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OPTIMIZATION OF PUMP JACK ELECTRIC DRIVE OPERATION TAKING INTO ACCOUNT RESERVOIR FLOW RATE

The object of the study are the processes occurring in electric drives of sucker rod pumping units, which are used for mechanical oil production.

When designing oil pumping units, one of the issues is the synthesis of control systems for the electric drives of pump jacks. In the synthesis of the control system, it is important to take into account as many factors as possible that influence the performance of the well and its equipment. When synthesizing the criterion for optimal operation of the system, it is necessary to take into account changes in the technical condition of the oil production facility and the conditions of the reservoir operation. This is implemented using a control system that adjusts the electric drive speed according to the real operating conditions of the well. This approach allows improving the productivity of the oil pumping unit and increasing its economic efficiency.

The main focus is on the synthesis of a control system for the electric drive of a pump jack, which makes it possible to balance the intensity of fluid inflow into the well and its withdrawal. An optimization criterion is proposed, which takes into account different fluid inflow rates, thus adapting the electric drive operation to the specific conditions of each well.

The study used the theory of classical variational calculus. To find the minimum of the functional, and therefore the optimal parameters of the control system, a quadratic form, which is a Lyapunov function, was used.

The study resulted in the formulae for calculating the key operational parameters of the pump jack electric drive. The synthesized optimization criterion links the unit's productivity to parameters such as the number of crankshaft rotations, the rod pump delivery coefficient, and the load torque of the drive motor.

In practice, the proposed optimization criterion for the electric drive operation will enable improving the efficiency of the oil pumping unit through the rational selection of the drive motor's speed and torque. This, in turn, will enhance the well's operating conditions and extend its operational life.

Keywords: *sucker rod pumping unit, optimization criterion, control system, pump jack, electric drive, well, functional.*

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1. Introduction

Technology of oil extraction from a well into which it flows from a productive reservoir requires equilibrium between the intensity of reservoir fluid inflow into the well and the intensity of its withdrawal by the sucker rod pump.

The main elements of the sucker rod pump are a pump, string of pipes and rods, pump jack, electric motor and starting and control equipment (Fig. 1). The plunger of the sucker rod pump is driven by a pump jack with the help of a string of rods, which converts the rotary motion of the electric motor into a reciprocating motion. For driving SRPU pump jacks, three-phase squirrel-cage induction motors, deep-bar or double-cage, which have an increased starting torque, are most commonly used.

The productivity of oil pumping unit is defined by the number of piston strokes per minute and its technical condition, which can worsen with time, and this, respectively, can have an adverse effect on the productivity. The inflow intensity, or the reservoir flow rate, depends on the level of fluid that filled the well and condition of the bottom-hole area of the reservoir. In each specific condition of well operation, the fluid level in it corresponds to the dynamic equilibrium between inflow and withdrawal and is termed the dynamic level. Obviously, the best operation mode of the well is the one when the dynamic level is established at the depth of the plunger pump immersion [1, 2], thereby ensuring improved technical and economic performance indicators of the whole pumping unit.

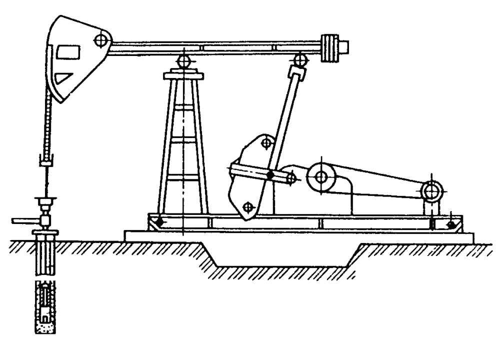

Fig. 1. Sucker rod pumping unit

A high efficiency of periodic operation can be achieved only on condition of creating and using advanced automated control systems

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for electric drives of pump jacks. As empirically confirmed [3–7], to regulate the inflow rate of periodic wells, the frequency-controlled electric drive is proposed to be used. Using the frequency-controlled electric drive of the pump jack implies the change of the fluid withdrawal rate, which is implemented by the automatic control system. In most cases, such systems operate based on a predefined algorithm with the corresponding set-point values that are determined when exploring the well [5, 8, 9]. At the same time, for a separate drive of the pump jack, it is necessary to adapt the control algorithm, since each well has its own characteristics. In connection with the above, the issue of synthesizing an optimality criterion is relevant, based on which the control system of the electric drive of the pump jack will provide operating modes of the sucker rod pumping unit adapted to different values of the fluid inflow into the well.

Thus, *the aim of research* is to synthesize a criterion by which it is possible to optimize the operation of the electric drive of the pump jack.

2. Materials and Methods

Since the rate of fluid withdrawal from a low-flow well is typically larger than the rate of fluid inflow into the well, this decreases the efficiency of the oil production unit. For an equilibrium to be established between the fluid inflow and withdrawal, the pump productivity should be adjusted so that the error E would equal or tend to zero according to formula [10]:

$$E = Q_l - Q_p = Q_l - 1440 n_{cr} S D k_n = 0, \tag{1}$$

where Q_l, Q_p are the theoretical and actual daily flow rates of the well, respectively; n_{cr} is the number of swings of the pump jack (revolutions of the crank); S is the plunger stroke length; D is the area of the plunger; k_n is the rod pump delivery rate, which is equal to the ratio of the actual pump delivery to the theoretical one. In practice, this coefficient falls in the range of 0.65–0.9.

Taking into consideration that the speed of rotation of the crank is defined as:

$$n_{cr} = \frac{30}{\pi}\frac{\omega_m}{k_i},$$

where k_i is the gear ratio from the shaft of the induction motor to the axis of rotation of the crank; ω_m is the speed of the drive motor shaft, equation (1) transforms into:

$$E = Q_l - 1440 S D k_n \frac{30}{\pi}\frac{\omega_m}{k_i} = 0. \tag{2}$$

For equality (2) to be fulfilled, the control system of the electric drive of the pump jack must change the speed of the drive motor in accordance with the change in the current productivity of the unit. In order to assess the quality of such a system, let's use the following criterion:

$$I = \int_0^\infty V \mathrm{d}t = \int_0^\infty \left(E^2 + \gamma \left(\frac{dE}{dt} \right)^2 \right) \mathrm{d}t, \tag{3}$$

where γ is a certain set constant.

If to assume the pump delivery rate to be a constant value, then:

$$\frac{dE}{dt} = -1440 S D k_n \frac{30}{\pi k_i}\frac{d\omega_m}{dt} = -K\frac{d\omega_m}{dt}. \tag{4}$$

It is known [11, 12] that the transfer function of the induction drive motor, when it operates on the linear segment of the mechanical characteristic, with respect to the transfer function of the motor relative to the reference input will have the form:

$$W(p) = \frac{1}{T_e T_m p^2 + T_m p + 1}, \tag{5}$$

where T_m is the mechanical time constant; T_e is the electromagnetic time constant.

Then the differential equation for the speed error will look as follows:

$$T_e T_m p^2 E + T_m p E + E = 0, \tag{6}$$

where $p = d/dt$.

Let's denote $E = x_1$, $pE = x_2$. As a result, equation (6) will transform into:

$$p x_2 = -\frac{x_1 + T_m x_2}{T_e T_m}. \tag{7}$$

In order to minimize the criterion (3), let's use the auxiliary quadratic form:

$$Z = B_1 x_1^2 + 2B_{12} x_1 x_2 + B_2 x_2^2, \tag{8}$$

related with the form V as written:

$$\frac{dZ}{dt} = -V. \tag{9}$$

Therefore, there is a positive definite form V, that is, the function of x_j coordinates that is positive at any conventional true value of x_j. If, at the same time, $dV/dt < 0$, such form is known as Lyapunov function [13]. In its turn, if Z is the quadratic function that is positive at all x_j values, and its derivative is negative, Z is also a Lyapunov function.

The coefficients of the form (8) are unknown. In [14] it is shown that having found the unknown coefficients, let's find the expression for the integral (3). For this, let the expression (9) be written as follows:

$$\frac{\partial Z}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial Z}{\partial x_2}\frac{dx_2}{dt} = -V. \tag{10}$$

Considering (7) and the agreed notations, (10) will take the form:

$$\begin{aligned} &2(B_1 x_1 + B_{12} x_2) x_2 + 2(B_{12} x_1 + B_2 x_2)\left(-\frac{x_1 + T_m x_2}{T_e T_m} \right) = \\ &= 2B_1 x_1 x_2 + B_{12} x_2^2 - 2B_1 \frac{x_1^2}{T_e T_m} - 2B_1 \frac{x_1 x_2}{T_e} - 2B_2 \frac{x_1 x_2}{T_e T_m} - 2B_2 \frac{x_2^2}{T_e} = \\ &= -Q_l^2 + K Q_l x_1 - K^2 x_1^2 - \gamma K^2 x_2^2. \end{aligned} \tag{11}$$

To define the unknown coefficients, let's equate the expressions prefixing x_1^2, $x_1 x_2$, x_2^2 in the left and right parts of equation (11) and solve the resulting system of equations. As a result, let's obtain:

$$\begin{aligned} &B_1 = \frac{2T_m^2 K^2 + T_e T_m K^2 + 2\gamma K^2}{4T_m}; \\ &B_{12} = \frac{T_e T_m K^2}{2}; \\ &B_2 = \frac{T_e^2 T_m K^2}{4} + \frac{T_e \gamma K^2}{2}. \end{aligned} \tag{12}$$

According to [7], the expression for the integral (3) will appear as:

$$I = \int_0^\infty V \mathrm{d}t = B_1 \left(x_1^2 \right)_0 + 2B_{12} \left(x_1 \right)_0 \left(x_2 \right)_0 + B_2 \left(x_2^2 \right)_0, \tag{13}$$

where $(x_1)_0 = Q_l$, $(x_2)_0 = -K\dfrac{M_m - M_l}{J_\Sigma}$ are the starting conditions; M_m is the starting torque of the motor; M_l is the load moment; J_Σ is the total moment of inertia.

Finally, the expression (13) for integral evaluation will assume the form:

$$I = \int_0^\infty V\mathrm{d}t = \frac{2T_m^2K^2 + T_eT_mK^2 + 2\gamma K^2}{4T_m}Q_l^2 + \\ + T_eT_mK^2Q_l\left(K\frac{M_l - M_m}{J_\Sigma}\right)_0 + \\ + \left(\frac{T_e^2T_mK^2}{4} + \frac{T_e\gamma K^2}{2}\right)\left(K\frac{M_l - M_m}{J_\Sigma}\right)^2. \tag{14}$$

Therefore, to find the optimal value of the control system parameters that ensure the minimum of the functional (14), it is sufficient to use one of the methods of variational calculus. In particular, such parameters can be the speed and torque of the motor. Since during the swing period of the pump jack, the motor speed and torque are variable, their average values should be taken for the expression (14).

As an example, let's find the expression for the torque of the drive motor which ensures the minimum functional (14). To do this, the expression (14) is differentiated with respect to M_m, the result of which is as follows:

$$f(M_m) = \frac{K^3 \cdot Q_l \cdot T_e \cdot T_m}{J_\Sigma} - \\ - \frac{2K^2\left(\dfrac{T_m \cdot K^2 \cdot T_e^2}{4} + \dfrac{\gamma \cdot K^2 \cdot T_e}{2}\right)(M_l - M_m)}{J_\Sigma^2}. \tag{15}$$

Let's find the extremum of the function $f(M_m)$. To do this, let's equate (15) to zero and obtain an expression for the torque of the drive motor, which provides the set flow rate:

$$M_m = \frac{2\gamma \cdot K \cdot M_l + 2J_\Sigma \cdot Q_l \cdot T_m}{2\gamma \cdot K + K \cdot T_e \cdot T_m} + \frac{K \cdot M_l \cdot T_e \cdot T_m}{2\gamma \cdot K + K \cdot T_e \cdot T_m}. \tag{16}$$

The flow rate of the well is defined as:

$$Q_l = 1440 n_{cr} S D k_n + E. \tag{17}$$

Substituting (17) in (16) will result in:

$$M_m = \frac{2\gamma \cdot K \cdot M_c}{2\gamma \cdot K + K \cdot T_e \cdot T_m} + \\ + \frac{2J_\Sigma \cdot T_m \cdot (1440 n_{cr} \cdot S \cdot D \cdot k_n + E)}{2\gamma \cdot K + K \cdot T_e \cdot T_m} + \frac{K \cdot M_l \cdot T_e \cdot T_m}{2\gamma \cdot K + K \cdot T_e \cdot T_m}. \tag{18}$$

If to assume that the pump productivity is equal to the flow rate of the fluid into the well, then the error E is equal to zero. In this case, it is possible to write that:

$$M_m = \frac{2\gamma \cdot K \cdot M_l}{2\gamma \cdot K + K \cdot T_e \cdot T_m} + \\ + \frac{2880 J_\Sigma \cdot T_m \cdot n_{cr} \cdot S \cdot D \cdot k_n + K \cdot M_l \cdot T_e \cdot T_m}{2\gamma \cdot K + K \cdot T_e \cdot T_m}. \tag{19}$$

Thus, based on the expression (19), it is possible to determine the optimal value of the drive motor torque at a given load moment and the number of swings of the pump jack (productivity).

3. Results and Discussion

As an example, Fig. 2 shows the dependencies of the induction drive motor torque ($P_n = 15$ kW, $U_n = 380$ V, $n_0 = 1500$ rpm) on the rod pump delivery coefficient for different load moment values: 1 – 64 N·m, 2 – 50 N·m, 3 – 40 N·m, 4 – 30 N·m. In Fig. 3, similar dependencies of the induction drive motor torque for different numbers of crankshaft rotations: 1 – 10 rpm, 2 – 8 rpm, 3 – 5 rpm, 4 – 3 rpm. The swing frequency of the pump jack is changed by the frequency converter according to the scalar control law.

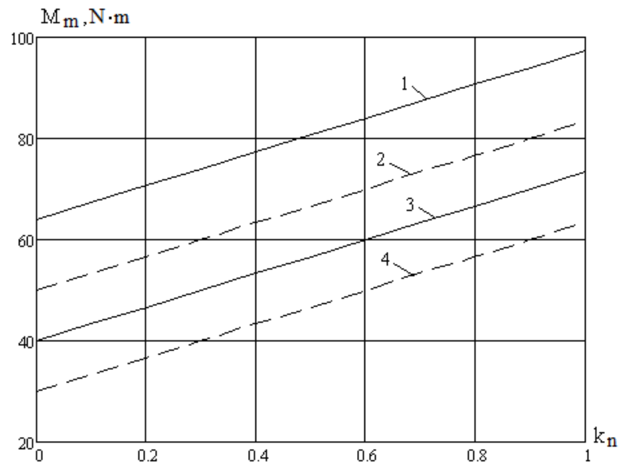

Fig. 2. Drive motor torque vs. rod pump delivery coefficient for different values of load moment: 1 – 64 N·m, 2 – 50 N·m, 3 – 40 N·m, 4 – 30 N·m

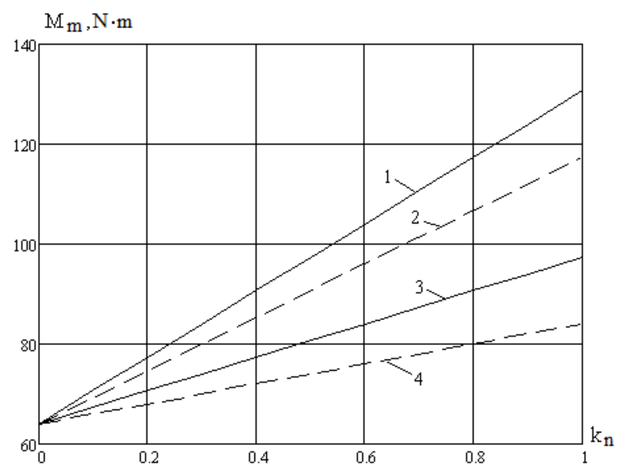

Fig. 3. Drive motor torque vs. rod pump delivery coefficient for different values of load moment for different numbers of swings of pump jack: 1 – 10 rpm, 2 – 8 rpm, 3 – 5 rpm, 4 – 3 rpm

Using the expression (19), it is possible to derive the equation for the number of crankshaft rotations:

$$n_{cr} = \frac{(M_m - M_l)(2\gamma + T_e \cdot T_m) \cdot K}{2880 \cdot J_\Sigma \cdot T_m \cdot S \cdot D \cdot k_n}. \tag{20}$$

Fig. 4 shows the dependencies of the number of swings of the pump jack on the delivery coefficient of the rod pump for different values of the average load moment: 1 – 64 N·m, 2 – 50 N·m, 3 – 40 N·m, 4 – 30 N·m.

In practice, the proposed optimization criterion of electric drive operation will make it possible to increase the efficiency of the oil pumping unit due to setting up the optimal values of the speed and torque of the pump jack drive motor. This, in turn, enhances the well's operating conditions and extends its operational life.

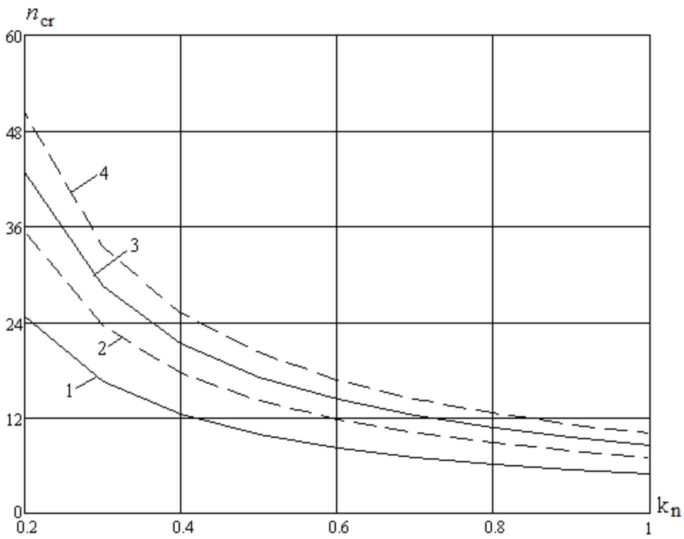

Fig. 4. Number of swings of pump jack vs. rod pump delivery coefficient:
1 – 64 N·m, 2 – 50 N·m, 3 – 40 N·m, 4 – 30 N·m

Prospects for further research are related to the synthesis of a criterion for the optimal functioning of a closed-loop control system taking into account random load variations.

4. Conclusions

The study resulted in the expression for the optimization criterion for the control system of the pump jack electric drive. The synthesized optimization criterion links the unit's productivity to parameters such as the number of crankshaft rotations, the rod pump delivery coefficient, and the load torque of the drive motor. On its basis, optimal values of electric the drive parameters can be selected for a specific well, which ensure rational operation of the sucker-rod pumping unit through balanced oil extraction from the well.

The research results can be used both for designing electric drives for oil production units and for the maintenance of the existing equipment of the well.

Conflict of interest

The authors declare that they have no conflict of interest concerning this research, whether financial, personal, authorship or otherwise, that could affect the study and its results presented in this paper.

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The study was performed without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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