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# DETERMINATION OF THE INFLUENCE OF PULP VISCOSITY ON THE ENRICHMENT PROCESS OF MAGNETITE SUSPENSIONS IN SCREW SEPARATORS

*The object of research is the process of enrichment of magnetite suspensions in screw separators, taking into account the particle size distribution, the compressibility of the liquid in the interparticle space, and the shape of the particles, which allows to assess the influence of these factors on the value of the effective viscosity of the suspension. The viscosity of the suspension is one of the properties for ore suspensions with a wide range of particle sizes and different concentrations of solids in operations and products. It determines the nature of the movement of the liquid, the state of the solids in it – the degree of its loosening, the difficulty of sedimentation, and energy consumption for transportation during the enrichment of magnetite ores in screw separators. The studies were conducted using analytical and experimental methods. It was experimentally established that the dependence of the viscosity of the pulp suspension on the particle size in the range of less than 1 mm has an inversely proportional relationship. At a volume concentration of solids in the pulp from 10 to 70 %, the viscosity of the suspension increases with a decrease in the volume concentration of the minus 0.1 mm class from 100 to 20 %. It has been established that at temperature regimes from 28 to 50 °C, the viscosity of magnetite suspensions increases with a decrease in the grinding fineness in different ways. The viscosity of magnetite suspensions depends on the mass fraction of solids: material with a fineness of 80 % minus 0.044 mm increases the viscosity of the suspension at concentrations above 40 %, and at a fineness of 80 % minus 0.031 mm – above 60 %.*

*The obtained scientific result, in the form of a calculation of the effective viscosity of the pulp, is based on a theory that takes into account the influence of particle size as a function of their average effective diameter, concentration and shape of suspended particles. From a practical point of view, the research results allow to develop optimal conditions for gravitational enrichment of magnetite ores in screw separators and avoid additional losses of valuable components.*

**Keywords:** suspension viscosity, screw separator, magnetite pulp, particle shape, solid concentration, temperature regimes.

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## 1. Introduction

Suspensions (pulp) in technological schemes for the enrichment of magnetite ores have a wide range of particle size in products – from 0.001 to 6(10) mm. At the same time, the concentration of solids in operations and processing products varies from 1 to 85 %. At the same time, the viscosity of the suspension is one of the most important properties that determine the nature of the fluid movement (Reynolds number), the state of the solid in it – the degree of its loosening, the difficulty of sedimentation, energy consumption for transportation, etc. Calculation of the effective viscosity of two-phase polymodal mixtures of solid particles, which include magnetite and hematite quartzites, is necessary for the development of methods for modeling the movement of particles in a screw separator and calculating their velocities during gravitational enrichment of iron ores. It should be noted that, despite the number of published theoretical and experimental studies, there is a certain qualitative and quantitative gap between them. Today, there are contradictions between the huge array of experimental data, the accuracy of which has increased dramatically in recent decades, and the level of theoretical models. Therefore, the

paper provides a critical analysis of existing views on the processes of viscosity influence on the flow of structured disperse systems and basic rheological models with their experimental justification, and shows new possibilities for describing the rheological behavior of a wide range of systems.

One of the parameters that affect the operation of screw separators is the viscosity coefficient of the medium. The viscosity coefficient of the medium is present in all formulas for calculating the rate of fall of grains of different sizes, given by various authors, and is also present in the denominator in the formula for calculating the Reynolds number, etc. However, the influence of the viscosity of the liquid phase on the gravitational enrichment process has been studied little. Existing viscometers are poorly adapted for measuring the viscosity of pulps that are prone to stratification. Therefore, the relationship between the structure and viscosity of disperse systems is considered.

The relationship between the structure and viscosity of dispersed systems has been established for a long time. In [1–5], the features of non-Newtonian flow of suspensions are explained by changes in their structure, in particular, the emergence and destruction of particle aggregates. There are also quite successful attempts to describe the

decrease in viscosity with increasing velocity within the flow models, where the existence of aggregates is not allowed [1–5].

To date, there is no single view among researchers on the mechanisms of flow of dispersed systems, which is confirmed by the variety of proposed rheological equations [2, 5–10].

This work does not use the methods of mechanics of integral media, which are common for theoretical rheology, considering that the anomaly of viscosity is determined by the structure of the system.

The idea that the structure exists where "viscosity changes with the change in shear rate" was taken as a basis, as well as the ideas of P. A. Reh binder about the decrease in viscosity as a result of the gradual destruction of the system structure [11]. In this case, the structuring of the suspension is usually understood as the formation of aggregates with coagulation contacts between particles.

Despite the large number of works and the variety of approaches in the field of rheology of structured dispersed systems [12, 13], there is still no satisfactory quantitative theory that connects the rheological properties of bodies with the parameters of their structure.

In the works [14, 15] it is directly stated that "rheological models, as is known, are not physical laws, but are empirical and semi-empirical approximations that describe flow curves in a certain interval of shear rates".

The analysis of the literature has shown that the variety of existing rheological models reflects the fundamental differences in the types of flow that are characteristic of different fluid systems.

However, the results of comparing the models with the experiment showed that systems with different physical and chemical natures could be described by the same rheological equation with approximately the same accuracy. At the same time, the same dispersed system is described by fundamentally different rheological equations. As a result, it is possible to choose a suitable rheological equation to describe any experiment, without taking into account the real flow mechanism. This state of affairs is quite acceptable for engineering research, but does not correspond to scientific and theoretical analysis.

Attempts to approximate experimental data over the widest possible range of shear rates have led to a large number of semi-empirical expressions, up to rheological equations with five or six fitting coefficients.

An alternative approach is to artificially divide the flow curves or viscosity curves into separate sections, which are described in different ways, but usually by a power law.

For example, in [12] it is shown that in a limited range of shear rates, experimental points are equally well described by different rheological equations (for example, the power law or the Cisco model). Moreover, for individual sections of the same flow curve, the above-mentioned models and the Cross model are equally suitable. In general, the choice of an appropriate rheological equation for the entire curve or for its individual section is left to the researcher. In this approach, individual rheological equations are considered as empirical formulas for approximating experimental data. The need to choose an adequate rheological equation among many possible ones led to the creation of certain rules, formal but quite logical [7]. The equation should provide an optimal approximation of the experiment in a wide range of shear rates, include a minimum number of independent constant parameters that are amenable to independent evaluation and have real physical meaning. The latter means that the rheological equation should be obtained from a microrheological model. *When choosing an adequate rheological equation, one should start with the formulas that have the smallest number of parameters and move on to others*, for example, three-parameter equations, only when the experiment does not agree well with the two-parameter model. This approach is based on the simple fact that equations with a larger number of parameters are easier to fit to a complex data series.

There are many formulas that reflect the dependence of the viscosity of heterogeneous mixtures on the concentration of particles. In most cases,

the calculated viscosity coefficient does not take into account the effect of particle size. For example, in the well-known Einstein formula [14], the viscosity of mixtures of a liquid and fine spherical particle linearly depends only on their volume concentration:

$$\mu = \mu_0(1 + k\varphi), \quad (1)$$

where  $\mu_0$  – the viscosity coefficient of water  $\mu_0 = 0.001$  Pa·s;  $k = 2.5$  – the coefficient 2.5 for spherical particles [13];  $\varphi$  – the volume concentration of particles, which is calculated by the formula:

$$\varphi = \frac{V_{dp}}{V_s}, \quad (2)$$

where  $V_{dp}$  – dispersed phase volume,  $m^3$ ;  $V_s$  – suspension volume,  $m^3$ .

However, this equation is experimentally confirmed only for a very dilute suspension with a volume concentration of up to 5 %, and the value of the coefficient  $k$  often exceeds 2.5 even for spherical particles.

A general scheme of the rheological behavior of systems with a coagulation structure was proposed in [11] for interpreting the flow of structured suspensions.

It is believed that after the complete destruction of the structure, a Newtonian flow occurs. For the described flow curve P. A. Reh binder introduces the concept of the highest viscosity of a practically undamaged structure and the lowest viscosity of a severely damaged structure.

It should be noted that the rheological curve describes a gradual transition from the solid state of the system to the fluid state. Therefore, the question of the equilibrium of this flow remains open. At the same time, the lowest viscosity of a severely damaged structure is not achieved in conventional viscometric measurements [13].

*A separate flow mechanism is assumed for each section of such a complete rheological curve.*

Approximation of these sections by straight lines is a useful approximation, but requires justification using a rheological model. In addition, there is an opinion [16, 17] that shear flow occurs at arbitrarily small shear stresses.

Over the past fifty years, numerous concepts have been proposed to describe the steady-state shear flow of dispersed systems, leading to a wide variety of functional relationships between the effective viscosity  $\eta$ , the shear stress  $\tau$ , and the shear rate  $\dot{\gamma}$  [18–29].

From a phenomenological point of view, the following initial mathematical equation is mainly used:

$$\tau^m = \tau_y^m + \eta_{pl}^n \cdot \dot{\gamma}^n. \quad (3)$$

Most of the known flow equations are obtained from equation (3), in which the values of the exponents  $n$  and  $m$  are specially changed:

– Bingham equation [24]:

$$\tau = \tau_B + \eta_B \dot{\gamma}; \quad (4)$$

– Caisson equation [24]:

$$\tau^{1/2} = \tau_C^{1/2} + \eta_C^{1/2} \dot{\gamma}^{1/2}; \quad (5)$$

– Herschel-Bulkley equation [24]:

$$\tau = \tau_H + \eta_H \dot{\gamma}^n, \quad (6)$$

where  $\tau_B, \tau_C, \tau_H$  – parameters that have the value of the ultimate shear stress.

Equation (3) for arbitrary values of  $n$  and  $m$  is known as the Shulman equation [13]. The Ostwald equation [13] is considered a special case of the Herschel-Bulkley equation at  $\tau_H = 0$ . The main disadvantage

of the Herschel-Bulkley, Ostwald and Shulman equations is that they lack a theoretical basis, i. e. a microrheological model that relates the rheological coefficients to the structural-rheological characteristics of the system, namely to the forces of interaction between particles, the structure and size of aggregates.

The justification of rheological equations is microrheological models that link rheological properties with the nature of particle motion in a viscous fluid, attractive and repulsive forces, and hydrodynamic interaction between particles.

Thus, the author of [20] suggested that the rheological properties of suspensions are determined by the volume concentration of the dispersed phase, the interval and magnitude of the interaction forces between particles, and the structure of the aggregates that are formed. The main factors affecting the flow of suspensions, the author attributed Brownian motion of particles, interparticle attractive and repulsive forces, and hydrodynamic interaction between particles. The latter, in his opinion, arose at high volume concentrations, when the flow field around one particle interacts with the flow field around a neighboring particle.

Today, *three types of microrheological models* can be distinguished.

*Models of the first type* use kinetic equations to describe the formation and destruction of aggregates consisting of two or more particles. The equations of state of the structure determine the equilibrium of the processes of breaking bonds between particles and their restoration.

*Models of the second type* include two sources of flow energy dissipation. The total energy dissipation is divided into structural ( $\tau_s \dot{\gamma}$ ) and viscous ( $\eta_{pl} \dot{\gamma}^2$ ) parts. It is assumed that the first part describes energy losses in the process of breaking bonds between particles, the second – energy losses during the flow of aggregates or individual particles by a dispersed viscous medium.

*Microrheological models of the third type* assume that energy losses occur only due to the viscous flow of the dispersion medium around flow units (particles or aggregates). The viscous flow of the suspension is the only source of energy dissipation.

The viscosity model is based on two assumptions. First, the power of energy dissipation depends only on the viscous friction between the moving aggregate and the dispersion medium flowing around the aggregate. Second, the size of the aggregate is determined by the balance between the hydrodynamic tensile forces leading to the rupture of the aggregate and the interparticle attraction forces.

In particular, the model developed in [30] describes the shear flow of rigid chains in which the particles are bound together by van der Waals forces. To provide hydrodynamic calculations, these chains were described as model cylinders with an axial ratio  $J$ . In fact, the limiting stress  $\tau_c$  arose in the final expression of the Caisson model due to the reduction of viscous friction losses due to the reduction of the size of the aggregate-cylinder.

A similar chain model is proposed for the flow of colloidal solutions of ferromagnets in a magnetic field. Equilibrium length of a chain of spherical particles:

$$(2l) \approx \dot{\gamma}^{-1/2}. \quad (7)$$

Whence  $\tau_B = 2/3 n r_0 F_s$ , where  $n$  – the number of particles bound into aggregates per unit volume,  $r_0$  – the distance between particles.

In [11], the concept of fractals is used to describe aggregates, supplemented by the concept of effective volume  $\Phi_{ef}$ . In essence, it is assumed that the real volume of particles in the aggregate is replaced by the volume of the sphere inside which the aggregate is located. It is clear that such an effective volume ( $\Phi_{ef}$ ) is much larger than the real volume of the dispersed phase, i. e.:

$$\Phi_{ef} = \Phi p^{(3/D-1)}. \quad (8)$$

From this it is concluded that the viscosity of the aggregated suspension is much higher than the viscosity of the same suspension, but divided into individual particles. Thus, a decrease in the number of particles in the aggregate and a corresponding decrease in the effective volume of the aggregate lead to shear thinning; in a shear flow field, the radius of the aggregate decreases according to the empirical law:

$$R(\dot{\gamma}) = \frac{R_0}{1 + (b\dot{\gamma})^m}. \quad (9)$$

For the most part, the mechanism of flow of structured dispersed systems is interpreted by researchers as a sequential breakdown of the initial structure under the action of increasing shear rate [13].

The presence of some structure is assumed even in those models where the adhesion forces between particles are completely neglected, for example, in the model of "layered" flow of particles of colloidal size, developed by the authors of the works [3, 22].

They assumed that during the flow at high speed, rows or chains of particles are formed, arranged in layers. At low shear stresses, the particles have a three-dimensional chaotic arrangement, which leads to a Newtonian flow with a high viscosity  $\eta(0)$ . With increasing shear stress  $\tau$ , such a system gradually transforms into a structure of two-dimensional hexagonal layers, while the viscosity gradually decreases. The final "layered" flow demonstrates a Newtonian regime with an extremely low viscosity  $\eta_\infty$ . With further increase in shear stress, the viscosity increased due to the instability of the layered structure, which leads to dilatant behavior.

For a more detailed consideration of the influence of suspension viscosity on the enrichment indicators of magnetite ores, models of the dependence of viscosity on the concentration of the dispersed phase are considered below.

Hydrodynamic interaction between particles occurs when the flow field around one particle interacts with the flow field around another particle.

Hydrodynamic interaction leads to an increase in the dissipation of viscous friction energy, as a result of which the viscosity increases nonlinearly with increasing particle volume concentration.

At low shear rates, the viscosity of a disordered system of particles will be high; with increasing speed, ordered layers of particles arise and the viscosity decreases.

The "soft sphere" model includes electrostatic repulsion between particles surrounded by a double ionic layer. This is often taken into account by some effective or hydrodynamic particle volume, which is replaced by the usual volume of the solid dispersed phase.

The authors of [9] considered the effective volume concentration as the total volume of the solid phase and the liquid trapped inside the aggregate. Then the destruction of the aggregates leads to the release of previously trapped liquid, a decrease in the effective volume concentration and, therefore, a decrease in viscosity with increasing velocity according to Einstein's equation.

In [23], aggregates (or flocs) are considered as separate flow units that include the trapped liquid. Such flocs in a dilute suspension can be replaced by equivalent flocs of a sphere with an impermeable surface. The volume concentration of the dispersed phase is replaced by the volume concentration of flocs, i. e., a conditional "hydrodynamic" volume of the dispersed phase is used.

The author of [21] explains the secondary electroviscous effect as an increase in the effective diameter of particles during collisions due to electrostatic repulsion forces. For this, a certain excluded (hydrodynamic) volume of particles is introduced, which is significantly larger for charged particles and, as is assumed, creates an additional source of energy dissipation in viscous flow.

The coefficient  $\eta$  represents the "characteristic viscosity".

In [24], the main models of the flow of suspensions are listed, which are based on strict hydrodynamic calculations. The forces of attraction

and repulsion are neglected. As mentioned above, for an infinitely dilute suspension, in which there is completely no hydrodynamic interaction between particles, Einstein's equation is valid. The derivation of this equation [24] is based on the calculation of energy dissipation during the flow of a viscous fluid around a separate sphere. A similar equation was obtained for the flow of a suspension of particles in the form of ellipsoids. In a suspension where the concentration is quite high, it is necessary to take into account the hydrodynamic interaction of the particles. As a result of the superposition of the flow fields of neighboring particles, the viscosity of the suspension increases. Then Einstein's equation takes on an expanded form:

$$\frac{\eta}{\eta_0} = 1 + 2.5\Phi + a_2\Phi^2 + a_3\Phi^3, \quad (10)$$

where the values of the coefficients  $a_2$  and  $a_3$  are different for different authors.

In the cell models, it is assumed that the flow disturbance caused by this isolated particle is concentrated in a small region near it (in the cell).

In the *Simkha cell model*, a rather complex expression for the relative viscosity was obtained in the form of a power series  $F$ . The concentration  $F_i$  corresponds to the limiting concentration at which the flow of the suspension stops. Usually, a value corresponding to a certain packing of spherical particles in the lattice is chosen.

The authors of [25] drew attention to the fact that these effects, which are observed under conditions of high concentration, can be explained in terms of pair interactions of individual particles. Later, it was noted [31] that the dependence  $\eta(\Phi)$  is poorly reproduced by different researchers. There are large variations in the viscosity value, which are attributed to non-hydrodynamic effects (Brownian motion and attractive forces between particles).

Therefore, none of the known models describes simultaneously plastic and pseudoplastic flow with sufficiently good accuracy. Analyzing all formulas, it is possible to notice that everywhere the viscosity coefficient depended on the volume concentration and did not take into account the effect of particle size. Although various experimental studies of coarse-dispersed suspensions used as a medium for mineral enrichment have shown that the effective viscosity is affected by both the concentration and the shape of the suspended particles and its size. The ignoring by some researchers of the role of particle size in the formation of pulp viscosity arose, apparently, as a result of transferring Einstein's thesis about the independence of the viscosity of low-concentration pulps from particle size to pulp with a high concentration of the solid phase.

Therefore, a theory that takes into account the effect of particle size as a function of the average effective diameter of pulp particles deserves special interest, and for different substances these functions may be different.

Analysis of research and publications [1–33] showed that the study of the influence of pulp viscosity on the enrichment process of magnetite suspensions in screw separators is an urgent scientific and technical problem. As a result of the analysis of microrheological models, it was confirmed that they link rheological properties with the nature of particle motion in a viscous liquid, forces of attraction and repulsion, and hydrodynamic interaction between particles. It was shown that the main unresolved problems of this process are the study of pulp viscosity taking into account the distribution of particles by size and shape and the presence of liquid in the interparticle space. This made it possible to assess the influence of these factors on the value of the effective viscosity of the suspension and explain the large scatter of experimental data.

Scientific research is aimed at determining the dependence of the viscosity of ore suspension at different concentrations on the size of mineral particles and temperature, which is one of the ways to solve this scientific and practical problem. The complexity of the problem is due to the fact that the dynamic viscosity of two-phase mixtures of mineral

particles and liquid at a constant value of the volume concentration of the solid phase can change by more than 1.5–2 times. It is proposed to study the influence of pulp viscosity on the enrichment process of magnetite suspensions in screw separators by mathematical modeling of this process.

The practical part of the research included experiments to determine the optimal conditions for gravitational enrichment of magnetite ores in screw separators, which allow developing a new technology and avoiding additional losses of valuable components.

The aim of research is to establish the dependence of the effective viscosity of magnetite suspension on the particle size and temperature of the pulp, as a two-phase mixture of mineral particles and liquid with different densities. It is also planned to develop a method for calculating the effective viscosity, which takes into account the distribution of particles by size and shape, the temperature of the liquid and its compressibility in the interparticle space.

To achieve the aim, it is necessary to conduct experimental studies and establish the relevant dependencies. At the same time, it is necessary to determine the factors that affect the indicators of gravitational separation of magnetite ore minerals in screw separators. Based on the analysis of the established dependencies, develop recommendations for their implementation in production.

## 2. Materials and Methods

To study the viscosity of iron ore pulp, a new research methodology was developed. The viscosity of suspensions was measured in two ways:

- 1) on a reconstructed laboratory flotation machine [33];
- 2) using a capillary reverse current viscometer of the VNZh type.

According to the first method, the viscosity of suspensions was measured on a reconstructed laboratory flotation machine. This machine in the bath had two identical horizontal impellers, each mounted on its own axis – the lower and upper. The lower impeller had a monolithic shaft driven by an electric motor with a constant speed. The lower impeller and its shaft were driving. The upper impeller (leading, driven) had an axis in the form of a hollow pipe, through which the shaft of the lower impeller passed. The shaft of the upper impeller was connected to a revolution counter through a gear transmission. In the absence of water in the bath, rotation from the lower impeller to the upper one was practically not transmitted.

During the studies, the frequency of rotation of the driving and driven shafts of the impellers was measured,  $s^{-1}$ . After that, the viscosity of the pulp was calculated based on the following provisions.

Understanding dynamic viscosity as the momentum flux that is transferred from layer to layer in a liquid per unit time through a unit plane at a velocity gradient equal to unity [27], let's obtain an expression for determining viscosity from experimental data:

$$\eta = \frac{\rho(\vartheta_1 - \vartheta_2)S}{\Delta l}, \quad (11)$$

where  $\rho$  – suspension density,  $kg/m^3$ ;  $\vartheta_1, \vartheta_2$  – speed of the driving and driven shafts, respectively,  $m/cm$ ;  $S$  – the cross-sectional area between the impeller planes,  $m^2$ ;  $\Delta l$  – distance between the driving and driven shafts of the impellers,  $cm$ .

In order to find the linear speed of the body along a circle, it is necessary to divide the length of the circle by the period. In this case, the length of the circle is  $2\pi$  multiplied by the radius.

After substituting the corresponding values:

$$\eta = 0.01(n_1 - n_2)\rho_1, \quad (12)$$

where  $n_1, n_2$  – frequency of rotation of the driving and driven shafts of the impellers, respectively,  $s^{-1}$ .



In this case, the density of the suspension was determined by the formula:

$$\rho_c = \frac{\rho_1 \rho_2}{(\rho_2 + k_1 (\rho_1 - \rho_2))}, \quad (13)$$

where  $\rho_1, \rho_2$  – density of the first and second components of the suspension, kg/m<sup>3</sup>;  $k_2 = 1 - k_1$  – fraction of the second component in the suspension, parts per million;  $k_1$  – the fraction of the first component in the suspension, parts per million.

According to the presented method, the calculation formula for the viscosity of the suspension had the form:

$$\eta = 2.1 \cdot 10^{-5} \rho_s n_d, \quad (14)$$

where  $\eta$  – suspension viscosity, mPa·s;  $\rho_s$  – suspension density, kg/m<sup>3</sup>;  $n_d$  – frequency of rotation of the driven shaft, s<sup>-1</sup>.

The measured viscosity values were supplemented with data and processed in the form of dependencies:

- the viscosity of the suspension (pulp)  $\eta$ :

$$\eta = f(\eta_0, \rho_s, \varphi_{sol}, f_{sp}); \quad (15)$$

- the relative viscosity of the suspension:

$$\frac{\eta}{\eta_0} = f(\rho_s, \varphi_{sol}, f_{sp}), \quad (16)$$

where  $\eta_0$  – water viscosity, mPa·s;  $\rho_s$  – suspension density, kg/m<sup>3</sup>;  $\varphi_{sol}$  – mass fraction of solids in the suspension, fraction of units;  $f_{sp}$  – specific surface area of solids in the suspension, m<sup>2</sup>/m<sup>3</sup> of the suspension.

According to the second method, when determining the effect of temperature on the pulp separation indicators, studies on determining viscosity were performed using a capillary reverse flow viscometer of the VNZh type (Ukraine). Viscosity measurement using the VNZh viscometer is based on determining the time of passage of a suspension of a certain volume from the measuring tank through the capillary. In this case, it is not the time of the end of the passage of the suspension through the capillary that is measured, but the time of filling the receiving tank with pulp (first the lower one, then the upper one).

At the same time, the determination of kinematic viscosity  $\eta_k$  was performed according to the formula:

$$\eta_k = \frac{g \cdot T \cdot K}{9.807}, \quad (17)$$

where  $g$  – acceleration of free fall at the measurement site;  $T$  – time of passage of a suspension of a certain volume from the measuring tank through the capillary;  $K$  – viscometer constant (in the range from 0.003 to 30 mm<sup>2</sup>/s<sup>2</sup> depending on the viscosity range from 0.6 to 30,000 mm<sup>2</sup>/s).

### 3. Results and Discussion

#### 3.1. Development of a mathematical model

The development of the model is based on the proposition that the dynamic viscosity of ore pulp as a two-phase mixture of mineral particles and liquid at a constant value of the volume concentration of the solid phase  $\varphi$  depends on the polymodal granulometric composition of the solid phase of the pulp.

The theoretical model and experimental data proposed for their description are analyzed [26].

The theoretical justification of the type of functional dependence is obtained on the basis of the following assumption. It is believed that adding the same number of particles to the initial dispersion me-

dium (liquid) and suspension with the same volumes will lead to the same relative values of the effective viscosities of the newly created volumes of the two media [26]. This assumption is apparently acceptable at a small concentration of monodisperse particles of the solid phase. *With increasing concentration of the solid phase, individual liquid jets flowing around single particles begin to interact with each other and the conditions of hydrodynamic similarity are violated even for monomodal particles.* The entire array of experimental data is described by a single functional linear dependence, the two coefficients of which take pairwise different values, which were determined by the results of comparison with the data obtained during the studies:

$$\bar{\mu} = \left(1 - \frac{\varphi}{0.73}\right)^{-1.525}; \quad (18)$$

$$\bar{\mu} = \left(1 - \frac{\varphi}{0.65}\right)^{-1.675}; \quad (19)$$

$$\bar{\mu} = \left(1 - \frac{\varphi}{0.56}\right)^{-1.8}. \quad (20)$$

It was assumed that the numerical value of the coefficient 0.73 corresponds to the maximum volume fraction of particles in their hexagonal packing. The value 0.65 corresponds to the maximum volume fraction of particles in their face-centered cubic packing. The value 0.56 corresponds to the maximum volume fraction of particles in their simple cubic packing.

From formulas (18)–(20) it follows that when the volume fraction of particles  $\varphi$  approaches the values of the maximum volume fraction of particles for each of the assumed clusters, the values  $\bar{\mu}$  tend to infinity. For example, for the smallest value of 0.56 this is physically reliable.

In the following, expressions (18)–(20) are used as information about the experimental range of values of the relative viscosity of two-phase mixtures (suspensions), to which magnetite pulp belongs.

The excess viscosity of a suspension is proportional not only to the volume fraction of the solid phase, but also inversely proportional to the volume fraction of the free liquid. Since Einstein's formula was obtained for an infinitely small volume fraction of the solid phase, it is possible to transform it approximately as follows:

$$\mu - \mu_0 = d\mu = \alpha \mu_0 \varphi \approx \mu_0 d \left[ \frac{\alpha \varphi}{(1 - \beta \varphi)} \right]. \quad (21)$$

Let's integrate this expression and obtain:

$$\bar{\mu} = \exp \left[ \alpha \varphi (1 - \beta \varphi) \right], \quad (22)$$

where  $\alpha$  and  $\beta$  are empirical coefficients selected from the condition of the best match with the data obtained in the studies.

Taking into account the above, let's obtain equations that are even to (18)–(20):

$$\ln(\bar{\mu}) = \frac{2.054\varphi}{(1 - 0.856\varphi)}; \quad (23)$$

$$\ln(\bar{\mu}) = \frac{2.5266\varphi}{(1 - 0.9714\varphi)}; \quad (24)$$

$$\ln(\bar{\mu}) = \frac{3.1502\varphi}{(1 - 1/103\varphi)}. \quad (25)$$

It follows from the numerical data that at  $0 < \varphi < 0.45$  the discrepancy between the pair dependencies (18) and (23), (19) and (24), (20) and (25) does not exceed 5 %. From this it is possible to conclude that the specific calculated dependencies (18)–(20) and (23)–(25), obtained on the basis of different physical and mathematical models, with the

appropriate selection of the values of the empirical coefficients, lead to practically identical results. From expressions (16)–(18) it follows that  $\varphi \leq 0.9066$ , at a volume fraction of particles, none of these dependences tends to infinity, which seems to be physically more reliable, compared to that according to equation (18) at  $\varphi = 0.56\bar{\mu} \rightarrow \infty$ .

The experimental data given in the literature, for example, the relative effective viscosity  $\bar{\mu} = \mu_p / \mu_f$ , which is defined as the ratio of the viscosity of a two-phase mixture  $\mu_p$  to the viscosity of the dispersion medium  $\mu_f$  of homogeneous spheres at a volume concentration  $\varphi = 0.4$ , are 7.4 and 16.0, i. e. they differ by more than twofold, without any comment on the physical reasons for such a discrepancy in the data obtained during the studies. In paper [28] for the same spherical particles the value  $\bar{\mu}$  is 6.2, i. e. the discrepancy with the maximum value  $\bar{\mu}$  given above is more than 2.5 times. The theoretical models discussed contain at least one constant, which is selected from the condition of the best fit between theoretical and practical data. With such a difference in practical data, the idea arises that the process of joint motion of particles of a monodisperse composition is actually the motion of particles of a polydisperse composition.

For example, in work [11], the speed of motion of monodisperse solid particles is experimentally investigated. In this case, the actual ratio of particle diameters (maximum to minimum) in the monomodal fraction varied in the range from 1.17 to 2.34.

Therefore, it is expected that the speeds of compressed particle motion within one fraction, determined by the average diameter, will differ slightly less than the speeds of free particle motion, which are proportional to the square of the particle diameter, should differ by a factor of 1.18 and 2.79, respectively.

The differences in the values of the effective dynamic viscosity of two-phase mixtures at a constant volume concentration of the solid phase  $\varphi$  are very significant. The idea that these values are completely determined by the maximum volume concentration  $\varphi_m$  for the actual particle size distribution of the solid phase is physically incorrect. This is due to the fact that the relationship between the value  $\varphi_m$  and the actually polymodal particle size distribution of the solid phase is not defined. In general, as shown above, the dynamic viscosity of a two-phase mixture at a constant volume concentration of the solid phase  $\varphi$  depends on the particle size distribution of the solid phase. Let's consider the limiting case when the differences in particle sizes are so large that the liquid and the particles of the fine fraction form a single carrier medium for the particles of the coarse fraction. According to this position, the effective viscosity of a suspension formed by particles of the fine fraction with a volume fraction  $\varphi_1$  is calculated by the formula:

$$\ln \bar{\mu}_{p1} = \frac{2.054\varphi_1}{(1-0.856\varphi)}. \quad (26)$$

When writing the last expression, it is taken into account that for small particles the volume fraction of free liquid is equal to  $(1-0.856\varphi)$ , in accordance with the empirical dependence obtained from experimental studies.

The relative viscosity of a suspension formed by particles of a large fraction with a volume fraction  $\varphi_2$  is calculated by formula (26). This occurs in a dispersion medium formed by small particles with a viscosity  $\bar{\mu}_{p1}$  and takes into account that for large particles the volume fraction of the free dispersion medium formed by small particles and liquid is equal to  $1-0.856\varphi_2$ . Taking this into account, the viscosity is the effective dynamic viscosity of the two-phase mixture, with respect to large particles and surfaces that limit the volume in which the two-phase mixture is located, let's obtain:

$$\ln \bar{\mu}_{p2} = \left\{ \left[ \frac{2.054\varphi_1}{(1-0.856\varphi)} \right] + \left[ \frac{2.054\varphi_2}{(1-0.856\varphi)} \right] \right\}. \quad (27)$$

The results of calculations according to formula (27) for a bimodal mixture of solid particles show the variation of the volume fractions of particles of two types of size. At  $\varphi = 0.6$  compared to a monodisperse two-phase mixture, the differences in the effective viscosity of the bimodal mixture depending on the particle size distribution can vary by more than 1.5 times.

Similar calculations can be performed for a polymodal mixture. In this case, the expression for the effective viscosity of a polymodal two-phase mixture, taking into account that  $\varphi_0 = 0$ , takes the form:

$$\ln \bar{\mu}_{pn} = \frac{2.054 \sum_{i=1}^n \varphi_i}{\left[ 1 - 0.856 \left( \varphi - \sum_{k=1}^{i-1} \varphi_k \right) \right]}. \quad (28)$$

The refinements of the model for calculating the effective viscosity of a two-phase mixture, which was developed, are based on the fact that during the movement of the pulp, the actual flow velocity around the particles is a liquid. In this case, the compression of the liquid flow in the interparticle space, which is higher than the flow velocity of the liquid flow at the frontal point of the particle, is taken into account.

In the case of a bimodal mixture, the value of the correction function  $f$  is equal to:

$$f = \left\{ \left[ \frac{\pi \left( \frac{6\varphi_1}{\pi} \right)^{\frac{2}{3}}}{1 - \frac{\pi}{4}} \right] \left[ \frac{\pi \left( \frac{6\varphi_1}{\pi} \right)^{\frac{2}{3}}}{1 - \frac{\pi}{4}} \right] \right\}^{\frac{1}{2}}. \quad (29)$$

Let's also assume that the increase in the derivative of the velocity along the transverse coordinate is equal to the value  $f$ .

The losses for moving a spherical particle in a laminar flow regime, according to the Stokes formula, are proportional to the product of the viscosity of the medium and the velocity of the particle. In fact, these two factors, the compressibility of the fluid flow in the interparticle space and the increase in energy losses for dissipation due to changes in the viscosity of the medium, are combined.

In the model being developed, the fluid motion compression factor applies only to the calculation of the velocity of flow around the particle. This was made possible by using the empirical dependence of the effective viscosity on the volume content of the solid phase. In this case, the increase in energy losses for dissipation due to changes in the viscosity of the medium is taken into account, compared to a homogeneous liquid. Taking these considerations into account, the value of the effective viscosity of a two-phase medium is defined as the product of expressions (27) and (29):

$$\bar{\mu}_{p2} = \left\{ \left[ \frac{\pi \left( \frac{6\varphi_1}{\pi} \right)^{\frac{2}{3}}}{1 - \frac{\pi}{4}} \right] \left[ \frac{\pi \left( \frac{6\varphi_1}{\pi} \right)^{\frac{2}{3}}}{1 - \frac{\pi}{4}} \right] \right\}^{\frac{1}{2}} \times \exp \left\{ \left[ \frac{2.054\varphi_1}{(1-0.856\varphi)} \right] + \left[ \frac{2.054\varphi_2}{(1-0.856\varphi)} \right] \right\}. \quad (30)$$

The results of calculations  $\bar{\mu}_{p2}$  according to formula (30) for a bimodal mixture of solid particles when varying the volume fractions of particles of two types of size expand the range of values of the relative effective viscosity of a two-phase medium. These results almost completely cover the range of experimental values given by the dependencies (18) and (20) or (23)–(25). The discrepancy between the minimum and maximum values of the effective viscosity is about 2, i. e. the calculated values are completely within the range of experimental values of the relative effective viscosity of two-phase mixtures.

The influence of the shape and orientation of particles on the value of the relative viscosity is then evaluated.

As was shown, for example, in [29], when analyzing the motion of particles of arbitrary shape, it is advisable to use the equivalent relative diameter of particles  $\theta_i$  (analogous to the particle sphericity coefficient), which is determined by the formula:

$$\theta_i = \frac{(2d_{si} + d_{mi})}{(3d_{vi})}, \quad (31)$$

where  $d_{si}$ ,  $d_{mi}$  and  $d_{vi}$  – equivalent diameters of spherical particles, determined by the area of the lateral surface, midsection and particle volume. For a spherical particle  $\theta = 1$ .

The expression for the effective viscosity of a bimodal mixture will take the form:

$$\bar{\mu}_{p2} = \left\{ \left[ 1 - \frac{\pi \left( \frac{6\phi_1}{\pi} \right)^{\frac{2}{3}}}{4} \right] \left[ 1 - \frac{\pi \left( \frac{6\phi_2}{\pi} \right)^{\frac{2}{3}}}{4} \right] \right\}^{-\frac{1}{2}} \times \exp \left\{ \left[ \frac{2.054\phi_1\theta_1}{(1-0.856\phi)} \right] + \left[ \frac{2.054\phi_2\theta_2}{(1-0.856\phi)} \right] \right\}. \quad (32)$$

To assess the influence of particle shape on the relative viscosity, let's consider the spatial orientation of elongated and flattened ellipsoid rotation. Using expressions (31) and (32) given in [29], let's obtain the value of the equivalent relative diameter of particles: for particles of the first type, relative to the semi-axes of the ellipsoid of rotation ( $a/b = 1-1000$ ); and for particles of the second type at ( $b/a = 1-0.001$ ). The values  $\theta_1$  refer to the case of particle orientation along the largest semi-axis of the ellipsoid or along the smallest semi-axis of the ellipsoid. The values  $\theta_2$  refer to the case of transverse particle orientation relative to the largest semi-axis of the ellipsoid of needle-shaped particles or to the smallest semi-axis of the ellipsoid of disk-shaped particles.

Table 1 shows calculations that allow to estimate the total effect of the particle size distribution, compression of the liquid flow in the interparticle space and the spatial orientation of the elongated ellipsoidal particles ( $a/b > 1$ ). The effect of the orientation of the compressed ellipsoidal particles ( $b/a < 1$ ) on the relative viscosity of the two-phase mixture when the particle is oriented along the largest semi-axis of the ellipsoid or along the smallest semi-axis of the ellipsoid, respectively, is also considered.

Table 1

Calculated values of the relative effective viscosity of the two-phase mixture (pulp) according to equation (23)

| Particles of the first and second types $\theta_1=0.956$ ; $\theta_2=1.118$ |     |      |      |      |      |      |      |      |       |
|---|-----|------|------|------|------|------|------|------|-------|
| $\phi_1$  |     | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.55 | 0.6   |
| $\bar{\mu}_{p2}$  | 0   | 1.23 | 1.44 | 2.27 | 3.73 | 6.90 | 15.3 | 25.1 | 45.4  |
|   | 0.5 | 1.25 | 1.50 | 2.20 | 3.35 | 5.39 | 9.41 | 13.0 | 18.5  |
|   | 1   | 1.21 | 1.53 | 2.10 | 3.26 | 5.63 | 11.4 | 17.7 | 30.1  |
| Particles of the first and second types $\theta_1=1.118$ ; $\theta_2=0.956$ |     |      |      |      |      |      |      |      |       |
| $\bar{\mu}_{p2}$  | 0   | 1.21 | 1.44 | 2.10 | 3.26 | 5.63 | 11.4 | 17.7 | 30.1  |
|   | 0.5 | 1.25 | 1.50 | 2.21 | 3.38 | 5.51 | 9.80 | 13.7 | 19.9  |
|   | 1   | 1.22 | 1.59 | 2.27 | 3.73 | 6.90 | 15.3 | 25.1 | 45.4  |
| Particles of the first and second types $\theta_1=1.024$ ; $\theta_2=1.749$ |     |      |      |      |      |      |      |      |       |
| $\bar{\mu}_{p2}$  | 0   | 1.32 | 1.72 | 3.11 | 6.30 | 15.2 | 47.3 | 96.4 | 224.7 |
|   | 0.5 | 1.30 | 1.62 | 2.58 | 4.31 | 7.59 | 15.1 | 22.2 | 34.1  |
|   | 1   | 1.22 | 1.46 | 2.17 | 3.45 | 6.13 | 12.9 | 20.5 | 35.8  |
| Particles of the first and second types $\theta_1=1.749$ ; $\theta_2=1.024$ |     |      |      |      |      |      |      |      |       |
| $\bar{\mu}_{p2}$  | 0   | 1.22 | 1.46 | 2.17 | 3.45 | 6.13 | 12.9 | 20.5 | 35.8  |
|   | 0.5 | 1.29 | 1.63 | 2.63 | 4.50 | 8.45 | 18.0 | 28.2 | 46.8  |
|   | 1   | 1.32 | 1.72 | 3.11 | 6.30 | 15.2 | 47.3 | 96.4 | 224.7 |

If instead of dependence (23) to use dependence (24), under the same conditions as used above, it is possible to obtain the calculated values of the effective dynamic viscosity of the two-phase mixture, which are given in Table 2.

Table 2

Calculated values of the relative effective viscosity of the two-phase mixture (pulp) according to equation (24)

| Particles of the first and second types $\theta_1=0.956$ ; $\theta_2=1.118$ |     |      |      |      |      |      |       |       |       |
|---|-----|------|------|------|------|------|-------|-------|-------|
| $\phi_1$  |     | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.5   | 0.55  | 0.6   |
| $\bar{\mu}_{p2}$  | 0   | 1.27 | 1.59 | 2.63 | 4.88 | 10.8 | 31.9  | 64.7  | 155.4 |
|   | 0.5 | 1.28 | 1.59 | 2.51 | 4.19 | 7.67 | 16.1  | 25.1  | 42.1  |
|   | 1   | 1.24 | 1.53 | 2.40 | 4.17 | 8.49 | 22.2  | 41.7  | 91.1  |
| Particles of the first and second types $\theta_1=1.118$ ; $\theta_2=0.956$ |     |      |      |      |      |      |       |       |       |
| $\bar{\mu}_{p2}$  | 0   | 1.14 | 1.52 | 2.38 | 4.11 | 8.28 | 21.4  | 39.9  | 86.3  |
|   | 0.5 | 1.28 | 1.59 | 2.52 | 4.26 | 7.94 | 17.2  | 27.6  | 47.8  |
|   | 1   | 1.27 | 1.60 | 2.66 | 4.98 | 11.1 | 33.3  | 68.2  | 165.6 |
| Particles of the first and second types $\theta_1=1.024$ ; $\theta_2=1.749$ |     |      |      |      |      |      |       |       |       |
| $\bar{\mu}_{p2}$  | 0   | 1.38 | 1.90 | 3.91 | 9.59 | 39.7 | 150.4 | 425.5 | 1539  |
|   | 0.5 | 1.34 | 1.75 | 3.06 | 5.75 | 12.1 | 29.7  | 50.7  | 93.8  |
|   | 1   | 1.26 | 1.56 | 2.51 | 4.49 | 9.52 | 26.4  | 51.3  | 117.0 |
| Particles of the first and second types $\theta_1=1.749$ ; $\theta_2=1.024$ |     |      |      |      |      |      |       |       |       |
| $\bar{\mu}_{p2}$  | 0   | 1.25 | 1.55 | 2.48 | 4.42 | 9.27 | 25.3  | 48.9  | 110.4 |
|   | 0.5 | 1.34 | 1.76 | 3.14 | 6.18 | 14.1 | 40.0  | 76.5  | 164.6 |
|   | 1   | 1.39 | 1.91 | 3.98 | 9.88 | 32.1 | 160.8 | 461.5 | 1700  |

The data provided in Tables 1, 2 consistently demonstrate the influence of the particle size distribution, the compression of the fluid flow conditions in the interparticle space, the shape and orientation of the particles in space relative to the direction of particle motion. The last two series of calculations were performed for a bimodal mixture of solid particles.

The calculated estimates show that at a constant value of the volume fraction of solid particles in a two-phase mixture (pulp), the value of its effective dynamic viscosity strongly depends on the specific characteristics of the solid particles – its particle size distribution and particle shape and, depending on these values, can vary by tens of percent and reach multiple values.

The range of calculated values of the effective dynamic viscosity completely covers the range of experimental values.

The absence or incompleteness of data on the considered characteristics of the dispersed phase does not allow for a comparison of practical and calculated data on the speeds of the pulp movement. This complicates comparative calculations of the velocities of solid particles and the motion of suspended flows, without additional assumptions about the value of these characteristics of solid dispersed particles. When developing the model, it was assumed that the absolute linear dimensions of the particles do not affect the effective dynamic viscosity of the two-phase mixture, which affect the entries of the equation of particle motion and contact interaction between particles of different classes.

So, as a result of developing a model of the effective dynamic viscosity of magnetite pulps as two-phase mixtures, a method for calculating the effective viscosity of the pulp has been proposed, which takes into account the distribution of particles by size, the compressibility of the liquid in the interparticle space and the influence of the shape of the particles. The obtained relations allow to assess the influence of these factors on the value of the effective viscosity of two-phase mixtures. The model explains the existing scatter of experimental data.

### 3.2. Results of studies of the viscosity of ore suspension at different concentrations and sizes of mineral particles

To confirm the adequacy of the proposed model and to refine the obtained coefficients, the viscosity of pulps at different concentrations

and sizes of mineral particles was experimentally determined, and the influence of temperature and viscosity on the enrichment indicators of iron ores in screw separators was studied.

The results of studies of the viscosity of ore suspension using the first method are given in Table 3.

The large difference in the values of the viscosity of the suspension according to the formulas and according to the experimental data is explained by the influence of the density and specific surface area of the suspension. The correlation matrix and the matrix of the significance of the correlation coefficients are given in Table 4.

**Table 3**  
Results of studies of the viscosity of ore suspension

| Indicators  | Value   |         |         |
|---|---------|---------|---------|
|   | minimum | average | maximum |
| $\eta$ , mPa·s  | 0.69    | 2.31    | 12.4    |
| $\eta_0$ , mPa·s  | 0.68    | 1.0351  | 1.365   |
| $\rho_s$ , kg/m <sup>3</sup>                              | 1072    | 1752    | 3000    |
| $f_{sp}$ , 10 <sup>3</sup> m <sup>2</sup> /m <sup>3</sup> | 4.72    | 113.58  | 484.3   |
| $\Phi_{sol}$ , share of unit                              | 0.155   | 0.54    | 0.91    |
| $\eta/\eta_0$ , un.                                       | 1.015   | 4.545   | 12.4    |

**Table 4**  
Correlation matrix and matrix of the significance of the correlation coefficients

| Indicators    | $\eta$ | $\eta_0$ | $\rho_s$ | $f_{sp}$ | $\Phi_{sol}$ | $\eta/\eta_0$ |
|---------------|--------|----------|----------|----------|--------------|---------------|
| $\eta$        | 51.625 | 0.024    | 0.821    | -0.486   | -0.741       | 0.119         |
| $\eta_0$      | 0.206  | 51.625   | 0.003    | -0.02    | -0.122       | 0.059         |
| $\rho_s$      | 9.774  | 0.025    | 51.625   | -0.669   | -0.938       | 0.038         |
| $f_{sp}$      | -4.47  | -0.166   | -6.6812  | 51.625   | 0.703        | -0.092        |
| $\Phi_{sol}$  | -8.023 | -1.035   | -14.48   | 7.35     | 51.625       | 0.036         |
| $\eta/\eta_0$ | 1.004  | 0.495    | 0.325    | -0.773   | 0.303        | 51.625        |

The multiple dependence of the viscosity coefficient on the viscosity, density of the suspension, the specific surface area of the solid in it and the solid fraction has a correlation coefficient of 0.83 and a significance of 9.8. Taking into account the mutual influence of the indicators, this dependence is as follows:

$$\eta = -0.778 + 0.1\eta_0 + 0.517\rho_s - 0.141f_{sp} + 0.145\Phi - 0.103\rho_s\Phi + 0.582f_{sp}\Phi + 0.811\rho_sf_{sp}. \quad (33)$$

The correlation coefficient of the set (33)  $R = 0.85$ , significance 10.1.

The multiple dependence of the relative viscosity ( $\eta/\eta_0$ ) on the same indicators is weaker than the given dependence: it has a correlation coefficient of 0.26–0.33 and a significance of 2.2–2.8. This is explained by the independence of the viscosity of the liquid from the remaining indicators and their large influence on the viscosity of the suspension.

To confirm the obtained data, the viscosity of the suspension was also measured by the capillary method.

In this case, the viscosity of the suspension  $\mu_s$  was determined depending on the volume concentration of the solid phase in the pulp  $w$  according to the empirical Wanda formula, which is valid for suspensions in which the solid phase is provided by small particles with a size of less than 0.1 mm:

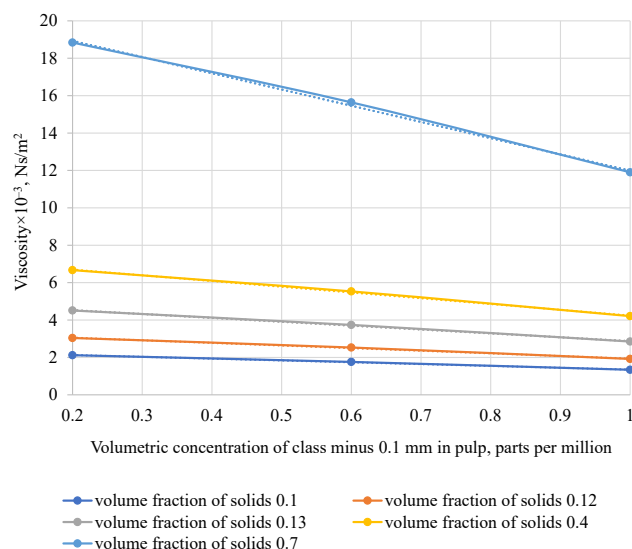
$$\mu_s = \mu_0 (1 + 2.5w + 7.34w^2 + 16.2w^3), \quad (34)$$

where  $\mu_0$  – viscosity of the liquid phase – water, which is equal to 0.001 Ns/m<sup>2</sup>.

The study of the influence of the volume concentration of the solid phase on the viscosity was carried out for an ore suspension with

a size of – 0.1 mm at a mass fraction of solids from 20 to 100 %. The experiments in each mode were repeated 5 times. The relative error in viscosity determinations was less than 3 %.

The results of the research are given in Fig. 1.



**Fig. 1.** Dependence of the viscosity of the suspension on the volumetric concentration of the class minus 0.1 mm in the pulp

The dependence of the viscosity of the suspension on the volumetric concentration of the class minus 0.1 mm in the pulp for different volumetric solid fractions has the form of the following linear equations:

– for 0.7:

$$\mu_s = -8.675 \cdot w_{0.1} + 20.665, R^2 = 0.998;$$

– for 0.4:

$$\mu_s = -3.075 \cdot w_{0.1} + 7.315, R^2 = 0.9982;$$

– for 0.3:

$$\mu_s = -2.075 \cdot w_{0.11} + 4.945, R^2 = 0.9983;$$

– for 0.2:

$$\mu_s = -1.4 \cdot w_{0.1} + 3.3367, R^2 = 0.9973;$$

– for 0.1:

$$\mu_s = -0.975 \cdot w_{0.1} + 2.325, R^2 = 0.998.$$

It was found that the dependence of the pulp suspension on the content of particles with a size of 0.1 mm has an inversely proportional law. For such a particle size in the range of the volume concentration of solids from 10 to 70 %, the viscosity of the suspension increases with a decrease in the volume concentration of the 0.1 mm class from 100 to 20 %.

After mathematical processing of the results, equation (34) can be transformed specifically for the conditions under study:

$$\mu_s = \mu_0 (1 + 2.5w + 7.34w^2 + 16.2w^3) \cdot K_e, \quad (35)$$

where  $K_e$  – correction factor that takes into account the particle size, namely the volume concentration of particles with a size of 0.1 mm, while:

$$K_e = 1.38 - 0.36w_{-0.1} - 0.09w_{-0.1}^2. \quad (36)$$



Therefore, taking into account the introduction of the correction factor, equation (34) can be used for practical determination of viscosity and justification of its influence on the conditions of gravitational enrichment of magnetite ores.

Since screw separators can be installed in magnetite ore processing schemes to improve the quality of magnetite industrial products, the influence of suspension magnetization on its viscosity was further investigated. The relationship between the viscosity of a dispersed system by volume and the volume fraction of the solid phase according to Einstein is taken as a basis – formula (1).

From Einstein's theory it is known that dilute and stable dispersed systems are Newtonian fluids, in which the viscosity is linearly related to the volume fraction of solids and does not depend on the dispersion. The coefficient  $\alpha$  for particles whose shape differs from spherical is usually more than 2.5 due to the fact that the volume of the body of revolution is greater than the volume of the rotating particle.

The presence of magnetically oriented layers of water molecules on the surface of solid particles can be taken into account by increasing the volume fraction of solids by the volume of these layers:

$$\eta_{spe} = 2.5(1+k)\varphi, \quad (37)$$

where  $k = \varphi_0 / \varphi$  – ratio of the volume fraction of the layers to the total volume fraction of the solid phase  $\varphi$  in the suspension.

Since the volume of the surface layers is linearly related to the specific surface area of the solid phase, the viscosity of the suspension should increase with increasing solid dispersion at a constant fraction of it in the dispersed system. The increase in viscosity is due to the increase in concentration:

$$d\eta = \eta_s \cdot \alpha \cdot d\varphi. \quad (38)$$

After separation of variables and integration in the range from zero to the final concentration, let's obtain:

$$\ln\left(\frac{\eta}{\eta_0}\right) \alpha \varphi, \text{ or } \eta = \eta_0 l^{(\alpha\varphi)}, \quad (39)$$

where  $l = 2.7$  – base of natural logarithms.

The most common form of equation (39) is the equation obtained by expanding the exponent in a series:

$$\eta = \eta_0 (1 - \alpha\varphi + \beta\varphi^2 + \dots). \quad (40)$$

Most researchers accept  $\alpha = 2.3$ . But the experimental values of the coefficient  $\beta$  at  $\varphi^2$ , which reproduces the interaction between particles of the solid phase, often do not coincide with the theoretical value  $\beta = \alpha^2 / \varphi = 3.12$ . This is due to the fact that the interactions in the systems are not the same, and therefore the coefficient can be different for different systems in the range from 3 to 75. Large values  $\beta$  are often explained by the presence of surface layers of liquid on solid particles [33].

If in a suspension:

$$Q_{sol} = 1 - Q_{H_2O}, \quad (41)$$

where  $Q_{sol}$  and  $Q_{H_2O}$  – fraction, respectively, of solid and water, then the specific surface area of the solid is:

$$F_{sur} = \frac{6(1-\theta)}{d_r}, \quad (42)$$

where  $F_{sur}$  – surface area of solid particles  $m^2/m^3$  of suspension;  $d_r$  – diameter of particles, m;  $\theta$  – fraction of water in the suspension, part. units.

Accordingly, the diameter of the gaps (channels) between particles in the suspension:

$$d_k = \frac{2d_r\theta}{3(1-\theta)^2}. \quad (43)$$

The ratio of particle sizes to channel sizes is:

$$\frac{d_r}{d_k} = \frac{1.5(1-\theta)^2}{\theta}. \quad (44)$$

The sizes of particles and channels between them are equal at a mass fraction of liquid in the suspension  $\theta = 0.456$ . Taking from the reference [32] the size of a water molecule equal to  $3.85 \cdot 10^{-10}$  m, the area occupied by a water molecule on the surface of a solid particle  $S_{H_2O} = 11.635 \cdot 10^{-20} m^2$  is determined.

Assuming that water molecules in one layer occupy 0.78 of its area on a wetted surface, their mass is calculated:

$$g_{H_2O} = \frac{0.78 \cdot 18}{11.636 \cdot 10^{-20} \cdot 6.0247 \cdot 10^{23}} = 2 \cdot 10^{-7} g/m^2, \text{ or } 2 \cdot 10^{-10} m^3/m^2. \quad (45)$$

Dividing the volume of water per 1  $m^2$  of particle surface by the volume of one layer of molecules, the number of layers of liquid molecules between the particles is determined:

$$n_{H_2O} = \frac{d_r\theta}{12(1-\theta) \cdot 10^{-10}}. \quad (46)$$

The results of calculations according to equation (46) show that the flow obstruction conditions are affected by the content of large and small particles. For example, the same number of layers of water molecules  $175 \cdot 10^3$  for particles with a diameter of 50  $\mu m$  and 5  $\mu m$  is obtained, respectively, at a water content in the suspension of 80 and 98 %. That is, for small particles, obstruction begins at their lower concentrations. This confirms the previously stated and the results of studies on determining the influence of particle size and content less than 0.1 mm. This explains the statement that "the degree of mutual influence of grains in a suspended state depends on their hydraulic dimensions" [22]. Accordingly, the degree of mutual influence of grains in a suspended state increases from large to small. This explains the different increase in the viscosity of suspensions with different grinding sizes. Material with a particle size of 80 % minus 0.044 mm rapidly increases the viscosity of the suspension at concentrations above 40 %, and material with a particle size of 80 % minus 0.031 mm increases the viscosity of the suspension at a mass fraction above 60 %.

Suspensions in which the solid phase has magnetic properties, in addition to wettability, are structured by the magnetic field of the particles, which also cannot but affect the viscosity of the suspensions and this must be borne in mind in order to control the enrichment process.

It should be noted that the viscosity of the liquid phase of the pulp is highly dependent on its temperature. This, in turn, significantly affects the performance of screw separators. Higher pulp viscosity can lead to a decrease in the recovery and quality of the concentrate. Therefore, at the next stage of research, the effect of temperature on pulp viscosity was studied.

### 3.3. Results of the study of the influence of temperature on the pulp viscosity

At the beginning of this cycle of experiments, the viscosity of water and pulp with a particle size of 80 % of the class – 0.033 mm was measured at different temperatures – 60, 50, 40, 30, 20, 10, 5 °C. The measurement results are shown in Fig. 2.

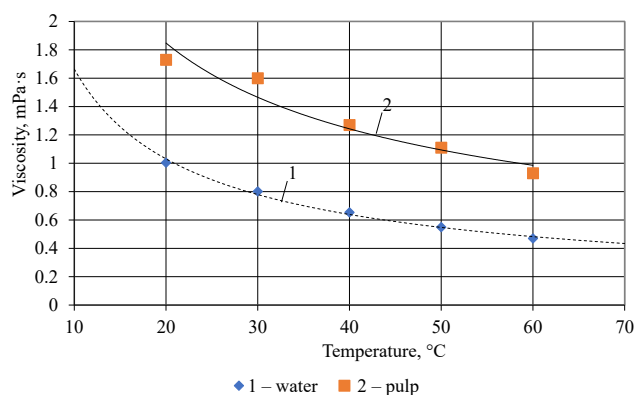


Fig. 2. Effect of water temperature and pulp with a particle size of 80 % of the class –0.033 mm on viscosity

As measurements showed, with increasing temperature, the viscosity of water and pulp decreases. This is explained by the essence of the concept of "viscosity," which characterizes the internal friction of liquids, i. e. the property of fluid bodies to resist the movement of one part relative to another. As a result, the macroscopic work spent on this movement is dissipated in the form of heat. The mechanism of internal friction in liquids is that molecules moving randomly transfer momentum from one layer to another, which leads to equalization of speeds.

Further research was carried out in two directions: 1 – theoretical calculations of the effect of viscosity on the helical surface and 2 – practical experiments at different pulp temperatures, different flow rates and different feed sizes.

Theoretical calculations were based on the fact that in screw separators the movement of particles occurs under the influence of gravitational force and the force of dynamic resistance of the medium, which is described in detail above.

According to the above, the speed of movement of particles of a given size is determined by the density of the medium and the dynamic viscosity coefficient. Taking into account the movement of particles in compressed conditions, their speed is determined as:

$$v_{com} = \frac{24}{d\Delta_{med}} \left( \sqrt{\mu_{com}^2 + \frac{d^3 \Delta_{med} (\rho - \Delta_{med})}{216}} - \mu_{com} \right), \quad (47)$$

where  $v_{com}$  – particle velocity in compressed conditions, m/s;  $d$  – particle size, m;  $\Delta_{med}$  – medium density, kg/m<sup>3</sup>;  $\rho$  – particle density, kg/m<sup>3</sup>;  $\mu_{com}$  – suspension viscosity, Ns/m<sup>2</sup>, which is determined by the dependence (34).

According to equation (47), a decrease in suspension viscosity leads to a significant increase in the pulp flow, and this, in turn, affects the separation of mineral particles on the screw separator. To confirm this conclusion, full-scale experiments were conducted at different flow rates for ore pulp with a size of 0.1 mm and a magnetic industrial product of 0.033 mm with a mass fraction of solids of 40 % and three temperature regimes – 5 °C, 30 °C, 50 °C. In this case, the viscosity of the suspension was measured by the capillary method. The results of the studies are given in Table 5. Technological indicators of gravity enrichment of magnetite pulp are given in Tables 6, 7.

The obtained practical results (Tables 5–7, Fig. 3, 4) confirm the statement that the viscosity of the pulp affects its speed. The practical results confirm the theoretical calculations and the adequacy of the derived mathematical model of the particle speed depending on the viscosity and particle size (47). Thus, when the kinematic viscosity coefficient decreases from 0.0152 to 0.00511 cm<sup>2</sup>/s, the flow speed increases by 1.38 times according to theoretical calculations (from 1.013 to 1.402 m/s), and by 1.33 times according to practical data (from 0.7 to 0.93 m/s). The difference between the theoretical and practical results is on average less than 5 % (3.6 %).

Table 5

Results of experiments on determining the pulp viscosity (mass fraction of solids 40 %) at different speed and temperature regimes (average values)

| Size      | Kinematic viscosity coefficient, cm <sup>2</sup> /s | Theoretical flow rate, m/s | Experimental pulp speed, m/s | Temperature, °C | Total iron recovery in concentrate, % |
|-----------|---|----------------------------|------------------------------|-----------------|---------------------------------------|
| –0.1+0.01 | 0.0152  | 1.013                      | 0.7                          | 32              | 68                                    |
| –         | 0.00511   | 1.402                      | 0.93                         | 32              | 66                                    |
| –0.033+0  | 1.567   | –                          | –                            | 5               | 62                                    |
| –         | 0.8   | –                          | –                            | 28              | 74                                    |
| –         | 0.556   | –                          | –                            | 50              | 66                                    |

Table 6

Technological indicators of gravity enrichment of magnetite pulp with a particle size of –0.1+0.1 mm at different temperatures

| Temperature | Material       | Separation rates, % |                  |             |
|-------------|----------------|---------------------|------------------|-------------|
|             |                | Yield               | Fe mass fraction | Fe recovery |
| 5 °C        | Concentrate    | 40.82               | 63.48            | 60.54       |
|             | Light fraction | 59.18               | 28.54            | 39.46       |
|             | Output product | 100                 | 42.8             | 100         |
| 28–30 °C    | Concentrate    | 50.89               | 69.03            | 82.08       |
|             | Light fraction | 49.11               | 15.62            | 17.92       |
|             | Output product | 100                 | 42.8             | 100         |
| 50 °C       | Concentrate    | 50.90               | 64.5             | 76.71       |
|             | Light fraction | 49.10               | 20.30            | 23.29       |
|             | Output product | 100                 | 42.8             | 100         |

Table 7

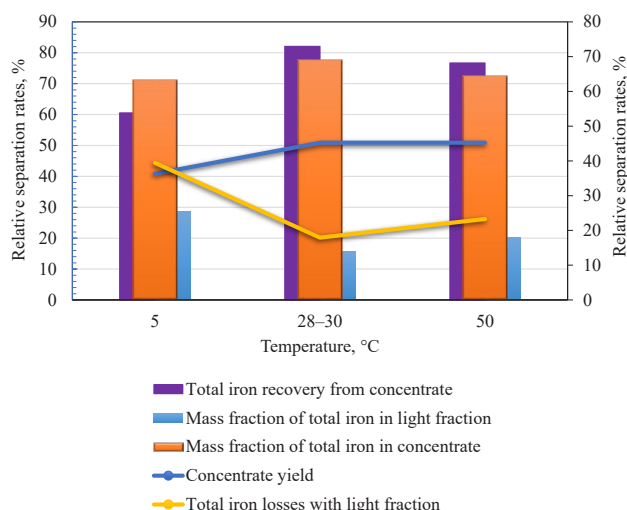
Technological indicators of gravity enrichment of magnetite pulp with a particle size of 80 % of the class – 0.033 mm at different temperatures

| Temperature | Material       | Separation rates, % |                  |             |
|-------------|----------------|---------------------|------------------|-------------|
|             |                | Yield               | Fe mass fraction | Fe recovery |
| 5 °C        | Concentrate    | 61.87               | 63.48            | 62.54       |
|             | Light fraction | 38.13               | 61.70            | 37.46       |
|             | Output product | 100                 | 62.8             | 100         |
| 28–30 °C    | Concentrate    | 68.38               | 68.03            | 74.08       |
|             | Light fraction | 31.62               | 51.49            | 25.92       |
|             | Output product | 100                 | 62.8             | 100         |
| 50 °C       | Concentrate    | 63.00               | 66.5             | 66.71       |
|             | Light fraction | 37.00               | 56.50            | 33.29       |
|             | Output product | 100                 | 62.8             | 100         |

A significant effect of temperature on the viscosity and, accordingly, gravitational enrichment indicators on screw separators of magnetite pulp of different sizes has been established.

As follows from the analysis of the obtained research results, the maximum iron extraction from the concentrate is observed at a temperature of 28–30 °C. At a temperature of 5 °C, the mass fraction and extraction of iron into the concentrate are minimal, both at the initial feed size of –0.1+0.01 mm and at the size of 80 % of the class –0.033 mm.

Thus, when the temperature decreases from 28–30 to 5 °C, the viscosity of the pulp with a size of –0.1+0.1 mm increases by 1.13 times. At the same time, the extraction of total iron into the concentrate decreases by 21.54 % – from 82.08 to 60.54 %. The quality of the concentrate also deteriorates – the mass fraction of total iron in the concentrate decreases from 69.03 % to 63.48 %.

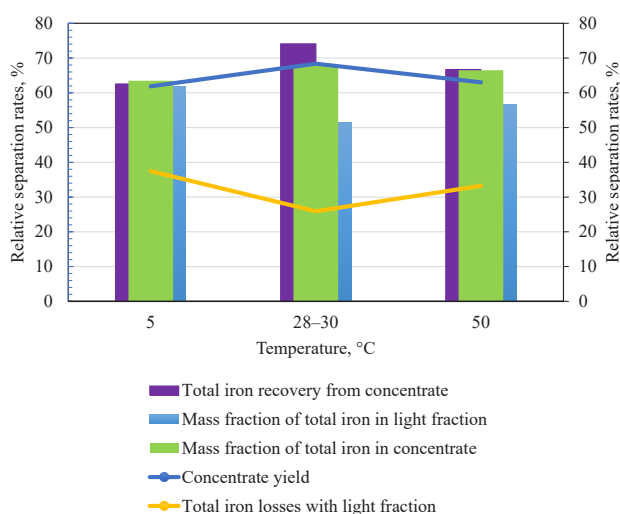


**Fig. 3.** Technological indicators of gravitational enrichment of magnetite pulp with a size of  $-0.1+0.1$  mm with a mass fraction of total iron of 42.8 % at different temperatures

A similar picture is observed for pulp with a size of  $-0.033+0$  mm.

Thus, when the temperature decreases from 28–30 °C to 5 °C, the viscosity of pulp with a size of 80 % of the class  $-0.033$  mm increases by 1.96 times. At the same time, the extraction of total iron into the concentrate decreases by 11.54 % – from 74.08 to 62.54 % with a simultaneous deterioration in the quality of the concentrate – the mass fraction of total iron in the concentrate decreases from 68.03 to 63.48 %.

When the temperature increases from 28–30 to 50 °C, the viscosity of the pulp with a particle size of  $-0.1+0.1$  mm decreases by 1.64 times. At the same time, the extraction of total iron into the concentrate decreases by 5.37 % – from 82.08 to 76.71 %. The quality of the concentrate also deteriorates – the mass fraction of total iron in the concentrate decreases from 69.03 to 64.5 %. When the temperature decreases from 28–30 °C to 50 °C, the viscosity of the pulp with a particle size of 80 % class  $-0.033$  mm decreases by 1.44 times. At the same time, the extraction of total iron into the concentrate decreases by 7.37 % – from 74.08 to 66.71 % with a simultaneous deterioration in the quality of the concentrate – the mass fraction of total iron in the concentrate decreases from 68.03 to 66.5 %.



**Fig. 4.** Technological indicators of gravitational enrichment of magnetite pulp with a particle size of 80 % of the class  $-0.033$  mm with a mass fraction of total iron of 62.8 % at different temperatures

Therefore, the viscosity of the pulp and its temperature have a significant impact on the enrichment process of ore magnetite pulp in

a screw separator. It can be argued that the highest enrichment rates of magnetite ore occur in a certain optimal range of pulp temperature – from 28 to 50 °C.

**Practical significance.** Controlling the temperature of the pulp in production conditions will help to avoid additional losses of valuable components, especially in the winter period, when the technical water has a rather low temperature, and as a result, high viscosity.

**Research limitations.** The use of the research results is limited to the specific conditions for measuring the viscosity of the suspension in industrial conditions. For practical implementation, it is necessary to install additional measuring devices in the production lines and adapt the developed models to the technologies of enrichment of ores of different sizes.

Today, *under martial law in Ukraine*, conducting additional research is complicated. This is due to factors such as restrictions on the use of electricity, disruption of logistics for shipping commodity concentrates to European and Asian countries, and in general, the suspension of the work of mining and enrichment plants. At the same time, the work of research laboratories is also disrupted. The performance of chemical analyzes and technological research is practically stopped.

**Further research** is aimed at expanding the experimental base, refining mathematical models and taking into account additional factors that affect the effective viscosity of suspensions. The use of modern methods of numerical modeling and improving technological approaches to optimize the composition and rheological properties of suspensions in different conditions is promising.

#### 4. Conclusions

As a result of the analysis of models in which the relationship between the structure and viscosity of dispersed systems is established, it is shown that the theoretical justification of rheological equations is microrheological models. These models link rheological properties with the nature of particle motion in a viscous liquid, forces of attraction and repulsion, and hydrodynamic interaction between particles.

It has been established that the main factors affecting the flow of suspensions include:

- Brownian motion of particles;
- interparticle forces of attraction and repulsion;
- hydrodynamic interaction between particles, which occurs at high volume concentrations, when the flow field around one particle interacts with the flow field around a neighboring particle.

The viscosity of an aggregated suspension is much higher than the viscosity of the same suspension, but divided into individual particles.

It is shown that the theory that takes into account the effect of particle size as a function of the average effective diameter of pulp particles deserves special interest. Since in most of the developed theoretical models the viscosity coefficient depends on the volume concentration and does not take into account the effect of particle size. Although various experimental studies of coarse-dispersed suspensions used as a medium for mineral enrichment have shown that the effective viscosity is affected by both the concentration and the shape of the suspended particles and its size.

It is shown that the idea of the effective dynamic viscosity of two-phase mixtures of mineral particles and liquid at a constant volume concentration of the solid phase is determined by the maximum volume concentration for the actual granulometric, which is physically incorrect. This is explained by the fact that there are very significant differences in these values, and depending on the particle size distribution they can vary by more than 1.5–2 times.

It is established that the excess viscosity of the suspension is proportional not only to the volume fraction of the solid phase, but also inversely proportional to the volume fraction of the free liquid.

It is proved that at a constant value of the volume fraction of solid particles in magnetite pulp, as a two-phase mixture of solid and liquid, the value of its effective dynamic viscosity depends on the characteristics of solid particles. It is determined by the polymodal granulometric composition and shape of particles and can vary by tens of percent, reaching multiple values.

A method for calculating the effective viscosity of the pulp for magnetite pulps, as two-phase mixtures of mineral particles and liquid, is proposed. The method takes into account the distribution of particles by size, the compressibility of the liquid in the interparticle space, the influence of the shape of the particles, which allows to assess the influence of these factors on the value of the effective viscosity of the suspension, and to explain the large scatter of experimental data.

An inversely proportional dependence of the viscosity of the pulp suspension on the content of particles with a size of 0.1 mm has been established. For this size in the range of the volume concentration of solids in the pulp from 10 to 70 %, the viscosity of the suspension increases with a decrease in the volume concentration of the 0.1 mm class from 100 to 20 %.

It is shown that the degree of mutual influence of grains in the suspended state increases from large particles to small ones. This allows to form conditions for reducing the mutual influence of particles on each other in the flow on the surface of the screw separator, namely, for small particles with a size of less than 0.1 mm, difficulty begins at lower concentrations.

The viscosity of magnetite suspensions increases with a decrease in the grinding size in different ways and depends on the mass fraction of solids. Material with a size of 80 % minus 0.044 mm quickly increases the viscosity of the suspension at concentrations above 40 %. Material with a particle size of 80 % minus 0.031 mm increases the viscosity of the suspension at a mass fraction above 60 %.

It has been proven that the viscosity of the pulp and its temperature have a significant impact on the enrichment process of ore magnetite pulp in a screw separator. The highest enrichment rates of magnetite ore occur in a certain optimal range of pulp temperature – from 28 to 50 °C. Controlling the temperature of the pulp in production conditions will help to avoid additional losses of valuable components, especially in winter, when the process water has a rather low temperature, and as a result, high viscosity.

## Conflict of interest

The authors declares that they have no conflict of interest in relation to this study, including financial, personal, authorship or other, which could affect the study and its results presented in this article.

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## Data availability

Manuscript has no associated data.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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