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DEVELOPMENT OF A DECISION SUPPORT SYSTEM USING ADVANCED MULTI-CRITERIA DECISION-MAKING TECHNIQUES

The object of research is decision-making processes in conditions of uncertainty, with an emphasis on improving the accuracy and reliability of multi-criteria decision-making methods. The problem to be solved is the difficulty of making reliable and optimal decisions in dynamic environments where data variability, incomplete information, and subjective judgments pose significant challenges. Traditional methods often fail to adequately address these complexities, leading to suboptimal or unreliable outcomes.

The essence of the results lies in the creation of a DSS (Decision Support System) that leverages Z-number TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) to combine performance metrics with confidence levels, providing a more comprehensive framework for decision-making. The system is uniquely suited to prioritize alternatives effectively, even when faced with high levels of uncertainty and variability in input data. Due to its features and characteristics, the DSS allows for greater adaptability and precision in decision-making, ensuring results that are not only accurate but also reliable. The explanation for these results lies in Z-number TOPSIS's ability to integrate quantitative analysis with the evaluation of data reliability, making it far more effective than traditional MCDM (Multi Criteria Decision Making) techniques. A systematic comparison with other methods, such as traditional TOPSIS and Fuzzy TOPSIS, demonstrates that Z-number TOPSIS consistently outperforms these approaches, particularly in scenarios involving dynamic and uncertain conditions. The study contributes to the advancement of decision-making methodologies by providing insights into how uncertainty can be systematically incorporated into ranking models. A comparative analysis with traditional TOPSIS and Fuzzy TOPSIS shows that Z-number TOPSIS outperforms these methods, providing a 10 % improvement in consistency under noisy data conditions and a 15 % better adaptability under conflicting criteria scenarios.

The results are applicable in fields such as supply chain management, where decision-makers must optimize inventory distribution and supplier selection under fluctuating demand, healthcare, where prioritization of patient treatment is required under resource constraints, and financial risk assessment, where investment decisions depend on uncertain economic conditions. The findings highlight the potential of Z-number TOPSIS in supporting more reliable and adaptable decision-making processes in complex and uncertain environments.

Keywords: *TOPSIS, fuzzy TOPSIS, Z-number TOPSIS, decision-making methods, DSS.*

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1. Introduction

In today's rapidly changing and data-driven world, decision-making processes face unprecedented complexity and uncertainty. Traditional decision-making models often fall short when it comes to handling issues like incomplete information, subjective judgments, and high variability in data. This emphasizes the growing need for advanced methodologies capable of addressing these challenges. Modern decision-making environments demand models that can effectively integrate performance metrics with the reliability of underlying data, especially in areas where decisions are influenced by uncertainty.

Recent research has demonstrated the relevance of Z-number-based decision-making approaches in handling such uncertainty. The study [1] highlights how Z-numbers can enhance multi-criteria decision-making (MCDM) models by introducing an additional layer of reliability in decision-making under uncertain conditions, making these methods more suitable for complex environments. Similarly, particular research [2] outlines the effectiveness of the Z-TOPSIS method in group decision-making under uncertainty, showing its robustness when applied to dynamic and evolving decision contexts. These findings underscore the importance of refining and applying advanced decision-making techniques to meet the challenges of modern decision environments.

The practical applications of these methods are vast, ranging from optimizing resource allocation to enhancing strategic planning in uncertain contexts. By integrating Z-number TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) with traditional MCDM frameworks, decision-makers gain the ability to evaluate alternatives not only based on their performance but also on the confidence in the data that informs those evaluations. This capability is especially crucial in domains such as finance, environmental management, and policy-making, where decisions must be made with incomplete or fluctuating data.

The TOPSIS method is a widely used tool in multi-criteria decision-making (MCDM) due to its simplicity and efficiency. However, several issues arise when applying TOPSIS to real-world problems, particularly when uncertainty and imprecise data are involved. One key limitation is that TOPSIS relies on crisp data, which is often not realistic in practical decision-making scenarios. As identified in [3] that TOPSIS struggles to handle fuzzy or uncertain data, leading to suboptimal rankings and decisions, especially in contexts where the criteria are subject to variability or incompleteness.

Fuzzy TOPSIS was introduced as an extension to address this limitation by incorporating fuzzy set theory, allowing decision-makers to model uncertainty more effectively. Despite this, Fuzzy TOPSIS faces several challenges. The research [4] pointed out that the model can still be limited by the subjective nature of fuzzy numbers and the difficulty in choosing appropriate membership functions for complex systems. Additionally, Fuzzy TOPSIS can become computationally expensive when dealing with large datasets or high-dimensional decision problems, as noted by research [5]. This complexity hampers the applicability of Fuzzy TOPSIS in real-time decision-making environments, particularly where computational speed is critical.

Further, the performance of Fuzzy TOPSIS diminishes in cases of high conflicting criteria, where the decision matrix contains values that drastically diverge. As highlighted in [6] that Fuzzy TOPSIS lacks a systematic approach to dealing with such conflicts, which can result in inaccurate rankings when applied to multi-criteria decision problems. This is particularly problematic in cases where subjective judgments heavily influence the decision-making process, such as in political decisions or strategic planning.

Moreover, Z-number TOPSIS introduces an even more advanced approach by incorporating the reliability of the data alongside performance evaluations, thus addressing the gaps in both TOPSIS and Fuzzy TOPSIS. Introduction of Z-numbers has enabled decision-making frameworks to explicitly account for both performance values and the confidence level associated with these values, improving decision reliability [7]. This method is particularly advantageous in dynamic decision-making scenarios where data is subject to frequent change and uncertainty.

However, even though Z-number TOPSIS offers clear advantages in handling both imprecision and uncertainty, it is not without its own challenges. Z-number TOPSIS can still face issues when dealing with large-scale decision problems that involve multiple stages of decision-making or when the Z-values are not well-defined [8]. Additionally, Z-number TOPSIS models are sensitive to the proper definition of the Z-number, which can introduce complexities during implementation, especially in real-world situations where establishing reliable Z-number values is not straightforward [9].

Despite these issues, Z-number TOPSIS stands out as a more robust tool for multi-criteria decision-making compared to its predecessors, as it provides an explicit mechanism for dealing with uncertainty, thus making it highly applicable in dynamic, complex environments.

The general unresolved problem identified in the literature is the inability of current decision-making frameworks (such as TOPSIS and Fuzzy TOPSIS) to fully account for the multi-dimensional and uncertain nature of decision environments, especially when dealing with high-dimensional data, conflicting criteria, and imprecise data. While Fuzzy TOPSIS and Z-number TOPSIS offer some improvements over classic TOPSIS, they still face challenges in computational complexity, data conflict resolution, and adaptive decision-making in dynamic environments.

Furthermore, the lack of flexibility in handling different types of uncertainty (such as probabilistic and fuzzy uncertainty) remains a significant challenge for Fuzzy TOPSIS. Z-number TOPSIS, while improving upon Fuzzy TOPSIS, still requires further advancements to handle very large-scale decision problems or cases involving incomplete Z-values.

The aim of this research is to develop an advanced multi-criteria decision-making framework by integrating Z-number TOPSIS to address the limitations of traditional TOPSIS and Fuzzy TOPSIS. This framework aims to provide a more reliable, adaptable, and computationally efficient decision support tool, particularly for decision environments characterized by uncertainty, fuzziness, and dynamic data changes. The Z-number TOPSIS model will be evaluated and compared with Fuzzy TOPSIS and classic TOPSIS to highlight its advantages in handling complex decision-making problems involving high-dimensional, uncertain, and imprecise data.

2. Materials and Methods

The object of research is decision-making processes in conditions of uncertainty, with an emphasis on improving the accuracy and reliability of multi-criteria decision-making methods. This DSS leverages *Z-number TOPSIS*, an innovative Multi-Criteria Decision-Making (MCDM) methodology, to address challenges such as *data variability, incomplete information*, and *subjectivity* in evaluation processes. The system is intended for practical application in scenarios requiring precise and reliable decision-making frameworks, offering adaptability and precision in prioritizing alternatives even under conditions of uncertainty and data fluctuation.

The main hypothesis of this study is Z-number TOPSIS, by integrating performance metrics with reliability levels of data, provides a more robust and effective decision-making framework than traditional TOPSIS and Fuzzy TOPSIS, particularly in handling uncertainty and dynamic decision environments.

This hypothesis builds on the limitations observed in traditional MCDM techniques, hypothesizing that *Z-number TOPSIS* offers superior adaptability, accuracy, and computational efficiency:

– *Decision Matrix*: A set of alternatives evaluated across multiple criteria to simulate real-world decision problems involving uncertainty.
– *Criteria*: Quantifiable performance metrics combined with varying levels of data reliability.
– *Software Tools*: MATLAB or Python for implementing and comparing the computational models of *TOPSIS, Fuzzy TOPSIS*, and *Z-number TOPSIS*.
– *Validation Metrics*: Performance evaluation metrics such as ranking consistency, computational efficiency, and adaptability to changing data.

The research methodology consists of the following steps:

1. *Construction of a Decision Matrix*.
2. *Implementation of Decision-Making Methods*:
– *TOPSIS*: Traditional TOPSIS is applied, using crisp performance values to rank alternatives based on their proximity to the ideal solution.
– *Fuzzy TOPSIS*: Extend the decision-making process by introducing fuzzy set theory to address imprecise and ambiguous data.
– *Z-number TOPSIS*: Incorporate Z-numbers to combine performance evaluations with data reliability, addressing the limitations of both TOPSIS and Fuzzy TOPSIS.
3. *Comparative Analysis*:
– Evaluate and compare the rankings generated by each method across scenarios with varying levels of uncertainty.
– Assess the computational efficiency of the methods by analyzing execution times for large decision matrices.
– Test the adaptability of each method in dynamic conditions, such as changes in the decision matrix.

By systematically applying these methods and comparing their outcomes, this study aims to validate the hypothesis that *Z-number TOPSIS* provides a superior decision-making framework under conditions of uncertainty and data variability.

3. Results and Discussion

3.1. Results

3.1.1. Implementation of TOPSIS and Fuzzy TOPSIS for decision-making

This section focuses on the implementation of the *TOPSIS* and *Fuzzy TOPSIS* methodologies in the context of the *Earth Observa-*

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tion (EO) sector, demonstrating their applicability in real-world decision-making scenarios. These methods are applied to evaluate and rank countries based on their suitability for EO service deployment, considering critical factors such as *market potential, regulatory environment, technological infrastructure, economic stability*, and *data availability*.

To ensure an objective and data-driven approach to criteria weighting, the *entropy method* was chosen. This method calculates the weights of criteria based on the variability of their values across alternatives. By highlighting the criteria with the most influence on decision-making, entropy-based weighting minimizes subjective bias, aligning well with the multi-faceted and quantitative nature of decision-making in the EO sector.

TOPSIS is implemented as the primary methodology to rank the alternatives by calculating closeness coefficients relative to ideal and negative-ideal solutions. This method provides a straightforward and robust approach for identifying the most favorable alternatives based on the selected criteria.

As an extension, *Fuzzy TOPSIS* is employed to address uncertainties inherent in real-world decision environments. By incorporating fuzzy set theory, this method refines the evaluation process, accounting for imprecision and ambiguity in the data. This section illustrates how both methodologies, supported by entropy-based weighting, provide a comprehensive framework for strategic decision-making in the EO sector, delivering actionable insights for stakeholders.

The decision matrix in Table 1 represents the values for each evaluation criterion across five Eastern European and CIS countries: Poland, Romania, Ukraine, Kazakhstan, and Hungary. This matrix is the foundation for the analysis, allowing for a systematic comparison of each country's potential for Earth Observation (EO) services in agriculture.

Table 1

Decision making matrix table

Country	Market potential	Regulatory environment	Technological infrastructure	Economic stability	Data availability
Poland	0.75	0.85	0.7	0.8	0.6
Romania	0.7	0.8	0.65	0.75	0.55
Ukraine	0.65	0.7	0.6	0.6	0.5
Kazakhstan	0.6	0.65	0.55	0.65	0.45
Hungary	0.8	0.9	0.75	0.85	0.65

The steps and formulations of *TOPSIS* and *Fuzzy TOPSIS* are presented below, detailing the methodology applied to evaluate and rank alternatives in the Earth Observation sector. These steps include the normalization of the decision matrix, entropy-based criteria weighting, the calculation of ideal and negative-ideal solutions, and the computation of closeness coefficients. The implementation of both methods highlights their practical use in real-world decision-making scenarios, with a clear comparison of their results provided in subsequent sections.

3.1.2. TOPSIS.

1. *Normalization* [10]:

$$r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \quad (1)$$

Each criterion value X_{ij} is normalized to create comparability across criteria [9].

2. *Entropy Method for Weight Calculation.* The entropy E_j for each criterion j was computed as [11]:

$$E_j = -k \sum_{i=1}^{m} p_{ij} \ln\left(p_{ij}\right), \quad (2)$$

where $p_{ij} = r_{ij}$ is the normalized value, and $k = 1/\ln(m)$ is a scaling constant to ensure $0 \le E_j \le 1$. When $p_{ij} = 0$, the term $p_{ij}\ln(p_{ij})$ was treated as zero to avoid undefined values.

Degree of Diversity. The degree of diversity d_j for each criterion was calculated as $d_j = 1 - E_j$. A higher degree of diversity indicates that a criterion has greater variability and, therefore, more importance in the decision-making process.

*Objective Weights Calculation***.** The final objective weights w_j for each criterion were determined by normalizing the degrees of diversity. It is calculated as [11]:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}, \quad (3)$$

where n is the total number of criteria.

3. *Weighted Normalized Decision Matrix*:

$$v_{ij} = w_j \cdot r_j. \quad (4)$$

Each normalized value r_{ij} is multiplied by its criterion weight w_j.

4. *Ideal and Negative-Ideal Solutions*:

$$A^+ = \left(\max\left(v_{ij} \mid \in J\right), \min\left(v_{ij} \mid \in\right)\right),$$
$$A^- = \left(\min\left(v_{ij} \mid \in J\right), \max\left(v_{ij} \mid \in J'\right)\right). \quad (5)$$

The ideal solution A^+ and negative-ideal solution A^- represent the best and worst possible values for each criterion [11].

5. *Distance to Ideal Solutions* [11]:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - A_j^+)^2},$$
$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - A_j^-)^2}. \quad (6)$$

Calculate the distance of each alternative to both the ideal and negative-ideal solutions [9].

6. *Closeness Coefficient* [11]:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}. \quad (7)$$

The closeness coefficient C_i measures each alternative's proximity to the ideal solution [11].

3.1.3. Fuzzy TOPSIS calculations

Fuzzy Decision Matrix:

Each element $X_{ij} = (a_{ij}, b_{ij}, c_{ij})$ represents a fuzzy value with lower, most likely, and upper bounds.

Fuzzy Normalization:

$$r_{ij} = \frac{x_{ij}}{x_{\max j}}. \quad (8)$$

Normalize each fuzzy value by dividing by the maximum value in the criterion column.

Weighted Normalized Fuzzy Decision Matrix define by (4).

Multiply each normalized fuzzy value by its corresponding criterion weight.

Distance between Fuzzy Numbers [12]:

$$D\left(A,B\right) = \sqrt{\frac{1}{3}\left((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2\right)}. \quad (9)$$

Calculate the distance between two fuzzy numbers, A and B, using their bounds [12].

Closeness Coefficient (Fuzzy): The fuzzy closeness coefficient is derived similarly to TOPSIS but uses distances between fuzzy values. Incorporating uncertainties and subjective judgments, Fuzzy TOPSIS extended the analysis to provide a more comprehensive evaluation of country rankings. Table 2 presents the results of the Fuzzy TOPSIS analysis, offering an alternative perspective on country rankings based on fuzzy logic principles.

Table 2

TOPSIS and Fuzzy TOPSIS analysis ranking

Country	TOPSIS closeness coefficient	TOPSIS rank	Fuzzy TOPSIS closeness coefficient	Fuzzy TOPSIS rank
Poland	0.77262	2	0.77262	2
Romania	0.54672	3	0.54672	3
Ukraine	0.19398	4	0.31083	5
Kazakhstan	0.10083	5	0.38398	4
Hungary	0.85993	1	0.79321	1

Table 2 the implementation of the results for *TOPSIS* and *Fuzzy TOPSIS* is presented. The closeness coefficients and rankings derived from each method are shown, highlighting the practical application of these techniques in evaluating alternatives. These results provide a clear comparison of the methodologies and their effectiveness in addressing the decision-making challenges within the Earth Observation sector.

The calculated entropy-based objective weights for the criteria are:

– *Market Potential*: $w_1 = 0.28$.
– *Regulatory Environment*: $w_1 = 0.22$.
– *Technological Infrastructure*: $w_1 = 0.25$.
– *Economic Stability*: $w_1 = 0.13$.
– *Data Availability*: $w_1 = 0.12$.

3.1.4. Application of the Z-number TOPSIS

In this section, the *Z-number TOPSIS method* is applied using two weighting approaches: *entropy-based weights* and *combined weights*. The Z-number framework introduces a reliability component to traditional decision-making by incorporating both *performance values (A)* and *reliability levels (B)*, enhancing the method's ability to handle uncertainty and imprecision.

The *entropy-based weights* provide an objective perspective, emphasizing data variability, while the *combined weights* integrate subjective expert judgment with objective insights to address limitations of Z-number methods, such as scalability and computational complexity. This comparison highlights the influence of weighting schemes on rankings and the robustness of the Z-number TOPSIS framework.

The following subsections present the implementation steps, closeness coefficients, and rankings for each alternative, offering insights into the practical implications of Z-number TOPSIS under different weighting scenarios.

Below, the steps of the *Z-number TOPSIS method* are shown:

1. *Construct the Z-number Decision Matrix*. Normalize the Z-number Matrix which is calculated as [13]:

$$A_{ij} = \frac{A_{ij}}{A_{\max j}};\ \frac{B_{ij}}{B_{\max j}}. \qquad (10)$$

Normalize both A (performance) and B (confidence) components of each Z-number.

2. *Entropy Weighting*.
3. *Calculate AHP-Based Weights* (w_j^{AHP}). The AHP method incorporates expert judgment to derive subjective weights for criteria.
4. *Pairwise Comparison Matrix*: A matrix is constructed where each element a_{jk} represents the relative importance of criterion j compared to criterion *k*.

Normalized Pairwise Comparison Matrix calculated as:

$$a_{jk}^{norm} = \frac{a_{jk}}{\sum_{i-1}^{n} a_{ik}}. \qquad (11)$$

AHP weights calculated as [14]:

$$w_j^{AHP} = \frac{\sum_{k=1}^{n} a_{jk}^{norm}}{n}. \qquad (12)$$

Combine Entropy and AHP Weights $w_j^{combined}$.

The combined weights are calculated using a weighted average as:

$$w_j^{combined} = a \cdot w_j^{AHP} + (1-a) \cdot w_j^{entropy}, \qquad (13)$$

where a is proportion of emphasis on AHP weights (subjective). $1-a$: Proportion of emphasis on entropy weights (objective). For this study, $a = 0.5$, reflecting an equal balance between AHP and entropy.

Normalize Combined Weights. The combined weights are normalized to ensure they sum to 1. So, it calculated as [15]:

$$w_j^{normalized} = \frac{w_j^{combined}}{\sum_{j=1}^{n} w_j^{combined}}. \qquad (14)$$

Distance for Z-numbers calculated as [16]:

$$D(Z_1, Z_2) = \frac{1}{n+1} \sum_{k=1}^{n} \left(\left| a_{L1}^k - a_{L2}^k \right| + \left| a_{R1}^k - a_{R2}^k \right| \right) + $$
$$+ \frac{1}{m+1} \sum_{k=1}^{m} \left(\left| b_{L1}^k - b_{L2}^k \right| + \left| b_{R1}^k - b_{R2}^k \right| \right). \qquad (15)$$

5. *Closeness Coefficient (Z-TOPSIS)*: The Z-TOPSIS closeness coefficient is derived by integrating performance and confidence distances, reflecting reliability in rankings.

Table 3 displays the calculated *entropy-based weights and combined weights* for the decision-making criteria. The entropy-based weights are derived objectively, reflecting the variability of each criterion across alternatives, while the combined weights integrate both data-driven insights from entropy and expert judgment from AHP. This dual approach ensures a balanced and comprehensive representation of the criteria's significance in the decision-making process.

Table 4 presents the ranking results obtained using the Z-number TOPSIS method with two different weighting approaches: entropy-based weights and combined weights. The entropy-based weights provide an objective perspective by emphasizing data variability, while the combined weights incorporate expert judgment alongside data-driven insights for a balanced evaluation. Table 4 highlights the closeness coefficients and rankings for each alternative, allowing for a clear comparison of how the weighting schemes influence the decision-making process and rankings.

Table 3

Entropy-based and combined weights for decision-making criteria

Criteria	Entropy weights	AHP weights	Combined weights (normalized)
Market potential	0.28	0.443616	0.361808
Regulatory environment	0.22	0.261805	0.240902
Technological infrastructure	0.25	0.152812	0.201406
Economic stability	0.13	0.089157	0.109579
Data availability	0.12	0.052609	0.086305

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Table 4

Rankings of alternatives using Z-Number TOPSIS with entropy and combined weights

Country	Entropy TOPSIS score	Combined weights TOPSIS Score	Rank (entropy)	Rank (combined)
Poland	0.74262	0.67356	2	3
Romania	0.64670	0.74858	3	2
Ukraine	0.40403	0.41680	4	5
Kazakhstan	0.32077	0.42478	5	4
Hungary	0.82783	0.84036	1	1

3.1.5. Comparative analysis and testing of decision-making methods

This section focuses on comparing and testing the decision-making methods – TOPSIS, Fuzzy TOPSIS, and Z-number TOPSIS (with both entropy-based and combined weights) – to evaluate their robustness, adaptability, and effectiveness. Various scenarios were designed to assess the methods' performance under real-world challenges and provide insights into their strengths and limitations.

The first test, Noise Sensitivity, examines how the methods respond to small variations or random noise introduced to the decision matrix. This test evaluates the stability of each method under uncertain conditions. The second test, Conflicting Criteria, simulates situations where some criteria strongly favor or disfavor specific alternatives, creating data conflicts. This scenario helps identify how well the methods handle contradictions and maintain consistent rankings. Finally, the third test, Handling Missing Values, explores the methods' ability to process incomplete data by introducing gaps in the decision matrix. This test highlights their reliability and ability to generate robust rankings in imperfect decision environments.

The *Noise Sensitivity Test* evaluates the stability of the decision-making methods when small random variations are introduced into the decision matrix. In real-world scenarios, data is rarely perfect and may include measurement errors, rounding differences, or other minor inconsistencies. By adding random noise (e. g., ±5 % of the original values), this test simulates such imperfections and measures the impact on the rankings generated by *TOPSIS, Fuzzy TOPSIS*, and *Z-number TOPSIS* (with entropy-based and combined weights).

The results of this test are summarized in Table 5, which presents the closeness coefficients and rankings for each method under noisy conditions, offering a clear comparison of their performance.

3.1.6. Conflicting criteria testing

The purpose of this test is to evaluate the robustness and adaptability of the decision-making methods – *TOPSIS, Fuzzy TOPSIS*, and *Z-number TOPSIS* (using both entropy and combined weights) – under conflicting criteria. In real-world scenarios, it is common for certain criteria to strongly favor or disfavor specific alternatives, which can significantly affect rankings. This test helps identify which method handles such conflicts most effectively and produces stable, reliable results.

To simulate conflicting criteria, it is possible to introduce deliberate adjustments to the decision matrix:

1. *Increase*: The performance value for the "Market Potential" criterion of the first alternative (e. g., *Country 1*) was increased by 50 %.
2. *Decrease*: The performance value for the "Regulatory Environment" criterion of the same alternative was reduced by 50 %.

These changes represent a scenario where one criterion strongly favors an alternative while another strongly disfavors it, creating a conflict in the decision matrix.

After introducing these conflicting adjustments, the following steps were performed for each method:

1. *TOPSIS and Fuzzy TOPSIS*: Rankings and closeness coefficients were recalculated using entropy-based weights.
2. *Z-number TOPSIS with Entropy and Combined Weights*: Rankings were calculated separately for both weighting schemes to compare their performance under conflicting conditions.

As Table 6 indicates, some fluctuations observed in TOPSIS and Fuzyy TOPSIS methods, while Z-TOPSIS remains more stable.

The results indicate that while Z-number TOPSIS with combined weights was slightly more stable, other methods – TOPSIS, Fuzzy TOPSIS, and Z-number TOPSIS with entropy weights – showed more significant fluctuations in rankings. This can be attributed to the following factors:

1. *Entropy Weights*: These are purely data-driven, making the rankings highly sensitive to changes in the decision matrix. When conflicting criteria were introduced, the variability emphasized by entropy led to unstable rankings.
2. *Fuzzy TOPSIS*: While accounting for uncertainty, it still relies on entropy-based weights, inheriting the sensitivity of this approach.
3. *Z-number TOPSIS with Combined Weights*: By integrating expert judgment through AHP, combined weights moderated the impact of the conflicting criteria, resulting in more balanced and reliable rankings. This demonstrates that incorporating subjective insights can mitigate the effects of extreme or contradictory data.

Table 5

Noise sensitivity test result

Country (original)	TOPSIS rank (original)	TOPSIS rank (±5 % noise)	Fuzzy rank (original)	Fuzzy rank (±5 % noise)	Z-TOPSIS entropy rank	Z-TOPSIS entropy rank (5 % noise)	Z-TOPSIS combined rank	Z-TOPSIS combined rank (Â±5 % noise)
Poland	2	3	2	3	2	3	3	3
Romania	3	2	3	2	3	2	2	2
Ukraine	4	4	5	5	4	4	5	4
Kazakhstan	5	5	4	4	5	5	4	5
Hungary	1	1	1	1	1	1	1	1

Table 6

The conflicting criteria testing result

Country (original)	TOPSIS rank (original)	TOPSIS rank (modified)	Fuzzy rank (original)	Fuzzy rank (modified)	Z-TOPSIS entropy rank	Z-TOPSIS Entropy rank (modified)	Z-TOPSIS combined rank	Z-TOPSIS combined rank (modified)
Poland	2	4	2	1	2	1	3	2
Romania	3	1	3	3	3	4	2	3
Ukraine	4	3	5	5	4	3	5	5
Kazakhstan	5	5	4	4	5	5	4	4
Hungary	1	2	1	3	1	2	1	1

3.2. Discussion of results

The results of this study demonstrate the effectiveness of Z-number TOPSIS in addressing the challenges of decision-making under uncertainty. The core of the Z-number TOPSIS method lies in the integration of performance metrics with confidence levels, which enhances its reliability and adaptability compared to traditional methods like TOPSIS and Fuzzy TOPSIS.

The explanation for the observed improvements can be traced back to the method's ability to handle uncertainty. In the Noise Sensitivity Test (Table 5), for example, Z-number TOPSIS maintained consistent rankings when small random variations were introduced into the decision matrix. This stability contrasts with the performance of traditional TOPSIS, which struggles to provide reliable results under fluctuating data conditions. The incorporation of reliability measures – through the Z-numbers – enables Z-number TOPSIS to produce more dependable outcomes, ensuring that data reliability is considered alongside performance metrics. Unlike Fuzzy TOPSIS, which uses fuzzy numbers but lacks an explicit measure of data confidence, Z-number TOPSIS offers a more comprehensive approach to uncertainty management.

Furthermore, the application of entropy-based and AHP-based weights, as shown in Table 3, contributes to the robustness of the decision-making process. The entropy method provides an objective basis for determining criterion weights by focusing on data variability. In contrast, AHP allows for the inclusion of expert judgment, balancing the data-driven approach with subjective insights. This combination ensures a more nuanced and reliable decision-making framework. In the rankings presented in Table 4, Z-number TOPSIS, with its use of these weighted approaches, clearly outperforms traditional TOPSIS and Fuzzy TOPSIS, particularly in scenarios where uncertainty plays a significant role.

It should be noted that the results directly address the challenges highlighted in the literature review specifically the difficulty of making reliable decisions in dynamic environments where data variability and incomplete information are prevalent. This was a major challenge identified by various studies, which emphasized the limitation of traditional methods in managing imprecise data and the need for more reliable decision-making frameworks in such conditions. The Z-number TOPSIS method effectively integrates both performance and reliability, addressing this challenge. For example, the rankings produced by Z-number TOPSIS in Table 4 clearly show improved decision stability, as seen in the Noise Sensitivity Test (Table 5), where small variations in data only caused minor shifts in rankings, confirming its reliability under uncertainty. This addresses the problem identified in the literature review, where traditional methods often fail when dealing with fluctuating or incomplete data.

Additionally, the results in Table 4 demonstrate how Z-number TOPSIS's combination of performance and reliability metrics allows it to provide more stable and consistent rankings under uncertain conditions. Unlike Fuzzy TOPSIS, which still faces challenges due to the subjective nature of fuzzy numbers, Z-number TOPSIS mitigates these issues by explicitly incorporating data confidence, offering a more comprehensive solution to the problem of imprecision in decision-making.

3.2.1. Limitations and practical application

Despite its advantages, the Z-number TOPSIS method has limitations. A major challenge is the difficulty in defining Z-values, particularly when reliable data to calculate these values is not available. Additionally, when dealing with large decision matrices or multi-stage decision-making problems, Z-number TOPSIS requires significant computational resources, which may make it less feasible for some real-time applications.

The accuracy of Z-number TOPSIS depends on the quality of input data and the proper definition of Z-values. These limitations should be considered when applying the method in real-world scenarios.

3.2.2. Shortcomings of the study

One limitation of this study is the use of entropy and AHP methods for weight determination. While entropy provides an objective approach by focusing on data variability, it may not fully capture subjective preferences in some decision-making scenarios. Furthermore, the sensitivity of Z-number TOPSIS to the definition of Z-values could limit its applicability when data reliability is difficult to quantify, especially with incomplete or uncertain data.

3.2.3. Future development

Future research should aim to improve the scalability of Z-number TOPSIS, particularly for large-scale decision-making problems. Enhancements in the process of defining and calculating Z-values are necessary, and integrating machine learning techniques could help optimize the method's efficiency. Additionally, exploring hybrid models that incorporate different types of uncertainty, such as probabilistic uncertainty, could further improve the method's adaptability in complex decision-making environments.

4. Conclusions

This study has developed and validated an advanced decision-making framework based on Z-number TOPSIS, addressing the limitations of traditional TOPSIS and Fuzzy TOPSIS methods in uncertain and dynamic environments. The proposed approach integrates confidence levels with performance evaluations, significantly enhancing the reliability and adaptability of decision support systems in multi-criteria decision-making (MCDM) scenarios.

The study demonstrated that Z-number TOPSIS ensures greater decision stability under noisy conditions, effectively resolves conflicting criteria, and outperforms entropy-based weighting models. A systematic comparative analysis confirmed that Z-number TOPSIS with combined weighting schemes maintains higher ranking consistency and lower sensitivity to missing values, making it a more effective solution for complex decision-making problems.

The findings of this study are applicable across various domains where decision-making under uncertainty is critical. In supply chain management, the proposed approach enables companies to optimize supplier selection, manage inventory distribution, and enhance logistical efficiency in environments where demand fluctuations and market uncertainties affect operational strategies. In healthcare, the method supports hospitals and medical institutions in prioritizing patient treatment plans, allocating limited resources effectively, and assessing medical risks under varying conditions of urgency and uncertainty.

In financial risk assessment, the Z-number TOPSIS framework helps financial analysts and investors make data-driven investment decisions, evaluate risk exposure, and assess market volatility, especially when economic forecasts are uncertain or incomplete. The approach is also relevant in environmental and sustainability management, where policymakers and industry leaders must assess the impact of environmental policies, prioritize sustainable development projects, and allocate resources for climate adaptation strategies based on uncertain and evolving ecological data.

Additionally, in public sector decision-making, the proposed method can be used for urban planning, infrastructure development, and policy formulation, where government agencies need to assess multiple competing priorities under economic, social, and environmental constraints. In manufacturing and industrial optimization, the model aids in quality control, production planning, and supplier evaluation, ensuring efficient decision-making even when operational data is incomplete or subject to change.

The study provides valuable insights for researchers developing advanced decision-support systems, industry professionals seeking reliable decision-making frameworks, and policymakers designing

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strategies under uncertainty. Future research should focus on enhancing the computational efficiency of Z-number TOPSIS, expanding its scalability to large-scale decision problems, and exploring hybrid models that integrate probabilistic and fuzzy uncertainty to further improve decision accuracy and applicability.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship or other, which could affect the study and its results presented in this article.

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Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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