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THE EFFECT OF NOISE ON RECURRENT DIAGRAMS OF ENERGY CONSUMPTION OF A METALLURGICAL ENTERPRISE

The most common problem faced by modern metallurgical enterprises is the improvement of their energy efficiency, which is based on the management of energy-saving projects. The paper deals with the analysis of the impact of external noise on recurrent diagrams based on short-term time series of daily energy consumption of a metallurgical enterprise. The object of this study is short time series of energy consumption of a metallurgical enterprise. The time series of energy consumption of PJSC «Electrometallurgical Plant «Dniprospetsstal» (Ukraine) for 2018–2021 were used as data. The subject of the study is the method of recurrent diagrams of short time series.

In the process of research, methods of short time series analysis based on recurrent analysis were used to study the characteristics of the system state on the example of a metallurgical enterprise. An analysis of the influence of external noise on recurrent diagrams of short-term chaotic time series was carried out using the developed software in the Matlab environment for constructing recurrent diagrams of energy consumption of a metallurgical enterprise.

The following tasks were solved in the work: software was developed for constructing recurrent diagrams in the Matlab package with the possibility of analyzing changes in the magnitude of quantitative indicators of recurrent diagrams under the influence of different levels of noise in time series.

The obtained results are recommended to be used to characterize the state of the system and analyze the influence of external noise. The practical value of the performed work is determined by the proven usefulness of recurrent analysis for estimating electricity consumption and the improvement of modeling of this process, which will allow increasing the accuracy of forecasting future dynamics verified by empirical data.

Keywords: recurrent analysis, network traffic, time series, recurrent diagram, energy consumption, nonlinear dynamics, metallurgy.

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1. Introduction

Energy consumption processes in the metallurgical industry belong to complex dynamic systems characterized by irregular behavior, the manifestations of which are random and deterministic chaotic processes [1]. Observations of such systems and their experimental research are represented by time series – discrete sequences of random variables. The main goal of time series analysis is to obtain information about the characteristics and mechanisms of system functioning. This is the basis for modeling such systems.

A new way of visualizing patterns of repetition in data was proposed, which was given the name recurrent diagram [2]. This made it possible to overcome difficulties in the study of non-stationary and rather short time series and opened the way to the introduction of modern tools for nonlinear data analysis. The main assumption in this theory is that the observed time series is the result of some dynamic process, the interaction of the relevant variables in time. Recurrence is a fundamental property of dynamical systems and can be used to characterize system behavior in phase space. Hidden regularities and structural changes can be graphically identified on recurrent diagrams, thus a topologically equivalent picture of the multidimensional behavior of the system can be reproduced using time series of one observed variable [3].

The problem with recursion charts is that the scope of their application and the conditions under which they give the most effective and interpretable results are not clearly defined. The advantage of this method is that the two-dimensional representation of recurrence can be used to study multidimensional trajectories in phase space and thus visualize the overall dynamic image [3]. Modern studies have shown that the recurrent diagram contains all the necessary information about the dynamics of the system. The work of many scientists has significantly improved the capabilities of this method over the last decade [4–7].

In paper [8], the errors associated with the noise of observations in quantitative repeatability analysis (RQA) were estimated. Based on this assessment, ways to minimize these errors were suggested. The threshold selection criterion, which is necessary for the optimal calculation of the recurrent schedule, is given.

The authors of the paper [9] describe the use of recurrent analysis for predicting nonlinear properties of complex dynamic systems. In [10], the impact of noise on recurrent diagrams and fault diagnosis was investigated by analyzing theoretical signals and vibration signals. Because of the noise, some phase points go beyond the hypersphere, which leads to $R_{i,i}$

the disappearance of the recurrent connection. The paper [11] analyzed the possibility of using recurrent analysis to detect signals and process observations. It is shown that the quality of signal detection using the numerical characteristics of the recurrent diagram deteriorates when noise is present in the observations.

The study [12] analyzed recurrent graphs of time series generated by discrete Gaussian noise processes. The researchers looked at two repeatability quantitative measures related to these lines, respectively: percent determinism and laminarity (IAM).

The method of quantitative assessment of the phase trajectory of the investigated process using recurrent diagrams and its quantitative analysis is proposed in [13], where the results of the application of this method to a model system and a real system are given. In previous studies, the authors conducted a recurrent analysis of the energy consumption of a metallurgical enterprise, but the effect of noise on recurrent diagrams was not determined.

Therefore, *the aim of research* is to analyze the influence of external noise on recurrent diagrams of short-term chaotic time series, which is performed using the developed software in the Matlab environment for constructing recurrent diagrams of energy consumption of a metallurgical enterprise.

2. Materials and Methods

Recurrence diagrams (RP) provide a qualitative display of the dynamics of this system. To obtain quantitative information, quantitative analysis of indicators (RQA) [14] is traditionally used, which is based on the distribution of diagonal, horizontal and vertical lines that are within the boundaries of a recurrent diagram.

Recurrent diagrams are presented as a graphical approach to the analysis of chaotic dynamics [3]. A recurrence diagram is a binary matrix that visualizes pairs (i, j) of states of a dynamical system at times i and j that are closer than a fixed threshold distance ε in phase space. Various patterns observed in the diagrams were associated with certain dynamic properties. For example, diagonal lines (Fig. 1) are associated with the predictability of dynamics.

Black dots are repeat dots, that is, places (i, j), where $|x_i-x_j| \le \varepsilon$. The green ellipse encircles a diagonal line of length 4, and the red ellipse encircles a vertical line of length 3.



Fig. 1. The structure of the 10×10 recurrent diagram

RPs were introduced to visualize the behavior of trajectories in phase space and are built on the basis of the matrix:

$$R_{i,j} = \Theta\left(\varepsilon_i - \left\|\overline{x_i} - \overline{x_j}\right\|\right), \ i, j = 1, \dots, N,$$
(1)

where x_i means the point in phase space at which the system is at time *i* and ε are a predefined threshold; $\Theta(x)$ is the Heaviside function.

The matrix consists only of values 1 and 0. The graphic representation is a grid of $N \times N$ points, which are coded as black for 1 and white for 0. A black point in RP means that the system returns to the ε -surrounding of the corresponding point in the phase space.

One of the main problems with a reliably constructed recurrent graph and, accordingly, its quantification of the constant dynamic component is noise, which is usually found in time series of observations. Accordingly, recurrent charts created by white noise contain few diagonals, while charts of deterministic or highly autocorrelated stochastic signals show more frequent and longer diagonals. This specified noise is known to cause deviations of the diagonal lines, despite the known deterministic features, and, accordingly, can lead to false conclusions. Although the lines that are interrupted can be further connected by increasing the repetition threshold, this approach leads to the appearance of thick lines on the graph. However, thick lines also affect the performance of recurrent charts, artificially increasing the number of diagonals and the length of vertical lines (for example, determinism (DET) and laminarity (LAM) indicators become higher).

The ratio of the number of recurrent points that are components of diagonal lines (of minimum length l_{min}) to the total number of recurrent points is called the measure of determinism, or the predictability of the system:

$$DET = \frac{\sum_{l=l_{\min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}.$$
 (2)

Processes with stochastic or chaotic behavior generate very short diagonals or none at all, while deterministic processes generate long diagonals and a small number of single, isolated recurrence points. That is, as the stochastic influence decreases, the value of the measure of determinism will increase. The length of the diagonal line reflects the period during which different segments of the trajectory pass sufficiently close to each other at different times.

Using the logistic mapping (3), the influence of noise on periodic diagrams of time series was investigated in [15]. This example is often used to illustrate how simple nonlinear equations can lead to complex and chaotic results:

$$x_{t+1} = rx_t (1 - x_t), (3)$$

where the value x_t varies from 0 to 1 and reflects the ratio of the population value at time t to the maximum possible, and x_0 denotes the initial population; r is a parameter that characterizes the rate of population growth and takes values from 0 to 4. At other values of x_t and r, the logistic equation gives negative values of the population size, which describes an unrealistic situation. With different values of the parameter r in the equation, qualitatively different types of behavior of the variable are obtained. That is, a change in the model parameter r in the range of 3.1-3.9 leads to a change in the nature of the dynamics of the time series x_t , namely to a transition from a stationary series to regular fluctuations and then to chaotic fluctuations. In order to obtain a noisy time series, random values obtained from arbitrary random number generators are added to those obtained by the formula:

$$x_{t+1} \to x_{t+1} + aR_{rand},\tag{4}$$

where R_{rand} – random number between 0 and 1; α – coefficient corresponding to the noise level in percent.

An arbitrarily chosen recurrent point does not contain useful information about the states at time i and j, only the entire set of recurrent points makes it possible to restore the properties of the system. In the case of determining circles in the form of a circle of fixed radius, the recurrence diagrams showed symmetrical structures relative to the main diagonal, since if x_i is close to x_i , then the opposite is also true: x_i is close to x_i . However, the picture will be complicated if the condition that the radii of the *i*-th and *j*-th circles are equal is not imposed. It is explained in the following way. In the case of defining an environment in the form of a circle with such a radius that it covers a strictly fixed number of states x_i , ε_i is chosen for each x_i (i=1,...,N) one separately $R_{i,i} \neq R_{i,i}$ since the environment defined for x_i does not necessarily coincide with the environment for x_i . This leads to asymmetry in the recurrence diagrams. Using a criterion such as ε_i , one can easily determine the recurrence density by choosing $\varepsilon = 15$. The locally determined radius ε_i is introduced so that its corresponding neighborhood covers 15 % of all phase space vectors. This approach is called a fixed number of nearest points, that is, it is due to a fixed number of phase space vectors that fall into the selected environment.

3. Results and Discussion

In Fig. 2, 3, recurrent diagrams are constructed according to the structure of the diagram shown in Fig. 1, based on data on the hourly energy consumption of a metallurgical enterprise.

In Fig. 2 recurrence diagrams were generated using a random number generator for normal processes (Fig. 2, *a*), respectively, and for a chaotic «randomly selected process» at r=3.6 (Fig. 2, *b*). The length of the time series is 50. All charts have a minimum value of $\varepsilon=0.05$.

There are no specific criteria for selecting parameters when creating recurrence plots, but it is important to use the same or similar parameters each time when analyzing and comparing time series properties. In this study, a time series with a length of 200 observations and recurrence plots with a value of ε =0.05 were used. The parameter *r* is 3.25 and 3.55 (regular, normal, chaotic) or 3.6 (chaos, dynamic). Fig. 3 shows diagrams for comparing the effect of noise level on the recurrent properties of time series, recurrent diagrams for noisy regular time series, as well as for noisy chaotic time series. The noise level here is 10 % (Fig. 3, a-c) and 15 % (Fig. 3, d-e). It can be seen that an increase in the noise level can lead to convergence of the recurrent properties of regular and chaotic processes, which is expressed in the similarity of the distribution of points on the corresponding recurrent diagrams.

When choosing the values of the threshold value ε , the smallest of the possible values was chosen. In the presence of a noise component, the threshold value is slightly increased. In the case when $\varepsilon_i \neq \varepsilon$, \forall_i , the threshold value is chosen, no more than 10 % of the value of the maximum diameter of the phase space $-\sigma$, if $\varepsilon_i \neq \varepsilon$, then the threshold value is calculated with respect to the recurrence density.

In the diagrams, white bars indicate that some states are exceptional or far from normal, transitions may have occurred. Since characteristic pronounced white zones appeared in the structure of the diagram, this reflects sharp changes in the dynamics of the process, non-stationarity. Let's change the value of ε to ε =30, ε =40 and ε =50 (Fig. 3, g-h) (limit distance or distance from the center of the selected circle to the border). The following pattern can be observed: the larger the value of ε , the more chaotic points appear around the diagonal line. This indicates strong fluctuations in the process. To reduce noise, it is more appropriate to choose the smallest possible value of ε , but this is not always possible for certain time series. As a result, recurrent charts did not undergo drastic changes.

Calculations were performed using the Matlab software package. For the study, energy consumption data for the winter and summer periods of 2018–2021 were taken [16]. The calculated quantitative indicators are presented in the form of Table 1, 2.

The existence of a line parallel to the main diagonal is an essential feature characterizing deterministic processes. The greater the number of points on the diagonal lines, the greater will be the deterministic component of the series. Quantitative measures *DET*, *DIV* and *ENTR* are based on this fact. The average length of the diagonal lines *L* is also an important indicator, because it takes into account the length of the different lines – as can be seen from the calculated *RQA* indicators, when taking into account the effect of noise, the value of *L* decreases and the process becomes more chaotic. The *DET* indicator counts all points on parallel lines, regardless of their length.



Fig. 2. Image of recurrent diagrams: a - for ordinary processes; b - for a chaotic «randomly selected process» at <math>r = 3.6

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Fig. 3. Recurrence diagrams for noisy time series at r=3.25 (a, d, g) and at r=3.55 (b, e, h), as well as for a noisy chaotic series at r=3.6 (c, f, i): a-c - the noise level is 10 %; d-f - the noise level is 15 %; $g - \varepsilon = 30$; $h - \varepsilon = 40$; $i - \varepsilon = 50$

Thus, for purely random processes, the value of *DET* is very small (close to zero), while for processes with some deterministic component, values of this measure that are significantly greater than zero (close to unity) will correspond. A decrease in the *DET* index is associated with an increase in the noise value.

The average value of the entropy measure also turned out to be high, which indicates the complexity of the obtained recurrent diagrams relative to the diagonal lines. Therefore, the investigated series contain some deterministic component, and when the influence of noise is taken into account, this value drops sharply.

The *RR* recurrence rate in studies without noise is close to 0.62. That is, the probability of repeating a certain state of the electricity consumption process during the day is 62 %. Under the influence of noise, this indicator decreases.

The average value of the *LAM* fading measure also decreases under the influence of noise. This indicator charac-

terizes the presence of states of freezing of the system, that is, when the movement of the system along the phase trajectory stops or moves very slowly.

The average length of the vertical lines, representing the time during which the system remains in a certain state, corresponds to 3.5 and 2, respectively, without noise and under the influence of noise.

Since the dynamics of the calculated indicators of complexity are indicators of changes in the state of the system, the obtained results can also be used to evaluate the dynamics of quantitative indicators of the system over time.

Limitations for the use of the research results in practice are associated with the lack of data on the utilization of the production facilities of the metallurgical enterprise, the lack of research on the dependence of recurring energy consumption diagrams on the utilization, which can significantly affect the results obtained.

Table 1

ROA indicators in the study of the effect of noise level on recurrent diagrams for the winter period

Noise level	RR	DET	L	DIV	ENTR	RATIO	LAM	TT				
Winter 2018												
0	0.4253	0.8756	3.9545	0.1111	10.9872	3.0402	0.8667	4.4318				
3.25	0.4253	0.8578	3.8636	0.1111	10.9872	3.1118	0.8756	4.1915				
3.55	0.4178	0.8643	3.8182	0.1429	10.9872	3.1488	0.8597	4.2222				
3.6	0.4216	0.8475	3.7727	0.1429	10.9872	3.1867	0.8700	4.0417				
Winter 2019												
0	0.2401	0.8425	2.8000	0.1667	7.4823	6.2976	0.6614	3.5000				
3.25	0.2439	0.8450	2.8667	0.1667	7.4823	6.1512	0.6667	3.7391				
3.55	0.2325	0.8374	2.8571	0.1667	6.9812	6.6125	0.6667	3.5652				
3.6	0.2363	0.8400	2.7333	0.1667	7.4823	6.4512	0.6720	3.5000				
Winter 2020												
0	0.3497	0.7622	2.6818	0.2000	10.9872	4.4831	0.7514	2.9574				
3.25	0.3459	0.7377	2.6667	0.2000	10.4867	4.7232	0.7377	3				
3.55	0.3497	0.7622	2.6818	0.2000	10.9872	4.4831	0.7297	2.9348				
3.6	0.3497	0.7622	2.6818	0.2000	10.9872	4.4831	0.7405	2.8542				
Winter 2021												
0	0.8715	0.9870	6.5455	0.0625	16.4909	1.2245	0.9718	7.4667				
3.25	0.8677	0.9869	6.5152	0.0625	16.4909	1.2302	0.9673	7.1613				
3.55	0.8677	0.9869	6.5152	0.0625	16.4909	1.2302	0.9695	7.2951				
3.6	0.8677	0.9869	6.5152	0.0625	16.4909	1.2302	0.9695	7.2951				

Table 2

RQA indicators in the study of the effect of the noise level on recurrent diagrams for the summer period

Noise level	RA	DET	L	DIV	ENTR	RATIO	LAM	TT				
Summer 2018												
0	0.4253	0.8400	2.9643	0.1429	13.9896	3.1867	0.7556	3.4694				
3.25	0.4291	0.8414	3.0000	0.1250	13.9896	3.1488	0.7621	3.4600				
3.55	0.4216	0.8296	3.0000	0.1429	13.4892	3.2654	0.7489	3.4792				
3.6	0.4216	0.8296	2.8929	0.1429	13.9896	3.2654	0.7534	3.4286				
Summer 2019												
0	0.4707	0.8313	3.1724	0.1667	14.4899	2.8750	0.8233	3.4167				
3.25	0.4707	0.8313	3.1724	0.1667	14.4899	2.8750	0.8233	3.4167				
3.55	0.4707	0.8313	3.1724	0.1667	14.4899	2.8750	0.8233	3.4167				
3.6	0.4745	0.8406	3.1333	0.1667	14.9902	2.8138	0.8207	3.4333				
Summer 2020												
0	0.6030	0.9436	3.9714	0.0588	17.4914	1.9029	0.9498	5.5091				
3.25	0.5992	0.9432	3.8333	0.0714	17.9916	1.9167	0.9495	5.3750				
3.55	0.6068	0.9439	4.0000	0.0588	17.4914	1.8893	0.9564	5.4821				
3.6	0.6068	0.9441	4.2123	0.0588	17.4910	1.7893	0.5364	5.5843				
Summer 2021												
0	0.4972	0.8403	3.3000	0.1250	14.9902	2.6717	0.7605	2.8571				
3.25	0.4972	0.8403	3.3000	0.1250	14.9902	2.6717	0.7605	2.8571				
3.55	0.4934	0.8391	3.2667	0.1250	14.9902	2.6990	0.7548	2.8551				
3.6	0.4896	0.8301	3.3103	0.1250	14.4899	2.7552	0.7490	2.8529				

The conditions of the war imposed restrictions on the use of more recent data on the functioning of the metallurgical enterprise for conducting research. Therefore, one of the promising directions of further research is the approbation of the presented method of recurrent analysis on the data of war and post-war states, establishing the dependence of recurrent diagrams of energy consumption on workload, determining new types of noise that affect recurrent diagrams and, as a result, the accuracy of forecasting.

4. Conclusions

In this work, the analysis of recurrent diagrams is carried out, which is related to the noise that occurs during time series observations in dynamic systems and reduces the accuracy of diagram construction. The influence of noise on recurrent diagrams representing the dynamics of energy consumption of a metallurgical enterprise in the summer and winter periods was studied.

During the research, it was established that the recurrent diagram undergoes changes and deviations of diagonal lines, which are caused by external noises. In other words, when the level of external noise is high, the recurrent diagrams become qualitatively similar to each other.

The utility of using the recurrent analysis method for forecasting electricity consumption and improving modeling capabilities has been demonstrated, which will lead to improved forecasts of future dynamics with verified empirical data.

The use of the recurrent analysis methodology is recommended to be applied to the time series of electricity consumption data in the future, which will potentially increase the energy efficiency of the metallurgical sector.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship, or any other, that could affect the study and its results presented in this article.

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The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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