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Valeriia Havrylko

DEVELOPMENT OF A MATHEMATICAL MODEL OF ACOUSTIC PROCESSES IN THE OPERA STUDIO OF THE KYIV CONSERVATORY

The object of this study is the acoustic characteristics of a concert hall, with a particular focus on the reverberation time, which significantly affects both the perception of sound by listeners and the performance of musicians.

The study emphasises the importance of mathematical modelling of acoustic processes in concert halls, especially in optimising reverberation time. In the context of modern materials and advanced acoustic design technologies, precise calculations and analyses are required to evaluate the impact of various elements on a room's overall acoustics. Poor design or material selection can result in listener discomfort and reduced sound quality, highlighting the critical role of scientific methods in analysing and modelling acoustic processes under specific conditions.

The research utilised mathematical models developed in MATLAB® software to calculate reverberation time based on different materials and their surface areas.

The key findings demonstrate that mathematical models can accurately simulate acoustic processes in a room, enabling predictions of acoustic characteristics based on defined parameters. The correlation between theoretical calculations and experimental data confirms that mathematical modelling is an effective tool for improving the acoustic quality of concert halls, considering the use of different materials.

The practical significance of the results lies in the ability to implement recommendations for optimising acoustic conditions in concert halls. The identified parameters can guide the design of spaces for musical events and enhance methods of acoustic design. Applying these findings in practice can improve sound quality, thereby increasing listener comfort and enhancing the overall perception of music.

Keywords: *mathematical modelling, MATLAB®, acoustic design, acoustic concert hall characteristics, reverberation time, sound absorption.*

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1. Introduction

The acoustic characteristics of rooms are critical to ensuring quality sound in various environments, such as concert halls, theaters and classrooms. The standard reverberation time (T_{60}) is the time it takes for the sound pressure level in a room to decrease by 60 dB after the sound source stops emitting sound. According to research [1], optimal values of the reverberation time can vary depending on the type of room and its purpose.

Numerous works, such as [2–4], consider the basics of acoustic modeling and its importance for the design of sound systems.

In [2], it is emphasized that taking into account the geometry of the room and the materials from which it is built are key to the accuracy of reverberation time calculations.

The value of mathematical modeling for optimizing the acoustic characteristics of concert halls is emphasized in [3, 4], which indicate that the accuracy of the calculations has a critical impact on the quality of sound perceived by listeners. However, according to [5], there is an insufficient number of studies that would compare the results of computer modeling with experimental data, which creates a significant gap in the knowledge of this field.

Thus, there is a need for new studies that would not only improve mathematical models, but also test them in practice in real rooms. This justifies *the aim of this research* is the creation of a mathematical model of the concert hall of the Opera Studio of the Kyiv Conservatory (Ukraine) to calculate the reverberation time and average sound absorption coefficient. The scientific part of the aim is to identify the patterns of the influence of materials on the acoustic characteristics of the room. The practical part of the aim includes the creation of recommendations for improving acoustic conditions in the concert hall, which can contribute to improving the sound quality and comfort for listeners.

2. Materials and Methods

The object of research is the acoustic characteristics of the concert hall of the Opera Studio of the Kyiv Conservatory (Fig. 1).

Fig. 1. General view of the Opera Studio hall [6]

In general, the hall has a complex curvilinear shape, consisting of a rectangular section located near the stage and a part that has a shape limited by a cylindrical one. Benoît boxes are located on the side walls, and an amphitheater is located at the back of the hall.

The study was conducted using two main approaches: experimental measurement and mathematical modeling.

Experimental measurements were carried out in real time in accordance with the requirements set out in the recommendations [7, 8] and in the standard [9]. The sound source was the pulsed broadband sound of a starting pistol shot. The reverberation time measurement points were selected taking into account the symmetry of the hall.

The conditions for conducting experimental measurements were as follows:

– the hall was empty (without spectators);
– air temperature ~ +20 °C;
– the orchestra pit and stage were open.

For measurements, sets of precision equipment from the companies "RFT" (Germany) [10], "Behringer" [11], etc. were used. Mathematical modeling was performed using MATLAB® software (Fig. 2).

```
%% given values
V_pr = 4533; % volume of the concert hall
S_pr = 3338; % area of the reflecting surfaces of the concert hall
f = [125 250 500 1000 2000 4000]; % standardized octave frequency band
S = [90 600 1415 15 125 234 250 80 22 659]; % area of absorbing surfaces in the room

%% calculating the average sound absorption coefficient of a room
alpha_125 = [0.15 0.27 0.01 0.03 0.05 0.2 0.76 0.17 0.04 0.02]; % k Sound absorption
coefficients of absorbers in the room (frequency 125 Hz)
alpha_250 = [0.11 0.31 0.01 0.02 0.58 0.3 0.47 0.53 0.03 0.02];
alpha_500 = [0.1 0.31 0.02 0.05 0.52 0.3 0.29 0.92 0.02 0.03];
alpha_1000 = [0.07 0.31 0.02 0.04 0.53 0.3 0.14 0.82 0.01 0.04];
alpha_2000 = [0.06 0.33 0.02 0.04 0.62 0.3 0.02 0.52 0.07 0.04];
alpha_4000 = [0.07 0.4 0.02 0.04 0.11 0.3 0 0.4 0.04 0.05];
A_125 = sum (S .* alpha_125); % total absorption fund of the room (frequency 125 Hz)
A_250 = sum (S .* alpha_250);
A_500 = sum (S .* alpha_500);
A_1000 = sum (S .* alpha_1000);
A_2000 = sum (S .* alpha_2000);
A_4000 = sum (S .* alpha_4000);
alpha_ser = [A_125/S_pr A_250/S_pr A_500/S_pr A_1000/S_pr A_2000/S_pr A_4000/S_pr];
% dependence of the average sound absorption coefficient on frequency

%% calculating room reverberation time
mu = [0 0 0 0 0.002 0.006]; % sound attenuation coefficient in air
T_rozr_125 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_125 / S_pr) + 4 * mu(1) * V_pr); % room
reverberation time (frequency 125 Hz)
T_rozr_250 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_250 / S_pr) + 4 * mu(2) * V_pr);
T_rozr_500 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_500 / S_pr) + 4 * mu(3) * V_pr);
T_rozr_1000 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_1000 / S_pr) + 4 * mu(4) * V_pr);
T_rozr_2000 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_2000 / S_pr) + 4 * mu(5) * V_pr);
T_rozr_4000 = 0.164 * V_pr / ((-1) * S_pr * log (1 - A_4000 / S_pr) + 4 * mu(6) * V_pr);

%% graphing
T_rozr = [T_rozr_125 T_rozr_250 T_rozr_500 T_rozr_1000 T_rozr_2000 T_rozr_4000];
T_opt = [1.56 1.43 1.3 1.3 1.3 1.43];
T_min = 0.9 * T_opt;
T_max = 1.1 * T_opt;
figure (1)
f_cat = categorical(f);
plot(f_cat, T_rozr,'b', f_cat, T_opt,'g', f_cat, T_min,'r--', f_cat, T_max,'r--')
ylim([1 2]);
xlabel('f, Hz')
ylabel('Reverberation time, s')
legend('T_р_о_з_р','T_о_п_т','T_м_і_н','T_м_а_к_с')
grid on
hold on
figure (2)
plot(f_cat, alpha_ser,'b')
ylim([0.1 0.2])
xlabel('f, Hz')
ylabel('Average absorption coefficient')
grid on
hold on
```

Fig. 2. Calculations for the mathematical model of the concert hall

Based on comparisons with experimental data, recommendations were made to improve the auditory picture in the room.

3. Results and Discussion

To calculate the optimal reverberation time for the opera studio room (Fig. 3, 4), the approximate formula is used:

$$T_{opt} = 0.4 \cdot \lg V - 0.15, \tag{1}$$

where V – the room volume. Then

$$T_{opt} = 0.4 \cdot \lg(4533) - 0.15 \approx 1.3 \text{ s.}$$

For this type of room, the purpose of which is to perform both opera singing and chamber music, it is necessary to provide sufficiently different reverberation times at different frequencies. For frequencies of 500, 1000 and 2000 Hz, the calculated number remains, at a frequency of 125 Hz an increase of 20 % is performed, and at 4000 Hz – by 10 %. The dependence is presented in the form of Table 1.

The average measured values of reverberation time in the hall's parterre are presented in Table 2.

From the graph in Fig. 5 it is clear that the values of T_{exp} at all frequencies do not fall within the optimal interval $[T_{\min}, T_{\max}]$.

The main way to shift the values of the reverberation time to the required interval is to change the total absorption fund of the room. Table 3 contains information on the sound absorption coefficients in materials with their indicated area and the calculated value of the total absorption fund. These values are included in the mathematical model of the Opera Studio concert hall (Fig. 2), which takes into account the geometric dimensions of the room, the calculation of the total absorption fund, reverberation time and the average sound absorption coefficient. The results of mathematical modeling are presented in Table 4 and Fig. 6, respectively.

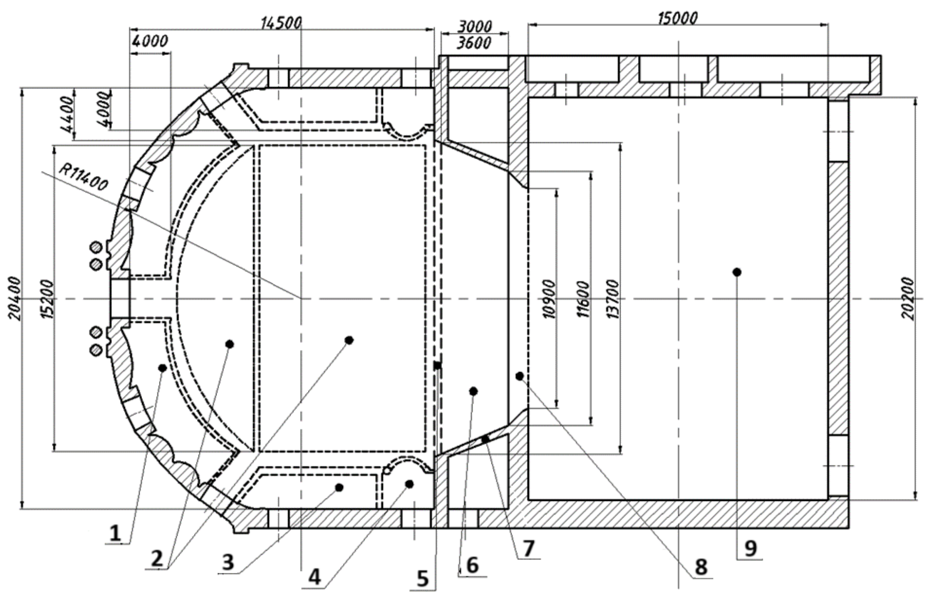


Fig. 3. Horizontal section of the first floor of the room:
1 – amphitheater; 2 – parterre; 3 – benoir; 4 – balcony; 5 – partition; 6 – orchestra pit; 7 – decorative grille; 8 – proscenium; 9 – stage box

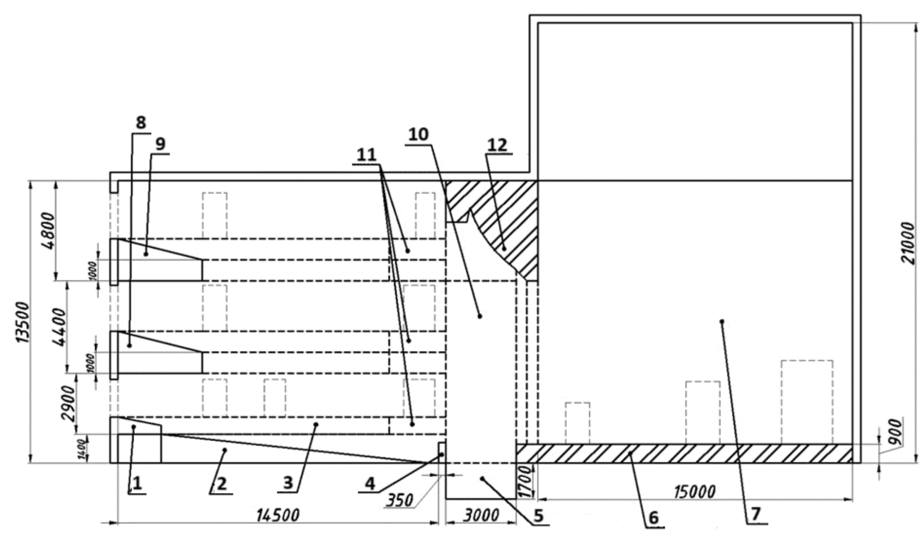


Fig. 4. Vertical section of the room:
1 – amphitheater; 2 – parterre; 3 – benoir; 4 – partition; 5 – orchestra pit; 6 – stage rise; 7 – stage box; 8 – 2nd tier belvedere; 9 – 3rd tier belvedere; 10 – decorative grille; 11 – balcony; 12 – ramp

As can be seen from Fig. 6, the values of T_{mat} at all frequencies are within the optimal range [$T_{\min}$, $T_{\max}$].

Table 1

Optimal reverberation time

f, Hz	125	250	500	1000	2000	4000
T_{opt}, s	1.56	1.43	1.3	1.3	1.3	1.43
$T_{min}=0.9\cdot T_{opt}$, s	1.4	1.29	1.17	1.17	1.17	1.29
$T_{max}=1.1\cdot T_{opt}$, s	1.72	1.6	1.43	1.43	1.43	1.6

Table 2

Average values of reverberation time in the parterre

f, Hz	125	250	500	1000	2000	4000
T_{exp}, s	1.35	1.22	1.17	1.14	1.12	0.93

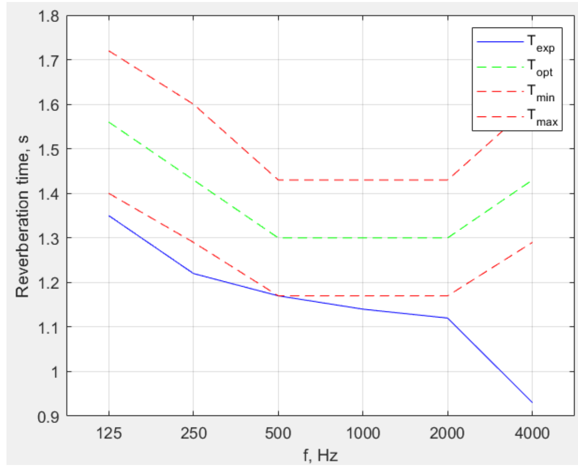


Fig. 5. Reverberation time obtained experimentally

Table 3

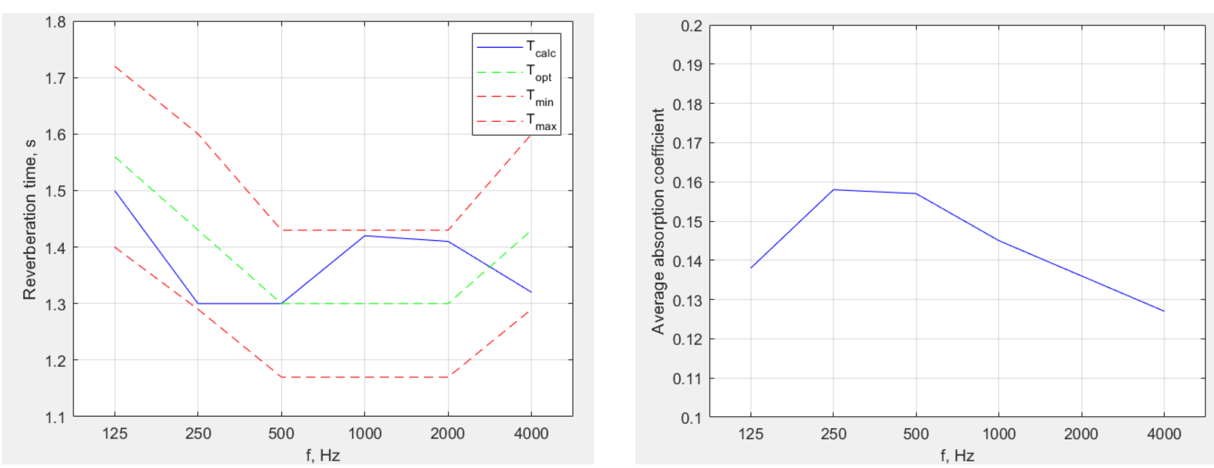
Total absorption fund of the room

Surface – absorber material	Area (m^2) or number of people	Main absorption fund											
		Frequency, Hz											
		125		250		500		1000		2000		4000	
		a	*A*	*a*	*A*	*a*	*A*	*a*	*A*	*a*	*A*	*a*	*A*
Floor on wooden beams	90	0.15	8.67	0.11	6.36	0.1	5.78	0.07	4.05	0.06	3.47	0.07	4.05
ADC plaster	600	0.27	162	0.31	186	0.31	186	0.31	186	0.33	198	0.40	240
Lacquered doors	15	0.03	0.45	0.02	0.3	0.05	0.75	0.04	0.6	0.04	0.6	0.04	0.6
Wall plastered and painted with oil paint	1415	0.01	14.15	0.01	14.15	0.02	28.31	0.02	28.31	0.02	28.31	0.02	28.31
Acoustic PA/D (panels/decors)	125	0.05	6.3	0.58	72.5	0.52	65	0.53	66.3	0.62	77.5	0.11	13.8
Stage armhole	234	0.20	46.76	0.30	70.14	0.3	70.14	0.30	70.14	0.3	70.14	0.30	70.14
Non-perforated plates 2 mm thick, gap 100 mm	250	0.76	19	0.47	117.5	0.29	72.5	0.14	35	0.02	5	0	0
Slotted plates, gap 100 mm	80	0.17	13.6	0.53	42.4	0.92	73.6	0.82	65.6	0.52	41.6	0.4	32
Mirror glass	22	0.04	0.8	0.03	0.6	0.02	0.4	0.01	0.3	0.07	1.5	0.04	0.9
Hard chair with plywood back and seat	659	0.02	13.2	0.02	13.2	0.03	19.8	0.04	26.4	0.04	26.4	0.05	33

Table 4

Calculation results

f, Hz	125	250	500	1000	2000	4000
T_{mat}, s	1.5	1.3	1.3	1.42	1.41	1.32
T_{min}, s	1.4	1.29	1.17	1.17	1.17	1.29
T_{max}, s	1.72	1.6	1.43	1.43	1.43	1.6
α_{av}	0.138	0.158	0.157	0.145	0.136	0.127
A_{tot}, Bes	460.7	526.7	525.5	484.9	454.5	425



a *b*

Fig. 6. Acoustic characteristics of the Opera Studio concert hall: *a* – reverberation time; *b* – average absorption coefficient

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The difference in values between Table 2 and Table 4 and Fig. 5 and Fig. 6 is explained by the following steps taken:

– reducing the number of seats in the stalls by two rows (25 listeners in each row) and replacing them with "hard chairs with plywood backs and seats";
– removing curtains from the entrance doors;
– reducing the amount of plaster on the side walls near the stage;
– removing carpets;
– adding non-perforated plates with a total area of 250 m^2, slotted plates with a total area of 80 m^2 and acoustic PA/D with a total area of 125 m^2.

The obtained results of the study have direct *practical significance* for improving the acoustic conditions in the Opera Studio of the Kyiv Conservatory. Calculations of the sound absorption fund and reverberation time allow to improve the sound quality in this room, which will facilitate the perception of musical and vocal performances. The use of these results helps to more effectively select and place sound-absorbing materials, which minimizes unwanted acoustic effects.

One of the main limitations of this study is that the calculations were performed for specific conditions of the concert hall, which takes into account only the average sound absorption coefficient of the materials used in this room. The model does not take into account the influence of external factors, such as temperature or humidity, which can affect the acoustic properties of the studio.

Due to the transition to distance learning and limited access to the rooms, most of the work was performed using mathematical modeling in MATLAB®. Despite these difficulties, the study demonstrated that even in difficult conditions, modeling methods can be successfully used to analyze acoustic processes, which makes the results useful both *for further research and for practical use*.

4. Conclusions

Thanks to the comparison of experimental data and mathematical modeling of the concert hall of the Opera Studio of the Kyiv Conservatory, recommendations were made regarding a significant improvement in the auditory picture in the room.

The reverberation time in the room will be 1.32–1.5 s, which is within the optimal indicators for concert halls of this type. The average sound absorption coefficient was determined at the level of 0.136–0.158, which corresponds to the recommended values for ensuring proper acoustics. This result was achieved by applying a number of measures aimed at adjusting the general sound absorption fund in the room.

On the considered example of a concert hall, it can be seen that the use of mathematical modeling helps to take into account more details at the design stage and assess the quality of the acoustic solution.

Conflict of interest

The author declares that she has no conflict of interest regarding this study, including financial, personal, authorship or other nature, which could affect the study and its results presented in this article.

Financing

The study was conducted without financial support.

Data availability

There are no related data in the manuscript.

Use of artificial intelligence

The author confirms that she did not use artificial intelligence technologies in creating the presented work.

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Valeriia Havrylko, *Department of Acoustic and Multimedia Electronic Systems, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine, ORCID: https://orcid.org/0009-0008-7036-6653, e-mail: lera.gavrilko.00@gmail.com*