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TESTING THE SUITABILITY OF VECTOR NORMALIZATION PROCEDURE IN TOPSIS METHOD: APPLICATION TO WHEEL LOADER SELECTION

The object of the research consists of testing the suitability of the vector normalization procedure (NP) in the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) method. One of the most problematic steps of the Multi-Criteria Decision Making (MCDM) process is related to the application of NPs by default to transform different measurement units of criteria into a comparable unit. This is because of the absence of a universal agreement that defines which NP is the most suitable for a given MCDM method. In the literature, there are thirty-one available NPs, each one of them has its strengths and weaknesses and, accordingly, can efficiently be applied to an MCDM method and even worst to another. Let's note that many NPs (e. g., NPs of sum, max-min, vector, and max) have been used by default (i. e., without suitability study) in the TOPSIS method. Consequently, outcomes of multi-criteria evaluation and rankings of alternatives considered in the decision problems could have led to inconsistent solutions, and, therefore, decision-makers could have made irrational or inappropriate decisions. That's why suitability studies of NPs become indispensable. Moreover, a description of the methodology, proposed in this research, is outlined as follows:

1) method of weighting based on an ordinal ranking of criteria and Lagrange multiplier (for determining criteria weights);

2) TOPSIS method (for ranking considered alternatives);

3) a statistical approach with 3-estimate (for comparing effects generated by the used NPs).

In the research, twelve different NPs are compared to each other in the TOPSIS method via a numerical example, which deals with the wheel loader selection problem. The results of the comparison indicate that, amongst the twelve different NPs analyzed in this suitability study, vector NP has the lesser effect on the considered alternatives' evaluation outcomes, when used with the TOPSIS method. The vector NP-TOPSIS approach can therefore be applied to solve multi-criteria decision problems. Its application further allows the decision-makers and users to better select efficient solutions and, consequently, to make conclusive decisions.

Keywords: *multi-criteria decision-making, wheel loader selection, normalization procedures, TOPSIS, statistical approach.*

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1. Introduction

Since the advent of the Multi-Criteria Decision Making (MCDM) theory in 1970 [1], normalization of initial performances' ratings of alternatives is necessary for solving the variety of measurement units of criteria. The normalization is an operation of compensation between different initial evaluations of an alternative on different criteria [2]. The compensation supposes the usage of comparable scale [3]. The normalized values are mapped either on the interval [0, 1], or on the interval [0, ∞[by applying the normalization of a decision making matrix [4].

On the other hand, let's mention that the normalization of the initial performances' ratings of alternatives does not concern all the MCDM methods. However, the MCDM methods, which require normalization of initial data, often use non-adapted normalization procedures (NPs). And this, in the absence of the universal agreement which permits of defining which NP is the most suitable for a given MCDM method [5]. In fact, usage of an inappropriate NP in a particular MCDM method can negatively influence as well the selected solution than the decision to be taken.

In the literature, influence of NPs on the ranking of alternatives and, hence, on the selection of the best solution is investigated by numerous authors. For example, author of [6], when comparing three NPs (vector, max, and max-min NPs) in Simple Additive Weighting (SAW), TOPSIS, and Elimination and Choice Translating the REality (ELECTRE) methods via a case study, finds that the obtained results show that an alternative comes first for one NP, and last for another. The same author in [7], by pursuing his research, concludes that the usage of NPs affects the multi-criteria evaluation results and, hence, violates certain conditions of consistent choice.

Authors of [8] performed a study of comparison of max and max-min NPs in an environmental problem. After the evaluation using the Weighted Sum (WS) method, obtained alternatives' ranking are not quite similar. In order to complete their study, [8] have then essayed two other possible NPs based on each criterion, and on a satiation thresholds. In reference to [8], one NP should especially be chosen in accordance with every criterion in a MCDM method.

Authors of [9] realized research on the improving of the accuracy of support vector machine algorithm, using the Data Envelopment Analysis (DEA) method. In their study [9], mean absolute error is used as a test of comparison of five NPs. Finally, their results indicated that the non-monotonic NP was the best choice for the DEA method, followed by the vector, sum, max, and max-min NPs, respectively.

Authors of [10] studied the influence of five NPs (vector, logarithmic, sum, max, and non-linear NPs) in COmplex PRoportional ASsessment-Grey (COPRAS-G) method, when evaluating and choosing:

1) best gear materials;

2) crash-worthiness characteristics of thin-valled structures.

According to these authors [10], results of comparison indicated that the logarithmic NP is a very good option, when used with the COPRAS-G method.

Authors of [11] assessed the influence of five NPs in Preference Ranking Organization METHod by Enrichment Evaluation type-2 (PROMETHEE-II) method on the basis of an empirical study related to the airport location selection. After the calculation, these two authors [11] indicated that the alternative, denoted as A_0, is ranked at the first place with two NPs (i. e., sum and vector NPs); and the alternative, denoted as A_1, is too ranked at the first place with three other NPs (i. e., max-min, max, and logarithmic NPs). In their study, authors of [11] did not specify: which NP is the best choice for the PROMETHEE-II method, nor which alternative amongst the top two was the best one for the airport location selection?

The studies carried out by the above-cited authors indicated that a MCDM method, applying different NPs when solving one same decisional problem, can provide different results of alternatives' ranking. Therefore, a NP, for being the most appropriate for a given MCDM method, must be chosen after a suitability study. Accordingly, the objective of this study consists to test the suitability of vector NP in TOPSIS method. For achieving this objective, let's compare vector NP with eleven different NPs each to other in TOPSIS via a numerical example, which deals with the choice of one wheel loader.

2. Materials and Methods

The usage of different NPs in MCDM methods has incited many researchers to undertake comparative studies. The aim of the comparative studies firstly consists to analyze the effects of different NPs on the rankings (rank orders) of the considered alternatives, when solving one same decision problem. Secondly, it consists to define which NP is the most suitable for a given MCDM method. The comparative studies of different NPs in TOPSIS method, disposable in the literature, are overviewed in the next subsection.

2.1. Overview of the comparative studies. The comparative studies of different NPs in TOPSIS, available in the literature, are shown in Table 1.

Table 1

Comparative studies of different NPs in TOPSIS

Source	Year	Compared normalization procedures													Used MCDM method(s)
		V	*JK*	*M-1*	*M-2*	*M-3*	*M*	*NL*	*S*	*L*	*NM*	*LG*	*EA*	*E*	
[6]	2000	*	–	*	–	–	*	–	–	–	–	–	–	–	SAW, TOPSIS, ELECTRE
[7]	2001	*	–	*	–	–	*	–	–	–	–	–	–	–	SAW, TOPSIS, ELECTRE
[12]	2005	*	–	*	*	–	–	*	*	–	–	–	–	–	TOPSIS
[13]	2006	*	–	–	–	–	–	–	–	*	–	–	–	–	TOPSIS
[14]	2009	*	–	*	–	–	*	–	*	–	–	–	–	–	TOPSIS
[15]	2012	*	–	*	–	–	*	–	*	–	–	–	–	*	TOPSIS
[16]	2013	*	–	*	*	–	*	–	*	–	*	–	–	–	TOPSIS
[17]	2013	*	–	*	–	–	*	–	*	–	–	–	–	–	TOPSIS
[18]	2014	*	–	*	–	–	*	–	*	–	–	–	–	–	TOPSIS
[19]	2014	*	*	–	–	–	*	*	–	–	–	–	–	–	PROMETHEE, GRA, TOPSIS
[20]	2014	*	–	*	–	–	*	–	*	–	–	–	–	–	TOPSIS
[21]	2015	–	–	*	–	–	*	–	*	–	–	–	–	–	SAW
		*	–	–	–	–	–	–	–	–	–	–	–	–	TOPSIS
[22]	2016	*	–	*	–	–	*	–	*	–	–	–	–	–	TOPSIS
[23]	2018	*	–	*	–	–	*	–	*	–	–	*	–	–	TOPSIS
[24]	2020	*	–	*	–	–	*	–	–	–	–	*	–	–	WSM, TOPSIS, ELECTRE
[25]	2023	*	*	*	*	*	*	*	*	–	–	*	*	–	TOPSIS

Notes: table is performed starting from the literature review; * – means that the NP was applied in the comparison; *V* – vector NP; *JK* – Jüttler-Körth's NP; *M-1* – max-version-1 NP; *M-2* – max-version-2 NP; *M-3* – max-version-3 NP; *M* – max-min NP; *NL* – non-linear NP; *S* – sum NP; *L* – linear NP; *NM* – non-monotonic NP; *LG* – logarithmic NP; *EA* – enhanced accuracy NP; *E* – exponent NP

After overviewing the suitability studies of different NPs in TOPSIS method (Table 1), it is possible to consider the comparison approaches used or reused in the aforementioned suitability studies. Thus, comparison approaches, used or reused to compare different NPs in TOPSIS method, are shown in Table 2.

Table 2

Comparison approaches of different NPs in TOPSIS method

Source	Year	Comparison approach(es) used or reused in TOPSIS method
[12]	2005	Multi-axial approach based on a solution algorithm
[13]	2006	Standard deviation and confidence interval
[14]	2009	Simulation experiment based on Ranking Consistency Index
[15]	2012	Simulation experiment based on Ranking Consistency Index
[16]	2013	Pearson's correlation coefficient
[17]	2013	Fuzzy reference models
[18]	2014	Consistency Conditions Set of [26]
[19]	2014	Kendall's concordance coefficient, and Spearman's rank correlation coefficient
[20]	2014	Time complexity and space complexity
[22]	2016	Simulation experiment based on random sets
[23]	2018	Ranking Consistency Index, Consistency Conditions Set of [26], Pearson and Spearman correlation coefficients, and plurality voting technique
[24]	2020	Spearman's rank correlation coefficient
[25]	2023	Pearson and Spearman correlation coefficients, Standard deviation, and Borda count technique

Note: table is performed starting from the literature review

The comparison approaches of different NPs in TOPSIS method, presented in Table 2, are well-known from the researchers working in MCDM field. On the other hand, comparison approaches, reused in certain above-cited suitability studies (Table 2), are:

– Ranking Consistency Index [15, 23];
– Consistency Conditions Set [23];
– Standard deviation [25];
– Spearman's rank correlation coefficient [23–25];
– Pearson's correlation coefficient [23, 25].

2.2. Critical analysis of the comparative studies. After reviewing the comparative studies relating to the suitability of different NPs in TOPSIS method, it is possible to do the remarks and critics below:

The four NPs, the most frequently compared in the majority of the reviewed studies, are the sum, max-min, max, and vector NPs (e. g., [14, 17, 18, 22]).

In certain reviewed studies, the linear, logarithmic, enhanced accuracy, non-linear of Peldschus et al., non-monotonic, exponent, and Jüttler-Körth NPs are too compared in TOPSIS method, but not simultaneously (Table 1).

The study reviewed in this article, having simultaneously compared the suitability of ten NPs in TOPSIS, is carried out and published by [25]. It is therefore considered as the first in the previous literature. Moreover, study of [16] is considered the second testing the suitability of six NPs in TOPSIS method. Besides, the mean number of NPs, compared in previous TOPSIS-based studies, was four (e. g., [17, 20, 24]). Finally, two only NPs are compared in TOPSIS method by [13]. It was about the vector and linear NPs.

Authors of [15], by using the Ranking Consistency Index (previously used in [14]) for comparing five NPs in TOPSIS, have indicated that the vector NP is the best one for TOPSIS. However, the conclusion: «it (i. e., vector NP) could deal with the general multi-attribute decision making problems with various problem sizes, data ranges, and attribute types effectively», expressed in [15] to the conditional form, is linguistically considered as a probable conclusion and, hence, may not be effective.

In the study [17] on the mean error estimation of TOPSIS, author has indicated that the max-min NP was shown to be the best choice for a small number of alternatives (five or less), and max NP is a better choice for a larger number of alternatives, when used with the TOPSIS method. Let's remark that the study presented by [17] is only conducted for two benefit criteria.

The two parameters, i. e., time complexity and space complexity, applied by authors of [20] for comparing four NPs in TOPSIS, have not been appropriate to determine the accuracy of multi-criteria evaluation results (rankings of alternatives). Apparently, both above-cited parameters have been applied for an aim of usability, and not for an aim of reliability analysis of the obtained evaluation results. As for the findings provided in [20], they indicated that the sum-based NP is considered to be the best one for the TOPSIS method.

At the end of their study (which has compared max-min, max, and sum NPs in SAW method, and vector NP in TOPSIS method), authors of [21] indicated that the usage of different normalization formulas, when solving one same decision problem, can lead to differences in result of evaluation. It is possible to note that in [21] have not applied none comparison approach in their study. This conclusion is relied solely on the multi-criteria evaluation outcomes.

In the framework of a study on the rank reversal of alternatives, authors of [22] compared four NPs in TOPSIS method using the simulation experiments (based on 4800 decision problems, randomly generated). The final results, obtained in [22], showed that TOPSIS method presents lower rank reversal rates when using the max NP.

The three metrics of comparison, i. e., Spearman and Pearson correlation coefficients, and Standard deviation, applied by authors of [25], have already been applied in [19, 16, 13], respectively.

The findings drawn by authors of [12, 25] have not sufficiently been conclusive. Because they have not precisely indicated which NP was the best one for the TOPSIS method.

On the contrary, findings of authors in [13–16, 18, 19, 23, 24] have been clear, and have straightforwardly shown that the vector NP is considered to be the best choice for the TOPSIS method.

2.3. Justification of the current study. For dispelling the doubts on the adequacy of vector NP for TOPSIS, and considering the number relatively non-exhaustive of NPs tested in TOPSIS, we have then decided of performing this study. The task of the present study consists to confirm or nullify the appropriateness of vector NP for the TOPSIS method. The comparison study, foreseen in this article, is presented as follows.

Defining an exhaustive set of twelve different NPs among those that exist in the literature.

Proposing another comparison approach (an alternate approach to the approaches already used and cited in Table 2) to compare the effect generated by each of the twelve different NPs on the results of alternatives' ranking.

The proposed comparison approach is based on the Theory of Mathematical Statistics. It includes three statistical estimates below:

1) standard error estimate (E_s);
2) relative error estimate (E_r);
3) variation coefficient estimate (C_v).

After the corresponding calculation, the NP, having the smallest E_s, E_r, and C_v values, is considered to be the most preferred for the TOPSIS method.

The present study is the first in the current literature to simultaneously compare the appropriateness of twelve different NPs in TOPSIS method.

2.4. Proposed research methodology

2.4.1. Elicitation of the criteria weights. In this article, method of weight based on ordinal ranking of criteria and Lagrange multiplier, developed by authors of [27], is used for determining the criteria weights. This method of weight, which takes into account the ordinal information from the decision maker(s), is presented by three steps below [27]:

Step 1 – Ask to a decision maker to provide ascending ordinal ranking of criteria.

Step 2 – Calculate the Lagrange multiplier (λ):

$$\lambda=\frac{2}{\left(\sum_{j=1}^{n}\frac{1}{\left[(n+1)-j\right]}\right)\cdot\left[n(n+1)\right]},\tag{1}$$

where n – number of criteria, and $j=1, ..., n$.

Step 3 – Determine the weight of criterion $j(W_j)$:

$$W_j=\frac{\lambda\left[n(n+1)\right]}{2\left[(n+1)-j\right]},\tag{2}$$

where λ – Lagrange multiplier, n – number of criteria, and $j=1, ..., n$.

The above-cited weight method is understandable and straightforward, easy to use, fulfills $\sum_{j=1}^{n}W_j=1$, and satisfies the axiom of transitivity: $W_j<W_{j+1}\left(j=1,\ldots,n-1\right)$.

2.4.2. TOPSIS method. TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) method is initially developed by authors of [28]. TOPSIS method is presented by the six following steps [29]:

Step 1 – TOPSIS method assumes that there are m alternatives and n criteria, and the initial performance rating of alternative $i\left(i=1,\ldots,m\right)$ with respect to the criterion $j\left(j=1,\ldots,n\right)$.

In this step, calculate the normalized decision matrix $R=\left(r_{ij}\right)$ from the initial decision matrix $A=\left(a_{ij}\right)$. The normalized value r_{ij} is computed according to the vector NP:

$$r_{ij}=\frac{a_{ij}}{\left(\sum_j a_{ij}^2\right)^{1/2}},i=1,\ldots,m \text{ and } j=1,\ldots,n.\tag{3}$$

The normalized decision matrix $R=(r_{ij})$ is outlined in expression:

$$R=\left(r_{ij}\right)\begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1n}\\ r_{21} & r_{22} & \ldots & r_{2n}\\ \ldots & \ldots & \ldots & \ldots\\ r_{m1} & r_{m2} & \ldots & r_{mn}\end{bmatrix}.\tag{4}$$

Step 2 – Calculate the weighted normalized decision matrix $V=\left(v_{ij}\right)$. The weighted normalized value v_{ij} is calculated according to the expression:

$$v_{ij}=W_j\cdot r_{ij},\ i=1,\ldots,m \text{ and } j=1,\ldots,n.\tag{5}$$

A set of criteria weights $W=\left[W_1,\ldots,W_n\right]$ satisfying $\sum_{j=1}^{n}W_j=1$, defined by a decision maker or indirectly determined from a calculation method [30], is accommodated to the decision matrix to generate the weighted normalized decision matrix $V=\left(v_{ij}\right)$, which matrix form is shown in expression:

$$V=\left(v_{ij}\right)\begin{bmatrix} W_1\cdot r_{11} & W_2\cdot r_{12} & \ldots & W_n\cdot r_{1n}\\ W_1\cdot r_{21} & W_2\cdot r_{22} & \ldots & W_n\cdot r_{2n}\\ \ldots & \ldots & \ldots & \ldots\\ W_1\cdot r_{m1} & W_2\cdot r_{m2} & \ldots & W_n\cdot r_{mn}\end{bmatrix}=$$

$$=\begin{bmatrix} v_{11} & v_{12} & \ldots & v_{1n}\\ v_{21} & v_{22} & \ldots & v_{2n}\\ \ldots & \ldots & \ldots & \ldots\\ v_{m1} & v_{m2} & \ldots & v_{mn}\end{bmatrix}.\tag{6}$$

Step 3 – The positive-ideal solution A_j^+ and the negative-ideal solution A_j^- are defined by the expressions:

$$A_j^+=\left\{\max j\ v_{ij}\ /\ j\in J,\ \ \min j\ v_{ij}\ /\ j\in J'\right\},\tag{7}$$

$$A_j^-=\left\{\min j\ v_{ij}\ /\ j\in J,\ \ \max j\ v_{ij}\ /\ j\in J'\right\},\tag{8}$$

where J and J' are the benefit and cost criteria sets, respectively.

Step 4 – from the N-dimensional Euclidean distance, S_i^+ is calculated in expression (9) as the separation measure of alternative $i\left(i=1,\ldots,m\right)$ from the PIS. S_i^- is calculated in expression (10) as the separation measure of alternative $i\left(i=1,\ldots,m\right)$ from the NIS:

$$S_i^+=\sqrt{\left[\Sigma\left(v_{ij}-A_j^+\right)^2\right]},\tag{9}$$

$$S_i^-=\sqrt{\left[\Sigma\left(v_{ij}-A_j^-\right)^2\right]},\tag{10}$$

where $i=1,\ldots,m$ and $j=1,\ldots,n$.

Step 5 – The relative closeness to ideal solution of alternative $i\left(C_i\right)$ is calculated as in expression:

$$C_i=\frac{S_i^-}{S_i^-+S_i^+},\ 0\le C_i\le 1.\tag{11}$$

Step 6 – Rank all the considered alternatives $i\left(i=1,\ldots,m\right)$ according to the decreasing order of C_i values. The maximum C_i value corresponds to the best solution of compromise.

As for the advantages of the TOPSIS method, they are cited in [31]:

1) comprehensive mathematical concept;
2) easy usability and simplicity;
3) computational efficiency;
4) ability to measure alternative performances in simple mathematical form.

Concerning the drawbacks of the TOPSIS method, they are cited as follows:

1) its standard form is deterministic [32];
2) its use of Euclidean distance does not consider the correlation of attributes [33];

3) it does not have a providing for weight elicitation, nor a consistency checking for judgments [34];

4) it does not consider the relative importance of distances from the positive-ideal and negative-ideal solutions [35].

On the other hand, TOPSIS method is one of the most widespread MCDM methods, it is confirmed by an important number of papers published in scientific journals [36]. Its applications number in the hundreds [37–46].

2.4.3. Statistical approach. In the current study, it is possible to re-utilize none of comparison approaches, cited in Table 2, since they have already been used for comparing the NPs in TOPSIS method. In counterpart, let's propose another completely different comparison approach. The proposed comparison approach, which is based on the Theory of Mathematical Statistics, is presented by three statistical estimates below:

1. Standard error estimate «E_s» [47]:

$$E_s = \frac{S}{\sqrt{m}}, \tag{12}$$

where S – standard deviation, and m – number of observations (subjects).

In the present study, m designates the number of alternatives.

2. Relative error estimate «E_r» [48]:

$$E_r = \frac{I_c}{\bar{X}}, \tag{13}$$

where I_c – confidence interval, and $\bar{X}$ – arithmetic mean value.

The confidence interval «I_c» is determined according to the formula below:

$$I_c = \frac{t_s \cdot S}{\sqrt{v}}, \tag{14}$$

where t_s – Student coefficient, S – standard deviation, and v – degree of freedom.

The values of Student coefficient (t_s) are determined from the Table of Student. They depend on the probability P and of the degree of freedom $v\,(v = m-1)$.

In this study, the probability used: $P = 0.95$, and the degree of freedom: $v = 5$ (in the numerical example of this article (Table 4), $m = 6$ alternatives). Therefore, on the basis of the values of P and v, $t_s = 2.02$. The found value of t_s will later be used in the comparison processes of multi-criteria evaluation results, provided by the TOPSIS method.

3. Variation coefficient estimate «C_v» [49]:

$$C_v = \frac{S}{\bar{X}}, \tag{15}$$

where S – standard deviation, and $\bar{X}$ – arithmetic mean value.

The three statistical estimates (E_s, E_r, and C_v), simultaneously applied in this study, successively evaluate the accuracy of the result of multi-criteria evaluation, generated by each of the twelve NPs, when used with the TOPSIS method. After the calculation, values of E_s, E_r, and C_v are ranked according to their increasing order.

Finally, the NP, having the smallest E_s, E_r, and C_v values, is considered to be the best choice for the TOPSIS method.

3. Results and Discussions

3.1. Normalization procedures. Currently, thirty-one NPs are identified by authors of [50]. Some of them are often used with the MCDM methods, which require a normalization of the initial performances' ratings of alternatives. In the current study, the twelve NPs to be compared each to other in TOPSIS method are outlined in Table 3.

Table 3

Normalization procedures

Normalization procedure	NP	Benefit criterion	Cost criterion
Vector non-linear normalization [51]	N1	$r_{ij} = \frac{a_{ij}}{\left(\sum_j a_{ij}^2\right)^{1/2}}$	$r_{ij} = 1 - \frac{a_{ij}}{\left(\sum_j a_{ij}^2\right)^{1/2}}$
Jüttler-Körth's normalization [52]	N2	$r_{ij} = 1 - \left\lvert \frac{a_{ij}^{\max} - a_{ij}}{a_{ij}^{\max}} \right\rvert$	$r_{ij} = 1 - \left\lvert \frac{a_{ij}^{\min} - a_{ij}}{a_{ij}^{\min}} \right\rvert$
Max linear normalization – 1st version [53]	N3	$r_{ij} = \frac{a_{ij}}{a_{ij}^{\max}}$	$r_{ij} = \frac{a_{ij}^{\min}}{a_{ij}}$
Max linear normalization – 2nd version [54]	N4	$r_{ij} = \frac{a_{ij}}{a_{ij}^{\max}}$	$r_{ij} = 1 - \frac{a_{ij}}{a_{ij}^{\max}}$
Max-min linear normalization [55]	N5	$r_{ij} = \frac{a_{ij} - a_{ij}^{\min}}{a_{ij}^{\max} - a_{ij}^{\min}}$	$r_{ij} = \frac{a_{ij}^{\max} - a_{ij}}{a_{ij}^{\max} - a_{ij}^{\min}}$
Non-linear normalization [56]	N6	$r_{ij} = \left(\frac{a_{ij}}{a_{ij}^{\max}}\right)^2$	$r_{ij} = \left(\frac{a_{ij}^{\min}}{a_{ij}}\right)^3$
Sum linear normalization [3]	N7	$r_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}$	$r_{ij} = \frac{(1/a_{ij})}{\sum_j (1/a_{ij})}$
Linear normalization [57]	N8	$r_{ij} = \frac{a_{ij}}{a_{ij}^{\max} - a_{ij}^{\min}}$	$r_{ij} = 1 - \frac{a_{ij}}{a_{ij}^{\max} - a_{ij}^{\min}}$
Non-monotonic normalization [34]	N9	$r_{ij} = e^{-z^2/2}$, where $z = \frac{a_{ij} - a_{ij}^{\max}}{\sigma_j}$	$r_{ij} = e^{-z^2/2}$, where $z = \frac{a_{ij} - a_{ij}^{\min}}{\sigma_j}$
Logarithmic normalization [4]	N10	$r_{ij} = \frac{\ln(a_{ij})}{\ln\left(\prod_j a_{ij}\right)}$	$r_{ij} = \frac{1 - \frac{\ln(a_{ij})}{\ln\left(\prod_j a_{ij}\right)}}{n-1}$
Enhanced accuracy normalization [58]	N11	$r_{ij} = 1 - \frac{a_{ij}^{\max} - a_{ij}}{\sum\left(a_{ij}^{\max} - a_{ij}\right)}$	$r_{ij} = 1 - \frac{a_{ij} - a_{ij}^{\min}}{\sum\left(a_{ij} - a_{ij}^{\min}\right)}$
Exponent normalization [15]	N12	$r_{ij} = \exp\left(\frac{a_{ij} - a_{ij}^{\max}}{a_{ij}^{\max} - a_{ij}^{\min}} \cdot k\right)$	$r_{ij} = \exp\left(\frac{a_{ij}^{\min} - a_{ij}}{a_{ij}^{\max} - a_{ij}^{\min}} \cdot k\right)$

Notes: table is performed starting from the literature review; the components of the formulas (Table 3) are defined as follows: a_{ij} and r_{ij} respectively are the initial and normalized performance ratings of alternative i with respect to the criterion j; $a_{ij}^{\max}$ and $a_{ij}^{\min}$ respectively are the max and min values of criterion j; σ_j – the standard deviation of criterion j (in non-monotonic normalization); n – the number of alternatives considered in decision matrix (in logarithmic normalization); and k – the multiplier coefficient$/k \in N$ (in exponent normalization)

3.2. Numerical example and results. The data of the numerical example are stated as follows: For the loading works of swarmed rocky materials, a Construction Enterprise launches an invitation to tender for acquiring one wheel loader whose bucket capacity is equal to 2.5 m^3. The schedule of conditions (specifications), placed at the disposal of the tenderers, fixes five criteria below:

1) criterion of unloading height (upper or equal to 2.80 m), denoted as C_1;
2) criterion of nominal loading cycle time, denoted as C_2;
3) criterion of gas-oil consumption, denoted as C_3;
4) criterion of motor power, denoted as C_4;
5) criterion of unit price, denoted as C_5.

The technical and financial offers, presented by the tenderers, are shown in Table 4. To solve this multi-criteria decision problem and, afterwards, to assess the comparison of the twelve different NPs (defined in Table 3) in TOPSIS method, it is possible to apply the methodology proposed in this present article.

Table 4

Initial decision matrix A (choice of one-wheel loader)

Wheel loaders a_i	Criteria C_j				
	C_1 m, Max	C_2 s, Min	C_3 l/h, Min	C_4 horse, Max	C_5 DA, Min
a_1	2.95	36	34	162	47 500 000
a_2	2.97	38	36	163	48 500 000
a_3	2.86	35	33	160	47 300 000
a_4	2.98	42	40	186	49 500 000
a_5	2.80	32	30	150	49 400 000
a_6	2.84	34	32	151	48 800 000

Notes: DA – Dinar of Algeria (Algerian currency); different calculations, performed in this study, are realized with the help of Software Excel; to better distinguish the values, resulting from the different calculations, it is possible to calculate with five ciphers after the comma

3.2.1. Results of the criteria weights. The weights of the criteria are determined according to the method of weighting based on ordinal ranking of criteria and Lagrange multiplier [27]. The results of the criteria weights are presented in Table 5.

Table 5

Results of the criteria weights

Criteria C_j	Definition of the criteria	Criteria weights	
		W_j	Values
C_1	Criterion of unloading height	W_1	0.08759
C_2	Criterion of nominal loading cycle time	W_2	0.10948
C_3	Criterion of gas-oil consumption	W_3	0.14599
C_4	Criterion of motor power	W_4	0.21898
C_5	Criterion of unit price	W_5	0.43796

3.2.2. Results of the normalized decision matrix R. The normalized decision matrix results, with vector NP, are given in Table 6.

Table 6

Normalized decision matrix R (with vector NP)

Wheel loaders a_i	Criteria C_j				
	C_1 Max	C_2 Min	C_3 Min	C_4 Max	C_5 Min
a_1	0.41516	0.40480	0.40450	0.40715	0.39976
a_2	0.41798	0.42729	0.42829	0.40966	0.40818
a_3	0.40250	0.39355	0.39260	0.40212	0.39808
a_4	0.41938	0.47226	0.47588	0.46747	0.41660
a_5	0.39405	0.35982	0.35691	0.37699	0.41576
a_6	0.39968	0.38231	0.38070	0.37950	0.41071

Max means criterion to be maximized, and Min means criterion to be minimized.

3.2.3. Results of weighted normalized decision matrix V. The weighted normalized decision matrix results, with vector NP, are outlined in Table 7.

Table 7

Weighted normalized decision matrix V (with vector NP)

Wheel loaders a_i	Criteria C_j				
	C_1 Max	C_2 Min	C_3 Min	C_4 Max	C_5 Min
a_1	0.03636	0.04131	0.05905	0.08915	0.17507
a_2	0.03661	0.04677	0.06252	0.08970	0.17876
a_3	0.03525	0.04308	0.05731	0.08805	0.17434
a_4	0.03673	0.05170	0.06947	0.10236	0.18245
a_5	0.03451	0.03939	0.05210	0.08255	0.18208
a_6	0.03500	0.04185	0.05557	0.08310	0.17987

3.2.4. Results of multi-criteria evaluation. The results of multi-criteria evaluation with vector NP, found by the TOPSIS method, are presented in Table 8.

Table 8

Results of multi-criteria evaluation (with vector NP)

Wheel loaders a_i	Distance from the PIS S_i^+	Distance from the NIS S_i^-	Relative closeness C_i	Rank
a_1	0.01549	0.01612	0.50996	2
a_2	0.01843	0.01183	0.39094	6
a_3	0.01549	0.01760	0.53188	1
a_4	0.02258	0.01947	0.46644	4
a_5	0.02121	0.02121	0.50000	3
a_6	0.02049	0.01702	0.45374	5

Notes: all the calculation steps of the TOPSIS method are to be repeated for every NP, used in this article. The final results of multi-criteria evaluation with all the used NPs are shown in the following corresponding tables

3.2.5. Results of relative closeness to ideal solution. The results of relative closeness to ideal solution (C_i), yielded by the TOPSIS method for every NP used, are outlined in Table 9.

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Table 9

Results of relative closeness's to ideal solution (with all the *NP*s used)

Wheel loaders a_i	Relative closeness C_i over normalization procedure NP											
	*N*1	*N*2	*N*3	*N*4	*N*5	*N*6	*N*7	*N*8	*N*9	*N*10	*N*11	*N*12
a_1	0.50996	0.53522	0.48387	0.51373	0.71621	0.48882	0.46395	0.23218	0.66428	0.48205	0.73310	0.55747
a_2	0.39094	0.39522	0.35988	0.39149	0.44726	0.32455	0.28990	0.53009	0.32257	0.50000	0.45492	0.18806
a_3	0.53188	0.57415	0.51179	0.53532	0.72235	0.57264	0.50000	0.15148	0.66580	0.34408	0.74215	0.64240
a_4	0.46644	0.40436	0.46602	0.46653	0.33202	0.39510	0.46402	1	0.33604	0.77777	0.29505	0.33202
a_5	0.50000	0.55916	0.48871	0.49042	0.27661	0.52226	0.50000	0.58313	0.27505	0.21171	0.33893	0.26959
a_6	0.45374	0.51498	0.43214	0.45013	0.35114	0.45218	0.43051	0.50027	0.47492	0.20388	0.39836	0.17860

Notes: the normalization procedures, denoted as *N*1, *N*2, *N*3, …, *N*12, listed and defined in Table 3, keep the same positions and definitions in Tables 9, 10, and 11

3.2.6. Results of ranks of the considered wheel loaders. The results of ranks of the considered wheel loaders, yielded by the TOPSIS method for every NP used, are shown in Table 10.

To better visualize and distinguish the ranks of the considered wheel loaders, obtained by each of the used NPs, let's construct the graph as illustrated in Fig. 1.

According to the Fig. 1, it is possible to observe that the wheel loader a_3 is ranked first for ten NPs. Whereas, wheel loader a_2 is ranked last for six NPs. On the other hand, other wheel loaders are ranked in different ranks; and this, in accordance with their respective results.

As a deduction, it is possible to state that the usage of different NPs, when solving one same decision problem, may lead to different alternatives ranking results (Fig. 1).

3.2.7. Results of statistical analysis. The results of statistical analysis, obtained by each of the twelve NPs used, are presented in Table 11.

3.3. Effect of normalization procedures. The found statistical approach results (Table 11), which are summarized and depicted in Fig. 2, are used for analyzing the effect generated by the twelve NPs in TOPSIS method.

Table 10

Results of ranks of the considered wheel loaders (with all the *NP*s used)

Wheel loaders a_i	Ranks of the wheel loaders over normalization procedure NP											
	*N*1	*N*2	*N*3	*N*4	*N*5	*N*6	*N*7	*N*8	*N*9	*N*10	*N*11	*N*12
a_1	2	3	3	2	2	3	4	5	2	3	2	2
a_2	6	6	6	6	3	6	6	3	5	2	3	5
a_3	1	1	1	1	1	1	1	6	1	4	1	1
a_4	4	5	4	4	5	5	3	1	4	1	6	3
a_5	3	2	2	3	6	2	1	2	6	5	5	4
a_6	5	4	5	5	4	4	5	4	3	6	4	6

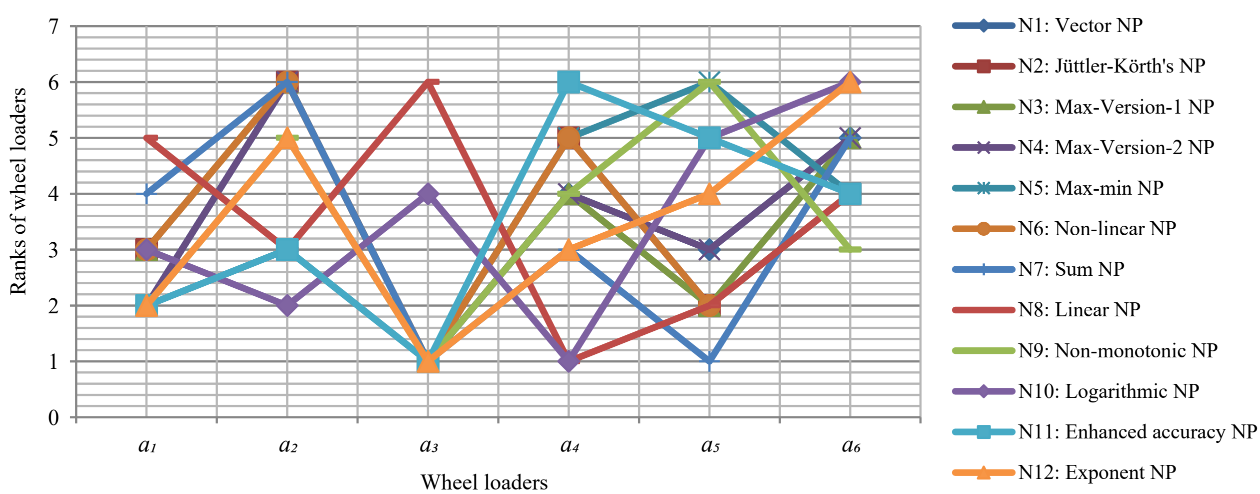

Fig. 1. Ranks of wheel loaders over normalization procedures

Table 11

Results of statistical analysis

NP	Standard error estimate		Relative error estimate		Variation coefficient estimate	
	E_s	Rank	E_r	Rank	C_v	Rank
*N*1	0.01875	1	0.08725	1	0.09659	1
*N*2	0.02912	4	0.12961	4	0.14348	4
*N*3	0.02028	3	0.09819	3	0.10871	3
*N*4	0.01902	2	0.08870	2	0.09818	2
*N*5	0.07364	10	0.34358	10	0.38033	10
*N*6	0.03331	6	0.16050	6	0.17768	6
*N*7	0.02929	5	0.14685	5	0.16257	5
*N*8	0.11192	12	0.49579	12	0.54882	12
*N*9	0.06511	8	0.31735	8	0.35129	8
*N*10	0.04218	7	0.31563	7	0.34942	7
*N*11	0.07321	9	0.32814	9	0.36324	9
*N*12	0.07962	11	0.48752	11	0.53967	11

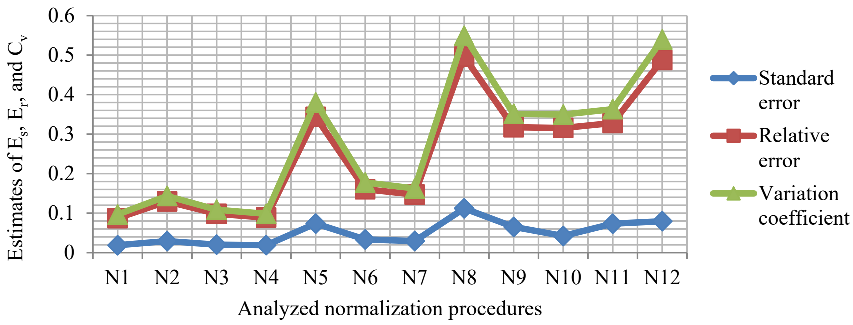

Fig. 2. Comparison of effect of normalization procedures

According to the Fig. 2, it is possible to observe that the results of statistical approach clearly show that the TOPSIS method is differently affected by each of the twelve NPs used. The vector NP (*N*1) has the smallest effect, compared to other NPs. It is followed by the max-version-2 NP (*N*4) and max-version-1 NP (*N*3), respectively.

On the other hand, linear NP (*N*8) and exponent NP (*N*12) have produced the highest effects on the ranking of alternatives (Fig. 2). Thereby, they are not recommended, when used with the TOPSIS method.

Besides, other NPs (Jüttler-Körth's NP (*N*2); max-min NP (*N*5); NP of Peldschus et al. (*N*6); sum NP (*N*7); non-monotonic NP (*N*9); logarithmic NP (*N*10); and enhanced accuracy NP (*N*11)) have obtained different estimates of E_s, E_r, and C_v and, hence, have produced different effects on the alternatives' ranking outcomes (Fig. 2).

3.4. Discussion of the results. The results found in this study, which deals with the suitability test of vector NP in TOPSIS method, are discussed below:

The findings in [6], according to which an alternative comes first for one normalization procedure and last for another, are verified in this current study. In fact, wheel loader a_3 has come first for ten NPs and last for linear NP (Table 10).

The suitability studies, available in the literature, have generally compared two to ten NPs in TOPSIS method (Table 1).

In the present study, we have increased the number up to twelve NPs (Table 3); and this, in order to verify if the usage of vector NP is truly justified for the TOPSIS method. Or is there another NP that outperforms it in terms of suitability?

The comparison results of the considered wheel loaders' rankings, when analyzing twelve NPs, indeed indicate that the vector NP is the best choice for the TOPSIS method (Table 11). Accordingly, its usage is justified for the TOPSIS method.

From the present study, max-version-2 NP (*N*4) and max-version-1 NP (*N*3) are revealed to be the second and third suitable choices for the TOPSIS method. Moreover, let's note that certain studies, performed in [16–18, 22], have indicated that max NP can be considered as a possible alternative to the vector NP, when used with the TOPSIS method.

Based on the comparison results (Table 11), it is possible to observe that the linear and exponent NPs have produced the highest effects, compared to other analyzed NPs. Thereby, linear and exponent NPs are not recommended, when used with the TOPSIS method.

The result found in the current study, indicating the appropriateness of vector NP for the TOPSIS method (Table 11), is quite in line with the results drawn by other researchers [13–16, 18, 19, 23, 24].

The resolution of the numerical example (Table 4) indicates that the wheel loader a_3 is the best alternative (Table 10). Accordingly, the Manager of the Construction Enterprise, as a decision-maker, can select it for ensuring the loading works of swarmed rocky materials.

The results of comparison of the twelve NPs in TOPSIS method, found in this suitability study, show that each analyzed NP produces an effect on the considered alternatives' rankings. Also, it is possible to observe that the effect generated by each NP is different compared to the effects generated by the other eleven NPs (Table 11). This thus means that each NP, existing in the literature, has its strengths and weaknesses and, hence, has its limits of applicability.

To avoid any misunderstandings about the application of a NP in a given MCDM method, it is necessary of carrying out suitability studies, which lead to the most judicious choice of a NP.

In the present suitability study, there is no limit of applicability of the obtained results. Because the statistical analysis, based on the standard error estimate; relative error estimate; and variation coefficient estimate, has clearly shown that the vector NP is considered to be the best choice for the TOPSIS method.

Indeed, the vector NP-TOPSIS, as an MCDM method, can practically be applied by decision-makers and researchers to solve multi-criteria decision problems, which criteria optimization objectives are to be maximized (benefit criteria) and minimized (cost criteria).

The current research should further be developed to choose the most appropriate NP for the TOPSIS method, when solving specific MCDM problems where all the considered criteria are independent from the optimization objectives, and do not constitute a blend of benefit and cost criteria.

The NPs available in the literature, which can be used to normalize the initial criteria values with the same optimization objective, are Z-score NP; Wu's reference-based NP; Aytekin's reference-based NP; NP equalizing the average to 1; NP equalizing the standard deviation to 1; comprehensive NP; range NP between –1 and +1; range NP between 0 and +1; and decimal NP.

The further comparison of these above-cited NPs in TOPSIS will enable to choose the most suitable NP for TOPSIS when solving specific MCDM problems where all the considered criteria have the same optimization objective.

4. Conclusions

When transforming the variety of measurement units of the criteria into a comparable scale, several MCDM methods (like WSM, WPM, MAUT, MAVT, SMART, AHP, TOPSIS, COPRAS, MOORA, ARAS, WASPAS, ROV, etc.) often apply NPs by default. Indeed, usage of NPs by default can negatively affect the considered alternatives ranking results and, hence, the decision to be taken.

On the other hand, there is not a universal agreement that permits of defining which NP is the most suitable for a given MCDM method [5]. Faced with this dilemma, suitability studies of NPs then become indispensable.

As it is well-known, TOPSIS method which is one of the widespread MCDM methods [43, 59] applies several NPs [36]. Thus, in [28] have initially proposed vector-TOPSIS, subsequently followed researches has proposed or applied other NPs for the TOPSIS method. By way of example, it is possible to quote the cases below:

– Linear NP-TOPSIS [57];
– Sum NP-TOPSIS [60];
– Max-min NP-TOPSIS for a small number of alternatives [17];
– Max NP-TOPSIS for a larger number of alternatives [17];
– Sum NP-TOPSIS [20];
– Combination of sum and vector NPs-TOPSIS [25].

To test the suitability of vector NP (initially proposed in [28], but by default) for the TOPSIS method, we have thus realized this study. This last consists to compare vector NP with eleven different NPs each to other in TOPSIS method via a numerical example, which deals with the wheel loader selection.

The results of comparison, resulting from the statistical analysis, clearly indicate that vector NP is the best choice for the TOPSIS method, followed by the max-version-2 NP (Table 11). Also, results of comparison indicate that the linear NP (*N*8) and exponent NP (*N*12) produce the highest effects on the rank order of alternatives (Table 11). Thereby, linear and exponent NPs are not recommended, when used with the TOPSIS method.

Before ending this conclusion, it is useful of noting that the present study is the first in the current literature to simultaneously compare the appropriateness of twelve different NPs in TOPSIS method.

As a recommendation: The decision-makers and practitioners can apply the comparison approach proposed in this study to choose the most suitable normalization procedure for the MCDM method that wish to utilize for solving the decisional problems in the presence of multiple criteria.

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Conflict of interest

The authors declare that they have no conflict of interest concerning this research, whether financial, personal, authorship or otherwise, that could affect the study and its results presented in this paper.

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Data availability

The paper has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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