ISSN 2664-9969

UDC 519.3:692
DOI: 10.15587/2706-5448.2024.298883

**Rebadj Chabane,
Sabah Moussaoui,
Mourad Belgasmia**

DOUBLE INCLINED CRACKS OVERLAPPING EFFECT ON MIXED STRESS INTENSITY FACTORS USING XFEM OBJECT-ORIENTED IMPLEMENTATION

The object of research is the Mixed Mode Stress Intensity Factor (MMSIF) of a two-dimensional (2D) plate.

With the emergence of modern technologies and advanced innovations which contribute to the development and improvement of the design, implementation and management of construction projects, it has become easier. However, it is very difficult to manufacture components free from unavoidable defects, such as cracks, which lead to material deterioration and ultimately shorten its service life. Based on the process of local enrichment region using partition of unity concept, the extended finite element method (XFEM) has overcome the limitations of the standard FEM method in terms of modeling and numerical simulation of discontinuities (cracks) while gaining its general advantages. This makes XFEM a powerful and widely used digital tool in recent years. One of the most frequently raised problems in the discontinuities field (cracks) is the phenomenon of juxtaposition of multiple cracks in a cracked isotropic plate, which must be studied to determine the extent of its effect on the crack stress intensity factor in order to obtain higher safety reliability. On this basis, an improved object-oriented programming (OOP) with extended finite elements was used because of its great importance and well-known benefits.

In this paper, the MMSIF of a 2D plate is determined to show the effect of the out-of-phase orientation of the angle, as well as the effect of the juxtaposition of two inclined cracks. As a result of the research, it is shown that, the convergence between the results obtained in this study with those reported in the literature, and to theoretical values is remarkable and their close agreement was noted. In the future, based on the object-oriented approach characteristics represented by flexibility, scalability, and modularity, which were explained in this research, this proposed approach can be enriched to include heterogeneous materials modeling, whether linear or nonlinear, crack propagation in dynamics, in addition to Complex 3D industrial problems.

Keywords: *extended finite element method (XFEM), mixed mode stress intensity factor (MMSIF), inclined center crack, overlapping effect, C++ object-oriented programming.*

Received date: 20.12.2023
Accepted date: 22.02.2024
Published date: 24.02.2024

How to cite

Chabane, R., Moussaoui, S., Belgasmia, M. (2024). Double inclined cracks overlapping effect on mixed stress intensity factors using XFEM object-oriented implementation. Technology Audit and Production Reserves, 1 (1 (75)), 12–18. doi: https://doi.org/10.15587/2706-5448.2024.298883

1. Introduction

In the field of civil engineering, the safety of structures is a major concern in engineering, which requires preventive maintenance as well as corrective interventions, which must be implemented to correct errors that may occur during construction, in order to ensure the reliability of constructed buildings and increase their lifespan.

These errors give rise to various forms of internal or external forces, which subsequently give rise to various forms of discontinuities, in particular cracks.

The discipline of discontinuities was learned in the field of fracture mechanics, and thanks to it, many theories and calculation methods were developed in this regard, in particular the extended finite element method (XFEM), which was introduced in the second millennium [1–6], as a practical numerical procedure for analyzing discontinuity problems. It is an extension of the finite element method (FEM) by enriching standard finite element (FE) shape functions with special enrichment functions (level set functions).

The concept of stress intensity factor (SIF) is considered one of the most important major advances in the field of linear elastic fracture mechanics (LEFM). This is a critical parameter that uniquely describes the stress. Field near the crack tip, as well as its design makes it possible to know the direction and growth rate of the cracks.

With the abundance of scientific research, many techniques have been developed to calculate SIF, including the stiffness derivative method [7]. The virtual crack extension method [8], the virtual crack closure technique [9], as well as the energy-based methods which are most frequently adopted such as the J-integral method [10, 11], and the contour integration method [12, 13].

The calculation of the stress intensity factor (SIF) by the extended finite element method (XFEM) has also taken its part in the scientific literature. In [14] presented a comparative study between extended finite element method (XFEM) and finite element method (FEM) in ABAQUS software; for stress intensity factors (SIF) calculation of a cracked aluminium plate at the angle.

In [15] is worked on SIF extraction through an experimental and theoretical analysis at the crack tip on specimens subjected to a three-point bending load. In [16] is focused on the calculation of SIF by presenting a new local mesh refinement approach. Using a combination of extended finite element method (XFEM), and hexahedral elements with variable nodes for three-dimensional (3D) linear elastic solid fracture analysis for straight and curved planar cracks.

A stress intensity factor computation for cracked plates using level set method combined with the extended finite element method (XFEM) presented in [17].

In [18] presents a crack stress intensity factor calculation in dynamic numerical model of crack propagation by mean of combination between each FEM, XFEM methods with M-method Integral.

Since the phenomenon of juxtaposition of several cracks (adjacent cracks) in a structure can be very dangerous, due to the possibility of connecting them together, creating a large crack that can cause a catastrophic failure, resulting in the reduction of resistance residual of the structural element.

Therefore, the study of this phenomenon in terms of stress intensity factors (SIF) becomes very important to determine the remaining breaking strength and capacity of the material and structure. Our research deals with the presence of double inclination cracks with an out-of-phase orientation in an isotropic plate element using an object-oriented C++ code [19] based on the XFEM method.

The aim of research is studying the effect of the overlap of these cracks on the mixed stress intensity factors (SIF). This will allow reducing structural damage, increase safety and further improve design by predicting crack growth rates by accurately calculating these factors.

2. Materials and Methods

2.1. Theoretical formulation. The extended finite element method (XFEM) is a numerical method based on the concept of partition of unity, which involves local enrichment of nodes near the discontinuity to describe the jump in a discontinuous displacement field across the crack as well as the remeshing is not required during crack growth.

The displacement vector, for 2D discontinuities modelling in XFEM framework is given by the following expression [20, 21]:

$$u^h(x)=\sum_{I\in N} N_I(x)u_I+\sum_{J\in N^{disc}}\tilde{N}_J(x)H_j(x)a_J+$$
$$+\sum_{K\in I(R)}\tilde{N}_K(x)\sum_{\alpha=1}^{4}B_{\alpha k}(x)b_{\alpha K}, \qquad (1)$$

where $N_I(x)$, $\tilde{N}_j(x)$, $\tilde{N}_K(x)$ is the FEM shape function; u_I, a_J, $b_{\alpha K}$ are the conventional degrees of freedom (Classical, Heaviside function and crack-tip enrichment function successfully); $H(x)$ is the modified Heaviside step function, which takes on the value:

$$H(x)=\begin{cases}+1, & \text{above the crack,}\\ -1, & \text{bellow the crack;}\end{cases} \qquad (2)$$

$B_\alpha(x)$ – asymptotic function for the crack tip in isotropic elasticity given by [22]:

$$B\equiv[B_1,B_2,B_3,B_4]=$$
$$=\left[\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\frac{\theta}{2}\sin\theta,\sqrt{r}\cos\frac{\theta}{2}\sin s\theta\right], \qquad (3)$$

where (r, θ) are the polar coordinates in the system of the local crack tip.

Elements that belonged to a circle $C(x_0,r)$ around the crack tip, whose center is the crack tip, and its radius r; are generally enriched with these functions [23].

The first term of Westergaard tip functions $\sqrt{r}\sin\theta/2$ represents a discontinuity in displacement field.

For the rest of the domain, the enrichment is chosen to be the modified Heaviside step function, so that the resulting displacement field contains a discontinuity at the crack location, Fig. 1.

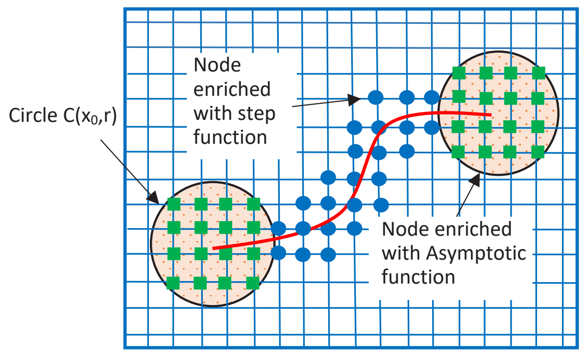

Fig. 1. Enriched node selection's for an arbitrary 2D crack problem by XFEM

As for Stress intensity factors (SIF) which are important numerical parameters in fracture mechanics, they characterize the strength of stress singularities near the crack tip, through which describing the evolution material's resistance to crack propagation.

For linear elastic fracture mechanics problems in 2D, the contour integral J is equal to the energy release rate G. The relation between the contour integral J and stress intensity factors mode I and II (KI and KII) for mixed-mode problems in 2D can be represented as [21]:

$$J=\frac{1}{E^*}(K_I^2+K_{II}^2), \qquad (4)$$

where

$$E^*=\begin{cases}\dfrac{E}{1-\upsilon^2}, & \text{for plane stain,}\\ E, & \text{for plane stress.}\end{cases} \qquad (5)$$

The contour integral J for the body with crack is given in equation (4) [10, 11].

Here Γ is a closed contour line encompassing the crack tip, n is its normal unit. W is the strain energy density, $T_i=s_{ij}n_j$ is the traction vector perpendicular to Γ in the outward direction, d_{1j} is Kroneker delta, s_{ij} is stress tensor, u_k is the displacement vector and $d\Gamma$ is differential element of arc length along the closed contour Γ:

$$J=\int_\Gamma\left(Wdx_2-T_i\frac{\partial u_i}{\partial x_1}\right)d\Gamma=\int_\Gamma\left(W\delta_{1j}-\sigma_{ij}\frac{\partial u_i}{\partial x_1}\right)n_j\,d\Gamma. \qquad (6)$$

The J-integral for the sum of the two states of a cracked body (1 and 2) with characteristics $(\sigma_{ij}^{(1,2)}, \varepsilon_{ij}^{(1,2)}, u_{ij}^{(1,2)})$ are defined as:

$$J^{(1,2)} = \int_{\Gamma} \left(\begin{array}{l} \frac{1}{2}(\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)})((\varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)})\delta_{1j} - \\ -(\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)}) \frac{\partial(u_i^{(1)} + u_i^{(2)})}{\partial x_1} \end{array} \right) n_j \mathrm{d}\Gamma. \quad (7)$$

On further solving, get:

$$J^{(1,2)} = J^{(1)} + J^{(2)} + I^{(1,2)}, \quad (8)$$

where $I^{(1,2)}$ is called the interaction integral for states 1 and 2, which is equal to [5]:

$$I^{(1,2)} = \frac{2}{E^*}\left(K_I^{(1)}K_I^{(2)} + K_{II}^{(1)}K_{II}^{(2)}\right), \quad (9)$$

where I (1, mode I) and (1, mode II) are interaction integrals.

According to equation (9), for two state (2) in XFEM, the SIF $KI^{(1)}$ mode I is evaluated by putting ($K_I^{(2)}=1$, $K_{II}^{(2)}=0$) (a pure mode I), and the SIF $K_{II}^{(1)}$ mode II by putting ($K_I^{(2)}=1$, $K_{II}^{(2)}=0$) (a pure mode II).

2.2. OpenXFEM++ numerical code overview. The object-oriented programming language (OOP) has useful resources and the potential to enrich its library that allows independent and concurrent execution. This language is often chosen because of the various benefits this design offers, including providing general-purpose codes that are understandable, maintainable, and extensible by making it easier to integrate new object types, solution techniques, and other features as soon as they become available.

The *OpenXFEM++* code [24], is an open source environment written in C++ language and developed within the framework of object-oriented programming (OOP) approach, based on an existing finite element code *FEMOBJ* [25], an object-oriented finite element package for static and dynamic nonlinear applications.

The extended finite element method (XFEM) is inherently a modular numerical tool, so object-oriented programming facilitates the use of this module. Therefore, evaluating *OpenXFEM++* code in terms of numerical efficiency depends on object-oriented programming mechanisms.

Knowing that every object-oriented application design requires finding classes and objects with specific structures and attributes, *OpenXFEM++* includes object, class, derived class, class members, function, etc. It also contains all the elements of object-oriented programming: methods, messages, constructors, destructors, encapsulation, polymorphism, inheritance, and class hierarchy, in addition to the basic techniques used in this type of programming.

Since the code consists of hundreds of classes and class templates with multiple default inheritances, some typical classes are discussed here with brief description. The data and functions kept to a minimum, with a simple presentation of member functions arguments.

2.2.1. Element class. *Element Class* is an abstract class which five class are publicly divided, which are: *Tri6_U* (6 nodes triangle elements), *Tetra4* (4 nodes tetrahedral elements), *Tri_U* (3 nodes triangle elements), *Quad4_U* (4 nodes quadrilateral elements), and *MITC4* (4 nodes quadrilateral shell elements), therefore creating the lists of enriched and non-enriched element.

One of the basic tasks of this class is to return enriched interpolation functions, ensuring interaction between finite elements and enriched finite elements, dividing into finite elements sub-triangles, as well as storing the local coordinates of the original element, using many methods similar to the *treatGeoMeshInteraction* method and the *PartitionMySelf* method.

2.2.2. Enrichment item. This class is concerned with the characteristics and features of the discontinuities that cause enrichment, which are called objects and each of which has its own geometry. It contains four objects: *CrackInterior*, *CrackTip*, *Hole*, and *MaterialInterface*.

The *CrackInterior* object contains methods to model the internal crack correctly, it updates the crack geometry with the function *UpdateMyGeometry()*, resolves the linear dependence of the enriched nodes with the function $H(x)$ using the function *resolveLinearDependency()* and also updates the enrichment after the cracks grow with the function *updateEnrichment( )*.

The *CrackTip* class is closely related to the CrackInterior class, especially the two-dimensional model. It uses the myTips function to read the information of the studied crack from the input file. The *buildIntegrationDomain2* function identifies the elements that intersect with the boundaries of the radius of the circle centered at the tip of the crack to Compute the interaction integrations, which is used Later to calculate SIFs.

Hole and *MaterialInterface* object implement holes, and material interface discontinuities between two Materials successively.

2.2.3. Enrichment function. The *EnrichmentFunction* class is an abstract class. Its primary role is to define specific functions that the enrichment function must implement through the *EvaluateYourSelfAt()* object, which is used to calculate the enrichment value of the node function and its gradient. The *EvaluateYourGradAt()* object is also used to calculate the derivatives of the enrichment function at a point, for example, the Gaussian point (GP) in order to formulate the enriched stiffness matrix.

The *EnrichmentFunction* class is divided into several classes, including: *DiscontiniousFunction* divided class implements a discontinuous function *H(x)* and is used to enrich nodes that belong to elements cuted by crack.

AsymptoticFunction divided class is a virtual base class, it implements the asymptotic functions, it is used to enrich nodes that belong to elements contain crack-tip for homogeneous media.

AbsSignedDistance divided class used model the material interface.

2.2.4. Others class. In addition, several other utility classes are also defined in a comprehensive finite element library, to have mathematical extraction of engineering variables:

- The *GeometyEntity* class is an abstract class and is concerned with the enrichment element geometry and Recognize different types of geometric entities: Vertex, PiecewiseLinear, PiecewiseParabolic, Circle and ellipse.
- Class *EnrichmentDetector* aims to make a selection of nodes to be enriched chosen by read from the input data file using a virtual method named *setEnrichedNodes*.

– As well as to determine the direction of crack growth *CrackGrowthDirectionLaw* class and its *MaxHoopStress* derived class are used, based on the maximum hoop stress criteria of Erdogan and Sih.

– The abstract class *CrackGrowthIncrementLaw* implement the crack growth increment law: Paris law, fixed increment and adaptive increment.

3. Results and Discussions

3.1. Out of phase angle orientation effect on inclined crack. The first example considers an inclined cracked plate element with Out of phase angle orientation, this squared plate is subjected to uniform tensile stresses $s=1$ on its two sides.

Fig. 2 shows a geometry and dimensions of cracked plate (W), containing one inclined internal cracks of length ($a/W=0.25$).

Whose elastic properties E modulus of elasticity and υ Poisson's ratio. The thickness is negligible compared to other plate's dimensions. Therefore, 2D analysis is performed with simple structured triangular elements to connect the geometric plate shapes.

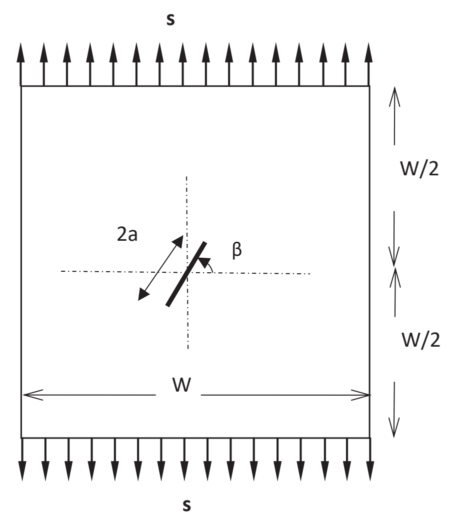

Fig. 2. Center inclined crack geometry in an infinite square plate under uniaxial tensile

The mixed stress intensity factor (SIF) mode I and mode II at crack tip are calculated depending on an angle (β) which is ranged from 0° to 180° with a pitch of 5° beginning from x axes to y axes and 90° in a counterclockwise direction respectively, and compared with theoretical SIFs under tensile load given by the formula [26]:

$$K_I = \sigma_{app}\sqrt{\pi a}\cdot\cos^2\beta,$$
$$K_{II} = \sigma_{app}\sqrt{\pi a}\cdot\cos\beta\cdot\sin\beta. \quad (10)$$

Table 1 lists the measured SIF for Mode I and Mode II for different β angle values, the error ratio compared to the theoretical solutions.

The Normalized SIF mode I and II results, presented in Table 1 show an obvious convergence with an acceptable error rate (around 5 %) comparing with those given by theoretical results.

Fig. 3 represents the normalized stress intensity factor values evolution in mode 1. The graph reveals a flagrant decrease in K_I until its annihilation within the interval of crack angle inclination ranging from 0° to 90°, which can be explained physically by the change in state of the crack through these lips, which pass, from a state of total open lips at 0° inclination angle, towards total closed lips at 90° inclination angle.

Table 1

Convergence of normalized SIFs for an inclined center crack

Angle β	Normalized K_I calculated	Theoretical normalized K_I	% error	Normalized K_{II} calculated	Theoretical normalized K_{II}	% error
30°	0.7308	0.7500	2.56	0.4210	0.4330	2.79
60°	0.2418	0.2500	3.28	0.4239	0.4330	2.10
120°	0.23928	0.2500	4.28	–0.41586	–0.4330	3.96
150°	0.72854	0.7500	2.86	–0.41996	–0.4330	3.01

Same observation for the second case but in the opposite direction, however, there is a pronounced increase in K_I evolution up to its maximum values in an interval of crack inclination angle going from 90° to at 180°. This can be explained physically by the opposite phenomenon of the previous case, hence the transition from a state of total closure to a state of total crack opening.

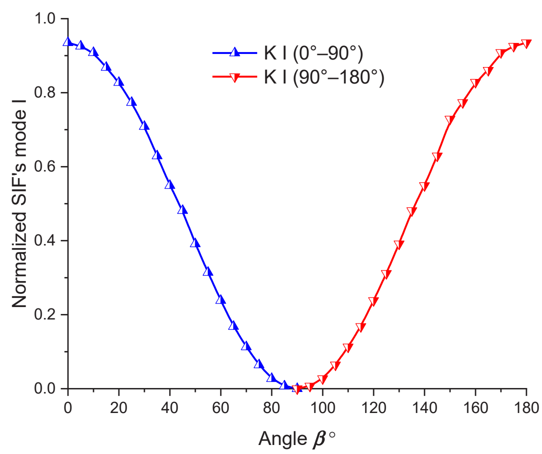

Fig. 3. Normalized SIFs mode I value vs angle of inclination β for an infinite plate with an inclined center crack under uniaxial tensile

For normalized stress intensity factor K_{II} mode II, the behaviour change completely for case a (Fig. 3), since the proportionality between K_{II} value and inclination angle β between (0° and 45°) is preserved; when beyond this range, the loss of the proportionality appears from 45° to 90°. An inversed phenomenon took place for case b (Fig. 4) inclination angle ranged between 90° to 180°, wherein all values are inside the negative range for normalized stress intensity factors mode II, with the remaining curve shape.

For the physical meaning, the deduction is that there is slip phenomena in the positive field, then a stability of crack lips ($K_{II}=0$ no slip), and reversal of slip in the negative direction all this appear in the range of 0° to 90°. The same phenomena will appear but in the opposite direction ranged from 90° to 180°.

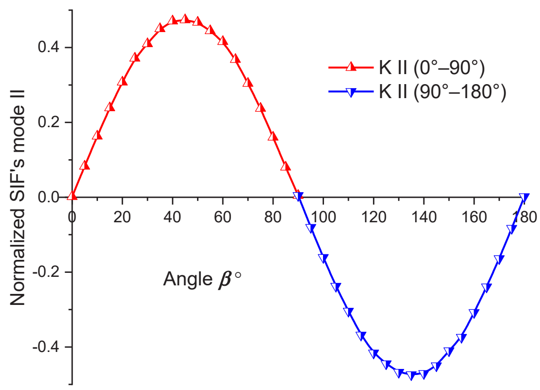

Fig. 4. Normalized SIFs mode *I* value vs angle of inclination β for an infinite plate with an inclined center crack under uniaxial tensile

3.2. Two similar inclined cracks. In this second part, an example of two inclined internal cracks of length ($a/W=0.25$) in a rectangular cracked plate ($L=2W$), subjected to uniform tensile stresses $s=1$ on the two upper and lower sides. The goal of this example is to illustrate the efficiency of *OpenXFEM++* implementation.

Table 2, summarizes a comparison between normalized mixed stress intensity factors values (NMSIF) mode *I* and *II* at the crack tip *A* and *C* calculated by setting an angle $\beta=\alpha=20°$, using C++ object oriented programming code, with those obtained by [27] using dual boundary element technique, as well as error rate (Fig. 5).

Table 2

Normalized SIFs convergence for an inclined center crack with an angle $\beta=\alpha=20°$ under tensile loading at tips *A* and *C*

Normalized SIFs	Tip-*A*			Tip-*C*		
	(C++)	Ref	C++/Ref (%)	(C++)	Ref	C++/Ref (%)
K_I	0.14385	0.14497	0.77	0.14386	0.14483	0.67
K_{II}	0.31649	0.32561	2.80	0.31650	0.32939	3.91

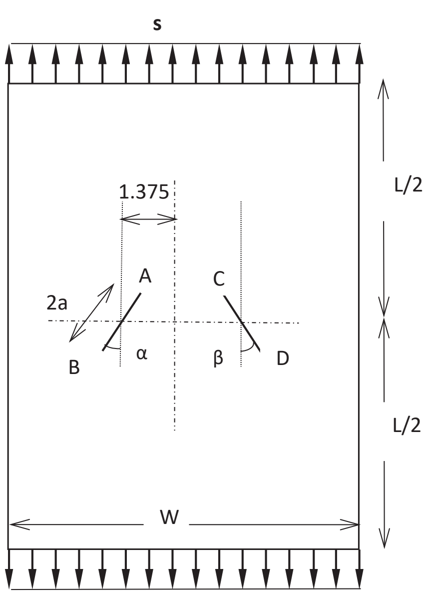

Fig. 5. Double inclined crack geometry in an infinite rectangular plate under uniaxial tensile

The same things for Table 3 which *summarize* a comparison between normalized mixed stress intensity factors values (NMSIF) mode *I* and *II* at the crack tip *B* and *D* (Fig. 6).

Table 3

Normalized SIFs convergence for an inclined center crack with an angle $\beta=\alpha=20°$ under tensile loading at tips *B* and *D*

Normalized SIFs	Tip-*B*			Tip-*D*		
	(C++)	Ref	C++/Ref (%)	(C++)	Ref	C++/Ref (%)
K_I	0.10265	0.10479	2.04	0.10265	0.10354	0.86
K_{II}	0.31935	0.33163	3.70	0.31936	0.33514	4.71

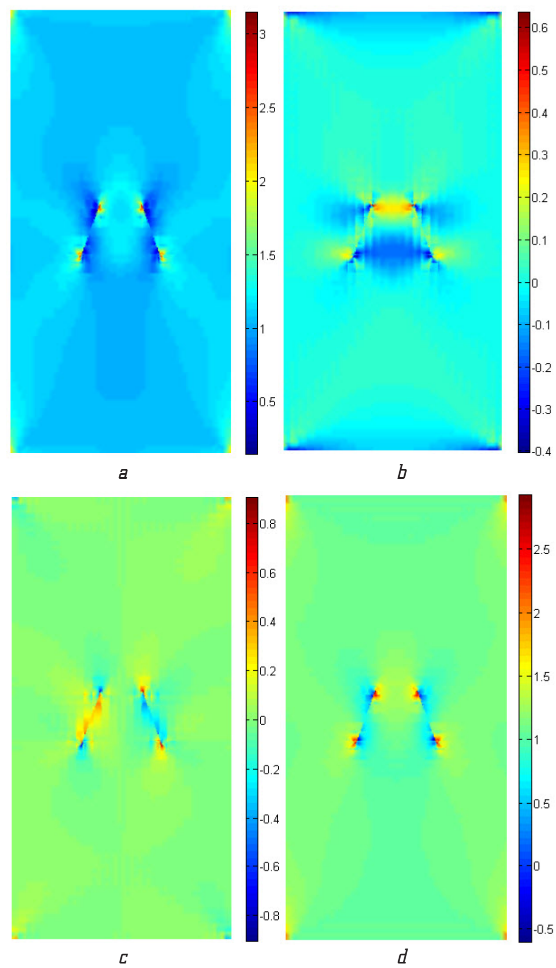

Fig. 6. Stress contour for a double inclined crack at inclination angle equal to $\alpha=\beta=12°$: *a* – Von Mises stress; *b* – s_{xx}; *c* – s_{xy}; *d* – s_{yy}

3.3. Overlapping effect. Let's take the previous example as basic principle, but by applying a small modification by mean of adding another crack just next to the first in order to study overlapping effect.

The stress intensity factor values influence at the crack-tip *A* was studied by changing the second crack's angle inclination β which ranged between 0° and 90° with a pitch of 5°. This is done by fixing the crack inclination *AB* at $\alpha=20°$.

The normalized mixed stress intensity factors values (SIFs) mode *I* and *II*, which correspond to crack tips *A* and *C* are plotted in Fig. 7 and Fig. 8, respectively.

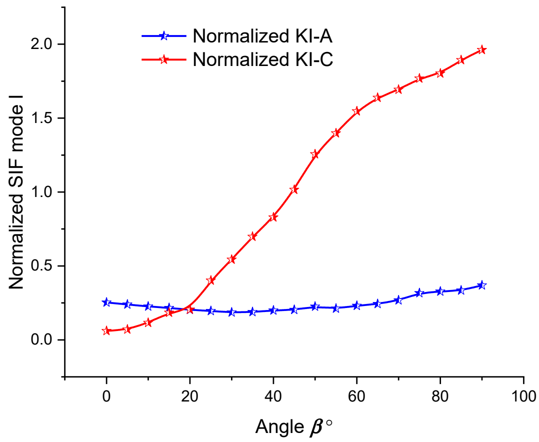

Fig. 7. Normalized SIFs mode *I* correspond to crack tips *A* and *C* vs inclination angle β for an infinite plate with two inclined center crack under uniaxial tensile

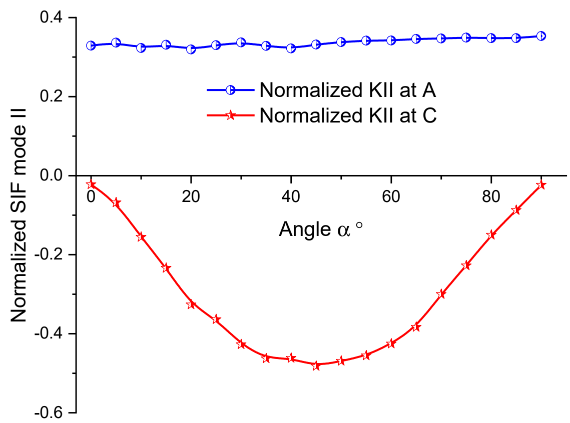

Fig. 8. Normalized SIFs mode *II* correspond to crack tips *A* and *C* vs inclination angle β for an infinite plate with two inclined center crack under uniaxial tensile

Stress intensity factors in mode *I* at both crack-tip *A* and *C* results given by object-oriented programming C++ code, using extended finite element method are shown in Fig. 7. It can be seen that on one hand, stress intensity factors values increase at crack-tip *C*. On the other hand, it is moderately stable at crack-tip *A*, which means that there is no overlapping effect of crack *C–D* on crack *A–B*.

Through Fig. 8 which contains the stress intensity factors value mode *II* calculated at both crack-tip *A* and *C*.

A quasi-stability of stress intensity factor values at crack tip *A* is observed, with a variation range not exceeding 9 %. Regardless of stress intensity factors curve values at the crack tip *C*, that has a decreasing and increasing trend in the interval form 0° to 45° and from 45° to 90° of crack's angle inclination β, respectively.

3.4. Discussion. Concerning our results, it is possible to see two major aspects:

– The first is the obvious convergence with an acceptable error rate (around 5 %) of the normalized stress intensity factors values modes *I* and *II* calculated using XFEM object-oriented implementation, compared to those given by the theoretical results.

– The second aspect is the absence of any effect of crack angle inclination variation in juxtaposition case of two inclined cracks, concerning normalized stress intensity factors values (K_I/K_{II}) at static state.

The absence of this effect is due to one limitation inheriting this study, which is the fact that dynamics effect is not taken into account in this research, leading to neglect mass matrix.

In the future, it will be necessary to carry out studies on the influence of juxtaposition of two inclined cracks in the case of crack propagation.

4. Conclusions

An object-oriented programming called OpenXFEM++ based on the extended finite element method is applied in this study by modeling a 2D internal oblique crack to illustrate the improvement of the code through its efficiency, applicability and extensibility.

In this paper, tensile loading was applied to a two-dimensional (2D) plate with inclined cracking to calculate mixed-mode SIFs (K_I/K_{II}). The effects of different factors such as the direction of the out of phase angle, as well as the effect of the juxtaposition of two inclined internal cracks, were taken into account, giving the following conclusion: the undeniable robustness and efficiency of the implementation used as well as the extended finite element method (XFEM) for the numerical modeling of discontinuities.

The results obtained with XFEM agree remarkably with those reported in the literature and thus justify the exceptional performances of the method for SIF calculation.

Negative values of the mode *II* stress intensity factor (K_{II}) are mainly linked to the sliding phenomenon of cracked lips.

The phenomenon of juxtaposition of two inclined cracks has almost no effect on the SIF in mixed mode (K_I/K_{II}) in the static state.

It is possible to note that all these results and remarks are valid only for linear static case; many surprises can arise if to apprehend these two physical phenomena.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The research was performed without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Belytschko, T., Mos, N., Usui, S., Parimi, C. (2001). Arbitrary discontinuities in finite elements. *International Journal for Numerical Methods in Engineering, 50 (4),* 993–1013. doi: https://doi.org/10.1002/1097-0207(20010210)50:4<993::aid-nme164>3.0.co;2-m
2. Belytschko, T., Gracie, R., Ventura, G. (2009). A review of extended/generalized finite element methods for material modeling. *Modelling and Simulation in Materials Science and Engineering, 17 (4),* 043001. doi: https://doi.org/10.1088/0965-0393/17/4/043001

3. Belytschko, T., Gracie, R. (2007). On XFEM applications to dislocations and interfaces. *International Journal of Plasticity, 23 (10-11),* 1721–1738. doi: https://doi.org/10.1016/j.ijplas.2007.03.003
4. Belytschko, T., Black, T. (1999). Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering, 45 (5),* 601–620. doi: https://doi.org/10.1002/(sici)1097-0207(19990620)45:5<601::aid-nme598>3.0.co;2-s
5. Mos, N., Dolbow, J., Belytschko, T. (1999). A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering, 46 (1),* 131–150. doi: https://doi.org/10.1002/(sici)1097-0207(19990910)46:1<131::aid-nme726>3.0.co;2-j
6. Moës, N., Belytschko, T. (2002). Extended finite element method for cohesive crack growth. *Engineering Fracture Mechanics, 69 (7),* 813–833. doi: https://doi.org/10.1016/s0013-7944(01)00128-x
7. Parks, D. M. (1974). A stiffness derivative finite element technique for determination of crack tip stress intensity factors. *International Journal of Fracture, 10 (4),* 487–502. doi: https://doi.org/10.1007/bf00155252
8. Hellen, T. K. (1975). On the method of virtual crack extensions. *International Journal for Numerical Methods in Engineering, 9 (1),* 187–207. doi: https://doi.org/10.1002/nme.1620090114
9. Rybicki, E. F., Kanninen, M. F. (1977). A finite element calculation of stress intensity factors by a modified crack closure integral. *Engineering Fracture Mechanics, 9 (4),* 931–938. doi: https://doi.org/10.1016/0013-7944(77)90013-3
10. Rice, J. R. (1968). A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. *Journal of Applied Mechanics, 35 (2),* 379–386. doi: https://doi.org/10.1115/1.3601206
11. Rice, J. R., Rosengren, G. F. (1968). Plane strain deformation near a crack tip in a power-law hardening material. *Journal of the Mechanics and Physics of Solids, 16 (1),* 1–12. doi: https://doi.org/10.1016/0022-5096(68)90013-6
12. *Fracture mechanics. 19: San Antonio* (1988). ASTM, Philadelphia, 939.
13. Stern, M., Becker, E. B., Dunham, R. S. (1976). A contour integral computation of mixed-mode stress intensity factors. *International Journal of Fracture, 12 (3),* 359–368. doi: https://doi.org/10.1007/bf00032831
14. Hedayati, E., Vahedi, M. (2014). Using Extended Finite Element Method for Computation of the Stress Intensity Factor, Crack Growth Simulation and Predicting Fatigue Crack Growth in a Slant-Cracked Plate of 6061-T651 Aluminum. *World Journal of Mechanics, 4 (1),* 24–30. doi: https://doi.org/10.4236/wjm.2014.41003
15. Aliha, M. R. M., Bahmani, A., Akhondi, Sh. (2016). Mixed mode fracture toughness testing of PMMA with different three-point bend type specimens. *European Journal of Mechanics – A/Solids, 58,* 148–162. doi: https://doi.org/10.1016/j.euromechsol.2016.01.012
16. Wang, Z., Yu, T., Bui, T. Q., Tanaka, S., Zhang, C., Hirose, S., Curiel-Sosa, J. L. (2017). 3-D local mesh refinement XFEM with variable-node hexahedron elements for extraction of stress intensity factors of straight and curved planar cracks. *Computer Methods in Applied Mechanics and Engineering, 313,* 375–405. doi: https://doi.org/10.1016/j.cma.2016.10.011
17. Dimitri, R., Fantuzzi, N., Li, Y., Tornabene, F. (2017). Numerical computation of the crack development and SIF in composite materials with XFEM and SFEM. *Composite Structures, 160,* 468–490. doi: https://doi.org/10.1016/j.compstruct.2016.10.067
18. Liao, Y., Zhang, X., Wang, B., He, M. (2023). Calculations of crack stress intensity factors based on FEM and XFEM models. *Australian Journal of Mechanical Engineering,* 1–11. doi: https://doi.org/10.1080/14484846.2023.2207243
19. *OpenXfem++*. Available at: https://sourceforge.net/projects/openxfem/
20. Khoei, A. R. (2015). *Extended finite element method: theory and applications.* Chichester: John Wiley & Sons, Inc, 584. doi: https://doi.org/10.1002/9781118869673
21. Mohammadi, S. (2012). *XFEM Fracture Analysis of Composites.* Wiley. doi: https://doi.org/10.1002/9781118443378
22. Khatri, K., Lal, A. (2019). Mixed mode stress intensity factors of slanted edge cracked plate with hole subjected to various in-plane loadings using XFEM. *International Journal of Materials and Structural Integrity, 13 (1/2/3),* 110. doi: https://doi.org/10.1504/ijmsi.2019.100388
23. Laborde, P., Pommier, J., Renard, Y., Salaün, M. (2005). High-order extended finite element method for cracked domains. *International Journal for Numerical Methods in Engineering, 64 (3),* 354–381. doi: https://doi.org/10.1002/nme.1370
24. Bordas, S., Nguyen, P. V., Dunant, C., Guidoum, A., Nguyen-Dang, H. (2007). An extended finite element library. *International Journal for Numerical Methods in Engineering, 71 (6),* 703–732. doi: https://doi.org/10.1002/nme.1966
25. Dubois-Pe'lerin, Y., Zimmermann, T. (1993). Object-oriented finite element programming: III. An efficient implementation in C++. *Computer Methods in Applied Mechanics and Engineering, 108 (1-2),* 165–183. doi: https://doi.org/10.1016/0045-7825(93)90159-u
26. Sih, G. C., Paris, P. C., Erdogan, F. (1962). Crack-Tip, Stress-Intensity Factors for Plane Extension and Plate Bending Problems. *Journal of Applied Mechanics, 29 (2),* 306–312. doi: https://doi.org/10.1115/1.3640546
27. Chen, W. H., Chen, T. C. (1995). An efficient dual boundary element technique for a two-dimensional fracture problem with multiple cracks. *International Journal for Numerical Methods in Engineering, 38 (10),* 1739–1756. doi: https://doi.org/10.1002/nme.1620381009

✉***Rebadj Chabane,*** *Postgraduate Student, Department of Civil Engineering, Mines Laboratory, Laarbi Tébessi University, Tébessa, Algeria, e-mail: chabane.rebadj@univ-tebessa.dz, ORCID: https://orcid.org/0000-0002-6943-8807*

Sabah Moussaoui, *Department of Civil Engineering, Civil Engineering Research Laboratory, Sétif 1 University, Sétif, Algeria, ORCID: https://orcid.org/0000-0002-8641-089X*

Mourad Belgasmia, *Professor, Department of Civil Engineering, Civil Engineering Research Laboratory, Sétif 1 University, Sétif, Algeria, ORCID: https://orcid.org/0000-0003-1409-02814*

✉*Corresponding author*