{B}$, $\vec{F}_r$ are proportional to the $\vec{B}_\parallel$-collinear component $\vec{M}$ of the vector $\vec{B}$.

Formula (10) is a direct reproduction of the general formula (8), adapted to the plane-parallel field of the inductor.

To obtain formulas (11), (12), mathematical calculations are required that are not presented in the literature and previous publications.

Let's expand formula (9) in polar coordinates, in which, as is known, the operator $\nabla = \frac{\partial}{\partial r}\vec{i}_r + \frac{1}{r}\frac{\partial}{\partial\alpha}\vec{i}_\alpha$:

$$\left(\vec{M}\nabla\right)\vec{B} = M_r\frac{\partial\vec{B}}{\partial r} + M_\alpha\frac{1}{r}\frac{\partial\vec{B}}{\partial\alpha},$$

$$\frac{\partial\vec{B}}{\partial r} = \frac{\partial(B_r\vec{i}_r)}{\partial r} + \frac{\partial(B_\alpha\vec{i}_\alpha)}{\partial r} =$$

$$= \frac{\partial B_r}{\partial r}\vec{i}_r + B_r\frac{\partial\vec{i}_r}{\partial r} + \frac{\partial B_\alpha}{\partial r}\vec{i}_\alpha + B_\alpha\frac{\partial\vec{i}_\alpha}{\partial r} = \frac{\partial B_r}{\partial r}\vec{i}_r + \frac{\partial B_\alpha}{\partial r}\vec{i}_\alpha,$$

$$\frac{\partial\vec{B}}{\partial\alpha} = \frac{\partial(B_r\vec{i}_r)}{\partial\alpha} + \frac{\partial(B_\alpha\vec{i}_\alpha)}{\partial\alpha} =$$

$$= \frac{\partial B_r}{\partial\alpha}\vec{i}_r + B_r\frac{\partial\vec{i}_r}{\partial\alpha} + \frac{\partial B_\alpha}{\partial\alpha}\vec{i}_\alpha + B_\alpha\frac{\partial\vec{i}_\alpha}{\partial\alpha} = \frac{\partial B_r}{\partial\alpha}\vec{i}_r +$$

$$+B_r\vec{i}_\alpha + \frac{\partial B_\alpha}{\partial\alpha}\vec{i}_\alpha - B_\alpha\vec{i}_r,$$

$$\left(\vec{M}\nabla\right)\vec{B} = \left(M_r\frac{\partial B_r}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_r}{\partial\alpha} - \frac{1}{r}M_\alpha B_\alpha\right)\vec{i}_r +$$

$$+\left(M_r\frac{\partial B_\alpha}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_\alpha}{\partial\alpha} + \frac{1}{r}M_\alpha B_r\right)\vec{i}_\alpha.$$

Let's obtain the required formula:

$$\vec{F} = \left(M_r\frac{\partial B_r}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_r}{\partial\alpha} - \frac{1}{r}M_\alpha B_\alpha\right)\vec{i}_r +$$

$$+\left(M_r\frac{\partial B_\alpha}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_\alpha}{\partial\alpha} + \frac{1}{r}M_\alpha B_r\right)\vec{i}_\alpha, \tag{13}$$

previously given by us in [13] without proof.

$$\vec{F}_r = \left(M_r\frac{\partial B_r}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_r}{\partial\alpha} - \frac{1}{r}M_\alpha B_\alpha\right)\vec{i}_r, \tag{14}$$

$$\vec{F}_\alpha = \left(M_r\frac{\partial B_\alpha}{\partial r} + \frac{1}{r}M_\alpha\frac{\partial B_\alpha}{\partial\alpha} + \frac{1}{r}M_\alpha B_r\right)\vec{i}_\alpha. \tag{15}$$

Let's connect the angular position of the vector $\vec{M}$ with the angular position of the vector $\vec{B}$, Fig. 3:

$$M_r = M\cdot\cos(\omega t - p\alpha - \psi), \tag{16}$$

$$M_\alpha = M\cdot\sin(\omega t - p\alpha - \psi). \tag{17}$$

Substituting expressions (4), (5), (16), (17) into (14), (15) and making the following transformations, let's obtain formulas (11), (12):

$$\frac{\partial B_r}{\partial r} = B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos(\omega t - p\alpha);$$

$$\frac{\partial B_r}{\partial \alpha} = B_0\left(\frac{r}{r_0}\right)^{p-1} p\sin(\omega t - p\alpha);$$

$$\frac{\partial B_\alpha}{\partial r} = B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\sin(\omega t - p\alpha);$$

$$\frac{\partial B_\alpha}{\partial \alpha} = -B_0\left(\frac{r}{r_0}\right)^{p-1} p\cos(\omega t - p\alpha);$$

$$F_r = M_r B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos(\omega t - p\alpha)+$$
$$+\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1} p\sin(\omega t - p\alpha)-$$
$$-\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1}\sin(\omega t - p\alpha)=$$
$$= M_r B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos(\omega t - p\alpha)+$$
$$+\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1}(p-1)\sin(\omega t - p\alpha)=$$
$$= B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\begin{pmatrix} M_r\cos(\omega t - p\alpha)+ \\ +M_\alpha\sin(\omega t - p\alpha)\end{pmatrix}=$$
$$= MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\begin{pmatrix}\cos(\omega t - p\alpha - \psi)\cos(\omega t - p\alpha)+ \\ +\sin(\omega t - p\alpha - \psi)\sin(\omega t - p\alpha)\end{pmatrix}=$$
$$= MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos\psi;$$

$$F_\alpha = M_r B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\sin(\omega t - p\alpha)-$$
$$-\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1} p\cos(\omega t - p\alpha)+$$
$$+\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1}\cos(\omega t - p\alpha)=$$
$$= M_r B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\sin(\omega t - p\alpha)-$$
$$-\frac{1}{r}M_\alpha B_0\left(\frac{r}{r_0}\right)^{p-1}(p-1)\cos(\omega t - p\alpha)=$$
$$= B_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\begin{pmatrix} M_r\sin(\omega t - p\alpha)- \\ -M_\alpha\cos(\omega t - p\alpha)\end{pmatrix}=$$
$$= MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\begin{pmatrix}\cos(\omega t - p\alpha - \psi)\sin(\omega t - p\alpha)- \\ -\sin(\omega t - p\alpha - \psi)\cos(\omega t - p\alpha)\end{pmatrix}=$$
$$= MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\sin\psi.$$

At $p=1$, formulas (10)–(12) show the absence of a displacement force, that is, the only force effect of a homogeneous RMF on the FP is torque $\vec{T}$. This case is considered in [20], it is characterized by high shock loads during oncoming collisions of FPs rotating and oscillating around their centers of mass, and is most studied at present.

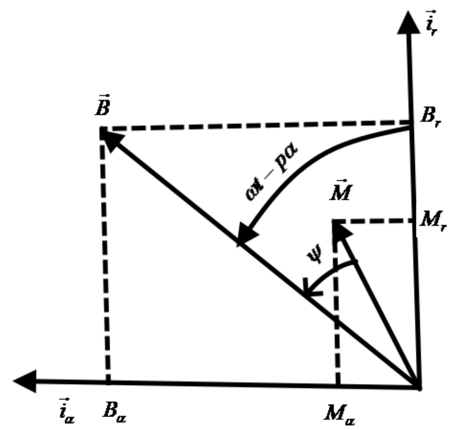


Fig. 3. Graph of the relationship between the angular position of the vector $\vec{M}$ and the angular position of the vector $\vec{B}$

Further analysis applies to cases $p\geq 2$.

For $p\geq 2$, formulas (11), (12) show the presence of a displacement force $\vec{F}$ acting on the magnetic particle, the direction of which is determined by the angle between the circular field induction vector $\vec{B}$ and the magnetic moment of the particle $\vec{M}$.

In the general case, at $\psi\neq 0$, the torque tends to decrease the angle ψ (which causes rotation and oscillation of the particle around its center of mass), and the displacement force drags the FP to move in a spiral, unwinding in the direction of the inductor bore surface. This spiral is the more «flat» the greater the resistance to particle movement from the medium in the working chamber of the inductor and, accordingly, the greater ψ. The intensity of particle motion in the tangential direction increases, and in the radial direction it decreases. When $\psi=0$ the inductor operates in the mode of a magnetic separator, simply attracting such particles to the surface of the boring. This mode can be observed when using isotropic soft-magnetic particles with negligible losses due to magnetization reversal and eddy currents.

For a hard-magnetic and saturated soft-magnetic particle, the displacement force modulus does not depend on the orientation of the ferromagnetic particle.

At $p=2$ and constant magnetization of the FP, the modulus of the biasing force is the same over the entire cross-section of the inductor bore.

In all cases, the force effect on the particle is not dependent on the α coordinate.

For soft-magnetic particles, the range of angle ψ is $|\psi|<90°$, while soft-magnetic particles, along with tangential motion, will always be attracted to the inner surface of the reactor working chamber (the «bagel» effect – Fig. 2).

For hard-magnetic particles with a high coercive force $M=\text{const}$, and the range of the angle ψ is $|\psi|\leq 180$, while in the case of obtuse values of the angle ψ such particles, along with tangential motion, will be repelled from the internal surface of the reactor and move towards the center reactor (the «bagel» effect may be weakened).

The bias force in inductors that are similar to each other and have the same value (the amplitude of magnetic induction on the surface of the bore) is less for inductors with a large bore diameter of the inductor, in particular, while maintaining the values of the magnetic moment of the particle, this force is inversely proportional to the bore radius of the cylindrical inductor.

It is known that in a constant magnetic field, FPs are aggregated into chain structures located along the field lines. Experimental studies record the destruction of such chain FP structures in the working chamber of the reactor when the rotation speed of the field of the external cylindrical inductor increases from zero to a certain threshold value. Formulas (10)–(12) explain the mechanism of this destruction for any number of inductor pole pairs. A row of particles lined up along the field line is a magnetized, in the general case, curved rod, and in the case of a uniform field ($p=1$) a straight rod. When the field pattern is rotated, the particles fall on other lines of force, because due to inertia and friction, the rod in each of its sections falls on a new line of force that forms a certain angle with the rod, that is, in each section of the rod a component of the magnetic field induction $\vec{B}_{\perp}$ normal to the rod appears and the tangent component $\vec{B}_{\parallel}$ to it changes. The component $\vec{B}_{\perp}$ excites the $\vec{T}$ and $\vec{F}_{\alpha}$ that grow with its growth, the component $\vec{B}_{\parallel}$ excites the change $\vec{F}_r$ that grows with its change. Accordingly, stresses that work on its bending arise in the rod (in a non-uniform field of varying magnitude along the length of the rod), the greater the rotation of the field pattern, the greater. Since $\vec{T}$ and $\vec{F}_{\alpha}$ tend to drag the rod along with the rotating field, one can expect that with an infinitely slow rotation of the field, the rod will rotate along with it (in the presence of some moderate friction – with some distortion of the shape and delay). In the case of a faster rotation of the field, the inertia of the rod composed of particles does not allow it to keep up with the field and the rod falls into that part of the field when the stresses reach a value that destroys this rod into a scattering of its constituent particles. The considered mechanism of destruction of chain structures formed along field lines also explains the observed [17] destruction of these structures during the transition from weak to stronger fields.

The above analysis of the obtained formulas (10)–(12) allows to draw the following practical conclusions and recommendations:

– for technological processes that require uniform processing of the product with ferromagnetic particles over the entire cross-section of the reactor, an inductor with the number of pole pairs $p=1$ is most suitable;
– in cases requiring the concentration of magnetic particles near the surface of the reactor working chamber, or their separation, the optimal solution is an inductor with the number of pole pairs $p=2$;
– in reactors in which it is necessary to reduce shock loads and increase the role of frictional particles (crushing by grinding bodies), it is advisable to use hard-magnetic spherical particles;
– in technological processes requiring the creation of shock loads, it is advisable to use elongated soft-magnetic particles;
– in all cases, the FP dynamics will include rotational-oscillatory FP movements around its own center of mass with simultaneous linear movements of the center of mass of the FP along complex trajectories;
– analytical consideration of the force effect of RMF on the FP can be used when selecting factors for computer modeling of technological processes based on the effect of a rotating magnetic field on magnetic particles in order to optimize them;
– while maintaining the similarity of the inductors and the equality of the amplitude of magnetic induction in them on the surface of the inductor bore, the magnetic displacement force does not preserve the similarity.

In practice, when designing a reactor, it is very important to predict the ratio of the displacement force and the weight of the ferromagnetic particle. Let's solve this problem using formulas (11)–(12) using the example of soft-magnetic steel and nickel ferromagnetic particles in a circular RMF inductor with a bore diameter of 0.1 m at $p=2$ and the value of the induction modulus $\vec{B}$ at the bore surface inductor 0.15 T ($B_0=0.15$ T).

Measurements carried out at the Kharkiv Laboratory of Magnetohydrodynamics of the Energy Institute (EI) showed that for wires made of carbon steels and nickel alloys, from which FPs were cut for devices with a vortex layer [20], the value of the saturation induction B_n is – in steel – 2 T, in nickel – 0.6 T, and is achieved at magnetic field strength $H_n=4\cdot10^4$ A/m in steel, and $3.2\cdot10^4$ A/m in nickel [10], that is, at a relative saturation magnetic permeability:

$$\mu_n = B_n / \mu_0 H_n. \tag{18}$$

In steel:

$$\mu_n = 2/(4\pi\cdot10^{-7}\cdot4\cdot10^4) = 40.$$

In nickel:

$$\mu_n = 0.6/(4\pi\cdot10^{-7}\cdot3.2\cdot10^4) = 15.$$

In this case, the induction of the external field $B=B_n/\mu_n$:
– in the case of steel:

$$\mathrm{B}=2/40=0.05 \text{ T}, \tag{19}$$

– in the case of nickel:

$$B=0.6/15=0.04 \text{ T}. \tag{20}$$

Taking into account (4), (5), it is possible to conclude that the FP, oriented along the field line, is in a state of saturation and strong magnetization in almost the entire volume of the inductor bore with $p=2$ at $B_0=0.15$ T.

The ratio of the displacement force modulus to the weight of a particle Q with density ρ (kg/m^3) with volume V and magnetization j (A/m) of the particle in a circular RMF of an external inductor with $p=2$:

$$F/Q = MB_0\frac{1}{r_0}\Big/Q = VjB_0\frac{1}{r_0}\Big/9.8\rho V = jB_0\frac{1}{r_0}\Big/9.8\rho, \tag{21}$$

that is, the ratio of the modulus of the displacement force to the weight of the particle is independent of the particle size.

Let's substitute into (21) the values of the corresponding values of the saturated FP, taking into account that the magnetization of the particle material is $j\approx B_n/\mu_0$. In the case of steel FP $\rho=7.8\cdot10^3$ kg/m^3, $B_n\approx2$ T; in the case of nickel FP $\rho=8.9\cdot10^3$ kg/m^3, $B_n\approx0.6$ T. Let's obtain in the inductor under consideration with a saturated FP:

– steel FPs:

$$F/Q=2\cdot0.15/(4\pi\cdot10^{-7}\cdot0.05\cdot9.8\cdot7.8\cdot10^3)=62.5;$$

– nickel FPs:

$$F/Q=0.6\cdot0.15/(4\pi\cdot10^{-7}\cdot0.05\cdot9.8\cdot8.9\cdot10^3)=16.5.$$

Thus, in a reactor with the inductor in question and the specified RMF and FP parameters, the magnetic displacement force is tens of times greater than the weight of the ferromagnetic particle.

For more accurate calculations of the force distribution in the bores of various inductors using formulas (11)–(12), including their computer modeling using the dipole approximation method, it is necessary to have magnetization curves of the materials used for the manufacture of soft magnetic FP.

To illustrate the application of formulas (10)–(12) to the operating conditions of reactors with a constant magnetic permeability of the FP, let's solve the problem posed in [17], where an incorrect answer was obtained to it, indicating the alleged presence of a homogeneous RMF (with the number of pole pairs circular field of the inductor $p=1$) bias force.

The authors of [17] sought to obtain formulas for the density of the displacement force $\vec{f}$ acting in a circular RMF on a ferromagnetic particle at $\psi=0$ and at a constant magnetic permeability $\mu=$const (which is assumed by the general formula they used for the force density, known to theoretical electrical engineering).

From the conditions of the problem and formulas (10), (11) it follows:

$$\vec{F}=\vec{F}_r=\left(MB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\cos\psi\right)\vec{i}_r, \tag{22}$$

which indicates the absence of a displacement force in the solution of the problem at $p=1$ and the presence of a radial displacement force at other values of p, since $\psi=0$ and $\cos\psi=1$. There is no tangential force in the conditions of this problem, since $\sin\psi=0$. Without limiting ourselves to this fact, let's present a complete solution to the problem, taking into account the change in the FP magnetization for the conditions $\psi=0$ and $\mu=$const.

Magnetic moment $\vec{M}$ (Am^2) of ferromagnetic particles during induction $\vec{B}_{FP}$ and tension in the body of the particle:

$$\vec{M}=\vec{j}V=\left(\vec{B}_{FP}/\mu_0-\vec{H}_{FP}\right)V=\left(\mu\vec{H}_{FP}-\vec{H}_{FP}\right)V=$$
$$=(\mu-1)\vec{H}_{FP}V=(\mu-1)\left(\vec{H}-N\vec{j}\right)V=$$
$$=\left((\mu-1)\vec{H}-(\mu-1)N\vec{j}\right)V, \tag{23}$$

where $\vec{j}$ – the magnetization vector of the FP material averaged over the particle volume (A/m); V – FP volume (m^3); $\vec{H}$ – intensity from the magnetic field source; N – FP demagnetizing factor. From here:

$$\vec{j}+\left((\mu-1)N\vec{j}\right)=(\mu-1)\vec{H}, \tag{24}$$

$$\vec{j}=\frac{(\mu-1)\vec{H}}{\left(1+(\mu-1)N\right)}=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}\vec{B}. \tag{25}$$

Vector modulus $\vec{j}$ with B-modulus of the induction vector $\vec{B}$ of the field created by the inductor:

$$j=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}B. \tag{26}$$

Vector module $\vec{M}$:

$$M=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}BV=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}B_0\left(\frac{r}{r_0}\right)^{p-1}V. \tag{27}$$

Returning to (22) taking into account (27), let's obtain a modified formula (22), taking into account the change in the FP magnetization under given conditions:

$$\vec{F}_r=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}B_0\left(\frac{r}{r_0}\right)^{p-1}VB_0\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{p-2}(p-1)\vec{i}_r=$$
$$=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}B_0^2(p-1)\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{2p-3}V\vec{i}_r. \tag{28}$$

Mechanical force density $\vec{f}$:

$$\vec{f}=\vec{f}_r=\vec{F}_r/V=\frac{(\mu-1)}{\left(1+(\mu-1)N\right)\mu_0}B_0^2(p-1)\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{2p-3}\vec{i}_r. \tag{29}$$

Or taking into account (3):

$$\vec{f}=\frac{(\mu-1)\mu_0}{\left(1+(\mu-1)N\right)}J_m^2(p-1)\frac{1}{r_0}\left(\frac{r}{r_0}\right)^{2p-3}\vec{i}_r. \tag{30}$$

In cases where the role of the demagnetizing factor is insignificant, $N=0$ can be taken, in which case formulas (28)–(30) and the corresponding calculations are significantly simplified.

Concluding this study, it is possible to note that this work examines the assessment of the force effect of a RMF on the FP in the dipole approximation for a plane-parallel quasi-stationary field of a single inductor harmonic. Issues of more accurate quantitative calculation of the force effect of RMF on FPs in reactors, as well as issues of magnetic interaction of FPs with each other and taking into account the magnetostrictive forces that arise during the magnetization reversal of soft-magnetic FPs, should be considered as separate topics and are beyond the scope of the article.

The obtained research results *can be applied in practice* in the process of designing reactors and equipment with RMF and FP technologies to assess and predict the nature of the force effects of RMF of a specific inductor on operating FPs.

Limitations: with a significant deviation of real conditions from the simplifications adopted when deriving formulas (10)–(12), the accuracy of estimates of the force effect of the RMF on the FP decreases and should be further investigated.

In the future, it is necessary to conduct an analytical study of the force effect of other types of inductors used in practice on the FP RMF, to implement and clarify the results obtained using computer and experimental research methods.

A separate task is to take into account the magnetic interaction of the FPs with each other and take into account

ISSN 2664-9969

the magnetostrictive forces that arise during the magnetization reversal of soft-magnetic FPs.

4. Conclusions

As a research result, the existing gap in the fundamentals of designing reactors and technological processes using FP in RMFs has been partially filled, namely, a theory of the force effect of circular RMF reactors with a cylindrical electric inductor on their executive elements – ferromagnetic particles – has been developed.

Based on the dipole approximation technique, general formulas are obtained and analyzed for estimating the force effect of a plane-parallel quasi-stationary circular rotating magnetic field of cylindrical electric external inductors on a magnetic particle.

Examples of the use of the obtained formulas and the ensuing engineering and practical recommendations for the design of reactors with RMF and FP technologies are given.

The research results will be useful both for specialists engaged in research, development and design of reactors based on the RMF and FP principles, as well as for their operation.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The research was performed without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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