UDC 621.74
DOI: 10.15587/2706-5448.2024.301156

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DEVELOPMENT OF THE HARDNESS MATHEMATICAL MODEL OF Ti-ALLOYED IRON FOR CAST PARTS USED IN CONDITIONS OF INTENSIVE ABRASIVE FRICTION

The object of research is wear-resistant cast iron, intended for cast parts that work under conditions of intense abrasive friction during operation. Examples of such parts can be mixer blades of various functional purposes, the operational properties of which include stability, which depends on the hardness, determined on the HRC scale. To give such cast parts wear-resistant properties, the cast iron from which they are made is alloyed with elements that contribute to the formation of carbides of different composition: W, V, Mo, Ti, etc. The main problem that prevents the purposeful selection of materials is incomplete knowledge about the effect of chemical composition on properties, in particular, wear resistance, which prevents a justified selection criterion.

Using regression analysis methods, a mathematical model was obtained, including a regression equation of the form HRC=f(C; C_{eq}; Ti), which relates the content of carbon, titanium and carbon equivalent in cast iron and hardness. The resulting model allows for purposeful selection of the chemical composition, which ensures a given value of HRC, on which wear resistance depends. Optimization of the chemical composition, performed according to this model, made it possible to determine that the chemical composition, which provides the maximum hardness of HRC=49, is outside the planning area: C=3.54 %, C_{eq}=3.95 %, Ti=3.56 %. It was established that the same value of hardness can be obtained inside the considered planning area, which has an arbitrary appearance, provided with available conditions of a passive experiment. According to the available experimental data, the values of the input variables equal to C=3.34 %, C_{eq}=3.727 %, Ti=0.73 % ensure obtaining hardness at the level of HRC=49. Such alternative options regarding composition and properties may indicate that the HRC=f(C; C_{eq}; Ti) response surface has a complex appearance that requires additional research.

Keywords: *wear resistance of cast iron, HRC hardness, alloying of cast iron, chemical composition of cast iron.*

Received date: 12.02.2024
Accepted date: 01.04.2024
Published date: 03.04.2024

How to cite

Barsuk, A. (2024). Development of the hardness mathematical model of Ti-alloyed iron for cast parts used in conditions of intensive abrasive friction. Technology Audit and Production Reserves, 3 (1 (77)), 00–00. doi: https://doi.org/10.15587/2706-5448.2024.301156

1. Introduction

Examples of cast parts that require increased hardness of working surfaces are the mixing bodies of mixers of various functional purposes. These include mixers used in agriculture [1], mixing and preparation equipment of foundries, etc. The conditions of operation of such parts in the process of operation allow to consider them in general as a typical part working on abrasive friction, regardless of the mixing media. Therefore, various research methods are used to analyze the predicted performance of such parts: modeling by the discrete element method (DEM) [2, 3], particle image velocimetry (PIV) moving in the field of their flow [4], the Advanced DEM-CFD method [5]. The results of such studies make it possible to form the optimal geometric profile of the parts according to the selected criteria [6], which, however, by itself cannot provide high operational characteristics without taking into account the quality of the material.

The specified characteristics of the material are obtained either at the stage of smelting the alloy, choosing rational combinations of alloying [7–9], varying the content of the main elements [10–12], or by surface heat treatment technologies [13]. The production of blades from white cast iron may be more preferable than from alloyed steels, and there are known production data on the effective use of titanium as an alloying element [14]. This is due to the fact that when low-carbon pre-eutectic cast iron is alloyed, its structure consists mainly of decay products of austenite, ledeburite, titanium carbide inclusions, and needle-shaped cementite. Such cast irons can be smelted in induction furnaces, which make it possible to ensure the formation of a given microstructure due to temperature control [15].

A set of mathematical models relating the chemical composition of Ti-alloyed cast iron with the tensile strength and wear resistance coefficient is given in works [16, 17]. However, the question of the influence of carbon content, carbon equivalent and titanium content on the hardness of cast iron, on which wear resistance depends, remained unresearched.

The object of research in the work is wear-resistant cast iron intended for cast parts working under conditions of intense abrasive friction during operation.

The aim of research is to build a mathematical model that allows predicting HRC hardness based on data on the chemical composition of cast iron.

2. Materials and Methods

In the work, a mathematical model of the form $HRC=f(C;\ C_{eq};\ Ti)$ was constructed, which was represented by a polynomial of the second degree:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_1^2 + a_5x_2^2 + \\ + a_6x_3^2 + a_7x_1x_2 + a_8x_1x_3 + a_9x_2x_3, \qquad (1)$$

where the following variables are accepted: x_1 – carbon content in the alloy (C, %); x_2 – carbon equivalent (C_{eq}, %); x_3 – titanium alloy content (Ti, %); HRC hardness is taken as the output variable (y).

To build the mathematical model, a sample of the production data given in [14] (Table 1) was used.

Table 1

Initial data sample [14]

Chemical composition, %					Hardness
C	*Si*	*Mn*	C_{eq}^{*}	*Ti*	*HRC*
2.25	1.31	0.95	2.615	0.32	44
2.37	1.4	0.99	2.76	0.63	44
2.26	1.03	1.01	2.539	0.94	45
2.21	1.53	0.95	2.641	1.69	44
2.4	1.16	1.14	2.714	2.1	45
2.68	1.08	0.6	2.986	0.37	45
2.69	0.93	1.07	2.937	0.75	45
2.53	1.5	1.02	2.949	1.26	46
2.73	1.1	1.21	3.024	2.94	48
3.13	1.6	0.78	3.587	0.28	48
3.34	1.4	1.1	3.727	0.73	49
3.24	1.4	0.98	3.631	1.54	48
3.34	1.72	0.98	3.827	2.5	48

Note: * – C_{eq} was calculated by the formula: $C_{eq} = C + 0.3Si - 0.03Mn$, %

Normalization of the values of the input variables was carried out according to the formulas:

$$x_i = \frac{x_i^* - \bar{x}_i}{I_i}, \qquad (2)$$

where x_i – the normalized value of the i-th independent variable (i=1 for C, i=2 for C_{eq}, i=3 for Ti); x_i* – the natural value of the i-th independent variable, %; $\bar{x}_i$ – the average value of the i-th independent variable, %; I_i – the interval of variation of the values of the i-th independent variable.

Table 2 shows the intervals of variation of the independent variables, and in the Table 3 – results of calculations of their normalized values.

Table 2

Variation of independent variables

Normalization parameters	*C*, %	C_{eq}, %	*Ti*, %
The lower limit of the interval, %	2.21	2.539	0.28
Upper limit of the interval, %	3.34	3.827	2.94
Average value, %	2.775	3.183	1.61
Variation interval, %	0.565	0.644	1.33

Table 3

Results of calculation of normalized values of independent variables

C, %	x_1	C_{eq}, %	x_2	*Ti*, %	x_3
2.25	–0.929	2.615	–0.882	0.32	–0.97
2.37	–0.717	2.76	–0.657	0.63	–0.737
2.26	–0.912	2.539	–1	0.94	–0.504
2.21	–1	2.641	–0.842	1.69	0.0602
2.4	–0.664	2.714	–0.728	2.1	0.3684
2.68	–0.168	2.986	–0.306	0.37	–0.932
2.69	–0.15	2.937	–0.382	0.75	–0.647
2.53	–0.434	2.949	–0.363	1.26	–0.263
2.73	–0.08	3.024	–0.247	2.94	1
3.13	0.6283	3.587	0.6273	0.28	–1
3.34	1	3.727	0.8447	0.73	–0.662
3.24	0.823	3.631	0.6957	1.54	–0.053
3.34	1	3.827	1	2.5	0.6692

The coefficients of equations (1) were estimated by the method of least squares:

$$A = \left(F^T F\right)^{-1} F^T Y, \qquad (3)$$

where F – the matrix of the experiment plan; F^T – the transposed matrix of the experimental plan; $(F^TF)^{-1}=C$ – the dispersion matrix; Y – the output variable matrix; A – the matrix of estimates of the coefficients of the regression equation (1).

To estimate the confidence interval, the lower and upper bounds of the output variables were calculated:

$$y_{\min} = y_{calc} - \frac{t_{cr}s}{\sqrt{N}}, \qquad (4)$$

$$y_{\max} = y_{calc} + \frac{t_{cr}s}{\sqrt{N}}, \qquad (5)$$

where s – the root mean square deviation determined from the formula:

$$s = \sqrt{\frac{S_R^2}{\varphi}}, \qquad (6)$$

where $S_R^2 = \sum_{i=1}^{13}\left(y_{calc\ i} - y_{exp\ i}\right)^2$ – the sum of the squares of the deviations of the experimental values of the output variables from the calculated values obtained by the regression equations; $\varphi = N-(k+1)$ – the number of degrees of freedom; N – the number of experiments (N=13); k – the number of estimated coefficients for independent variables (k=9); t_{cr} – the critical value of the Student's distribution for the chosen confidence probability and the number of degrees of freedom φ=3.

The validation of the mathematical model was checked by checking the degree of correspondence of each regression equation to the experimental data by estimating whether the actual experimental values fall into the confidence interval with a confidence probability of 99 % and 95 %.

3. Results and Discussion

The obtained mathematical model in the normalized form:

$$y_1 = 46.586 + 5.582x_1 - 3.062x_2 + 0.809x_3 - 22.761x_1^2 - 25.389x_2^2 + 0.516x_3^2 + 48.058x_1x_2 - 0.748x_1x_3 - 1.108x_2x_3, \qquad (7)$$

Fig. 1 presents the results of statistical processing of the obtained data for validation of the mathematical model at the selected confidence probability $P=0.99$.

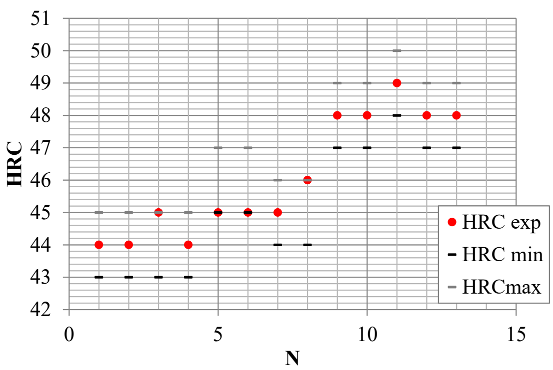

Fig. 1. Results of regression equation validation by comparing calculated and experimental data for $P=0.99$

Fig. 1 shows that with the selected confidence probability, all experimental points fell within the calculated range. This allows to conclude that the obtained mathematical model is capable of predicting hardness in the selected variable variation interval. However, if the confidence probability $P=0.95$ is set, two experimental points out of 13 were outside the calculation interval – points No. 5 and No. 6 (Fig. 2).

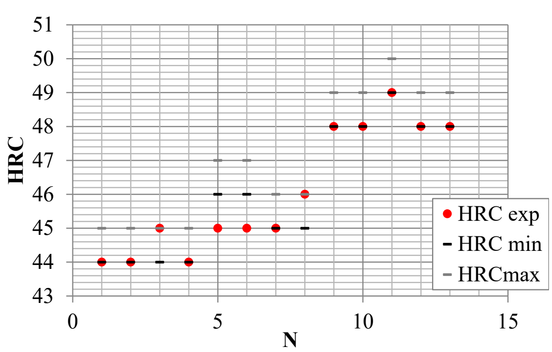

Fig. 2. Results of regression equation validation by comparing calculated and experimental data for $P=0.95$

The drop of two points allows to consider that at this level of confidence probability, the predictive ability of equation (7) is approximately 85 %, which can be considered a practically acceptable result.

For the practical application of equation (7), the normalized form is reduced to the natural form:

$$HRC = 46.586 + 5.582\frac{C-2.775}{0.565} - 3.062\frac{C_{eq}-3.183}{0.644} + 0.809\frac{Ti-1.61}{1.33} - 22.761\left(\frac{C-2.775}{0.565}\right)^2 - 25.389\left(\frac{C_{eq}^2-3.183}{0.644}\right)^2 + 0.516\left(\frac{Ti^2-1.61}{1.33}\right)^2 + 48.058\left(\frac{C-2.775}{0.565}\right)\left(\frac{C_{eq}-3.183}{0.644}\right) - 0.748\left(\frac{C-2.775}{0.565}\right)\left(\frac{Ti-1.61}{1.33}\right) - 1.108\left(\frac{C_{eq}-3.183}{0.644}\right)\left(\frac{Ti-1.61}{1.33}\right). \qquad (8)$$

The optimal chemical composition is found from the condition:

$$\frac{\partial y}{\partial x_i} = 0,\ y \to \max. \qquad (9)$$

Solution (9) leads to a system of equations of the general form, from which the values of the input variables are determined in the normalized form:

$$\begin{cases} \dfrac{\partial y}{\partial x_1} = a_1 + 2a_4x_1 + a_7x_2 + a_9x_3 = 0, \\ \dfrac{\partial y}{\partial x_2} = a_2 + 2a_5x_2 + a_7x_1 + a_8x_3 = 0, \\ \dfrac{\partial y}{\partial x_3} = a_3 + 2a_6x_3 + a_8x_2 + a_9x_1 = 0. \end{cases}$$

Table 4 shows the results of the calculation of the optimal chemical composition and expected optimal indicators of mechanical properties.

Table 4

The results of the calculation of the optimal chemical composition and expected optimal indicators of mechanical properties

Optimal chemical composition in standardized form			Optimal chemical composition in natural form, %			Optimal mechanical properties
x_1	x_2	x_3	C	C_{eq}	Ti	HRC
1.349	1.184	1.465	**3.54**	**3.95**	**3.56**	49

From the Table 4 it can be seen that the optimal chemical composition, which provides maximum hardness, is outside the planning area. It should be noted that this area has an arbitrary appearance, as the research was conducted using the methods of a passive experiment. Consequently, there is no reason to fully accept this result as unequivocal. To determine the actual optimal values of the input variables, it is first necessary to carry out the procedure of artificial orthogonalization [18], which allows to convert the passive plan into an active one and obtain accurate results of calculating the coefficients of the model (1).

The resulting equation in the form of (8) can be used to predict hardness only in the range of variation of the factors given in the Table 1, however, it is necessary to take into account the real area of planning, within which the obtained results will be adequate in the general case. This is the limitation of the study.

This research can be further developed either in the direction of a mathematical description of the planning area, within which the obtained model is operational, or in an in-depth study of the structure of cast iron corresponding to the considered area of chemical compositions. In the latter case, it is possible to identify the mechanisms of hardness formation by comparing the characteristics of the microstructure with it.

4. Conclusions

The obtained mathematical model of the form $HRC=$ $=f(C;\ C_{eq};\ Ti)$ makes it possible to carry out a targeted selection of the chemical composition that provides a given value of hardness. Optimization of the chemical composition, performed according to this model, made it possible to determine that the chemical composition, which provides the maximum hardness of HRC=49, is outside the planning area: C=3.54 %, C_{eq}=3.95 %, Ti=3.56 %. However, the same hardness value can be obtained within the considered planning area, and with titanium content close to the minimum values: C=3.34 %, C_{eq}=3.727 %, Ti=0.73 %. This indicates that the response surface has a complex appearance that requires additional research based on preliminary artificial orthogonalization of the passive experiment.

The resulting mathematical model can be used in two ways: design and operational. The first allows designers to assign the optimal chemical composition of cast iron based on the criterion of maximum hardness. The second allows to choose the batch of delivered cast parts that meets the given operational requirements for wear resistance, which depends on hardness.

Conflict of interest

The author declares that he has no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The research was performed without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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