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DEVELOPMENT OF A MODEL FOR COLORING RASTER ELEMENTS OF POLYNOMIAL TRANSFORMATION OF DIGITAL IMAGES

The object of the study is the process of pre-printing image preparation, in particular the final stage of preparing an image for printing – rasterization using polynomial transformation.

One of the problems in the process of preparing an image for printing is the lack of a program in computer graphics programs and a raster processor for constructing gradation characteristics and rasterization characteristics.

This work used scientific research methods, in particular the method of mathematical modeling, object-oriented programming and the MATLAB:Simulink software package. In the process of the study, rasterization models of polynomial transformation of digital images were built and simulators for simulation modeling were developed.

Gradation characteristics, rasterization characteristics and optical density of the raster elements were obtained, which quantitatively and qualitatively describes the raster tone reproduction of printed images. The developed model of coloring for determining the amount of paint on the surface of raster elements of polynomial transformation of images of light tones allows to correct the image based on the analysis of the properties of gradation characteristics, characteristics of screening and optical density in a wide range of tone reproduction.

Thanks to the proposed model, the informativeness of the analysis of tone reproduction is significantly expanded. This is a significant advantage over the model based on power transformation, which has limitations in terms of the reproduction of dark tones and causes the phenomenon of posterization.

Based on the obtained results of coloring raster elements of typical variants of polynomial transformation for a polynomial thickness value $H=1$ µm it was established that an increase in the thickness of the paint layer by 20 % of the nominal shifts the initial values of coloring. In particular: at $V=1.2$ the characteristics shift towards dark tones – the image darkens, and at $V=0.8$ the characteristics of coloring shift towards midtones – the image becomes lighter.

The results of the conducted studies of raster tones can be applied at the stage of preparing digital images for rasterization in computer publishing systems.

Keywords: *polynomial transformation, coloring raster elements, simulator, gradation characteristics, coloring characteristics, optical density.*

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1. Introduction

In publishing and printing processes, computer publishing systems are widely used to prepare images for printing. At the final stage, rasterization is performed using a raster processor (RIP), in the working window of which it is possible to specify the shape of raster elements (square, round, oval), ruler, etc. It is worth noting that the selected shape of raster elements forms a rasterization algorithm, which significantly affects the quality of the printed image [1, 2]. The operator corrects the image in the absence of the original and observes it on the monitor screen, and the printed image is reproduced on white paper in black paint, which is significantly different from the original and, accordingly, makes correction difficult. The procedure for preparing the image for printing consists in placing a gradation wedge with a uniform step of brightness change next to the scanned image. Then the image is printed and re-scanned and additional correction is performed, for example, using the Curves tool. Therefore, the procedure for preparing for printing becomes more complicated and requires time and materials [3, 4].

In computer graphics programs and raster processors, there is no program for constructing gradation characteristics and rasterization characteristics. This significantly limits the capabilities of the operator and technologist when preparing an image for printing [5, 6]. Accordingly, the quality of preparation depends on their skills and experience. The lack of quantitative indicators for assessing the properties of images limits the information capabilities when preparing it for printing. Therefore, modeling the coloring raster elements, constructing their gradation characteristics and determining the optical density and analyzing their properties is a relevant task.

Existing methods of image processing and preparation of text and graphic information using computer publishing systems differ significantly from classical methods [7, 8] in other industries. Therefore, these methods cannot be directly applied for high-quality

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preparation of images for printing. In available sources [9, 10] little attention is paid to modeling the processes of preparation for printing, which are multi-stage and describe various processes and parameters. In particular, these are scanning, correction, rasterization, demodulation, optical density, etc. For example, in fundamental works on digital image processing [11, 12] only a few pages are devoted to printing.

Images scanned by a package are mostly too dark and, accordingly, with unclear tone separation. To ensure quality, they need to be given brightness and appropriate contrast (adjusted) [13, 14]. In printing, simple methods of nonlinear stretching, dynamic range stretching, which are based on power-law gamma transformation of images, are most often used. It is used to adjust scanners, monitors, printers and when preparing images for printing [15, 16]. However, the use of power transformation has limitations in terms of reproducing dark tones and causes the phenomenon of posterization – the appearance of visually noticeable transitions (stripes) in black areas, which distorts the image.

Therefore, it is relevant to develop a method for calculating and constructing gradation characteristics of the coloring raster elements, determining their optical density and analyzing their properties.

Thus, *the aim of research* is to develop a coloring model to determine the amount of paint on the surface of raster elements of the polynomial transformation of digital images of light tones.

2. Materials and Methods

The object of research is the process of pre-printing image preparation, in particular the final stage of preparing the image for printing – rasterization using polynomial transformation.

The following scientific methods were used in the research:

– mathematical modeling methods for constructing models of rasterization of polynomial transformation of digital images to expand the capabilities of the process of preparing images for printing in order to improve the visual quality of the print;

– object-oriented programming in the development of simulators for simulation modeling of rasterization of polynomial transformation of digital images in order to determine the amount of paint on the surface of raster elements;

– interactive software package MATLAB:Simulink in calculating and constructing gradation characteristics, rasterization characteristics and optical density graphs in which there is no posterization phenomenon.

There are used: mathematical modeling methods for constructing models of rasterization of polynomial transformation of images; object-oriented programming for developing simulation simulation simulators for rasterization of polynomial image transformation; MATLAB:Simulink software package.

3. Results and Discussion

To solve the problem of modeling the coloring raster elements, the proposed polynomial transformation of light tones was used:

$$L = \frac{b + L_0}{L_0 + d}, \text{ if } 0 \le L_0 \le 55, \quad (1)$$

where L_0 – the linear scale [6]; b, d – the transformation parameters [6].

For the study, typical variants of the normalized polynomial transformation of light tone images were studied:

$$L_1 = \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.65, \quad (2)$$

$$L_2 = \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.75, \quad (3)$$

$$L_3 = \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.9, \quad (4)$$

$$L_4 = \frac{2 \cdot L_0}{L_0 + 1.3} \cdot 1.15, \text{ if } 0 \le L_0 \le 1, \text{ then } 0 \le L_i \le 1, \quad (5)$$

where the scales M_i of the polynomial transformation tuning to the limit $0 \le L_i \le 1$.

Let's perform the rasterization operation of typical variants of the polynomial transformation of light tone images, the result of which is the relative areas – rasterization algorithms:

$$S_1 = 1 - \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.65, \quad (6)$$

$$S_2 = 1 - \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.75, \quad (7)$$

$$S_3 = 1 - \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 0.9, \quad (8)$$

$$S_4 = 1 - \frac{2 \cdot L_0}{L_0 + 0.3} \cdot 1.15, \quad (9)$$

$$S_0 = 1 - L_0, \text{ if } 0 \le L_0 \le 1, \text{ then } 0 \le S \le 1. \quad (10)$$

In which the raster transformation S_0 [6] of the linear scale L_0 is defined for comparison.

The coloring V [6] is determined – the amount of paint on the surface of the raster elements:

$$V = S \cdot H, \quad (11)$$

where H – the relative thickness of the paints [6].

The nominal value of the paint thickness depends on the type of paint and paper, for example, for offset printing the paint thickness is within $0.7 \le H \le 1.3$ µm, and its average value $H_c = 1.0$ µm:

$$H = \frac{0.7 \le H_n \le 1.3}{1.3}. \quad (12)$$

The thickness of the paint layer on the print significantly affects the quality of the printed product, which is estimated by the density of the print. If the amount of paint on the surface of the raster elements (11) is known, then it is possible to determine the raster optical density:

$$D = V \cdot D_m, \quad (13)$$

where D_m – the optical density of the plate [6], which can be measured with a densitometer in production conditions, or its value can be set for a given type of paper.

It should be noted that the best densitometer models quantitatively determine the optical density of the plate and the thickness of the paint layer [6], which significantly increases the accuracy of determining the optical density according to expression (13). Based on the above and expressions (2)–(12), a structural diagram of a model simulator for rasterizing polynominally transformed images of light tones and determining the optical density of the coloring has been developed, which is presented in Fig. 1.

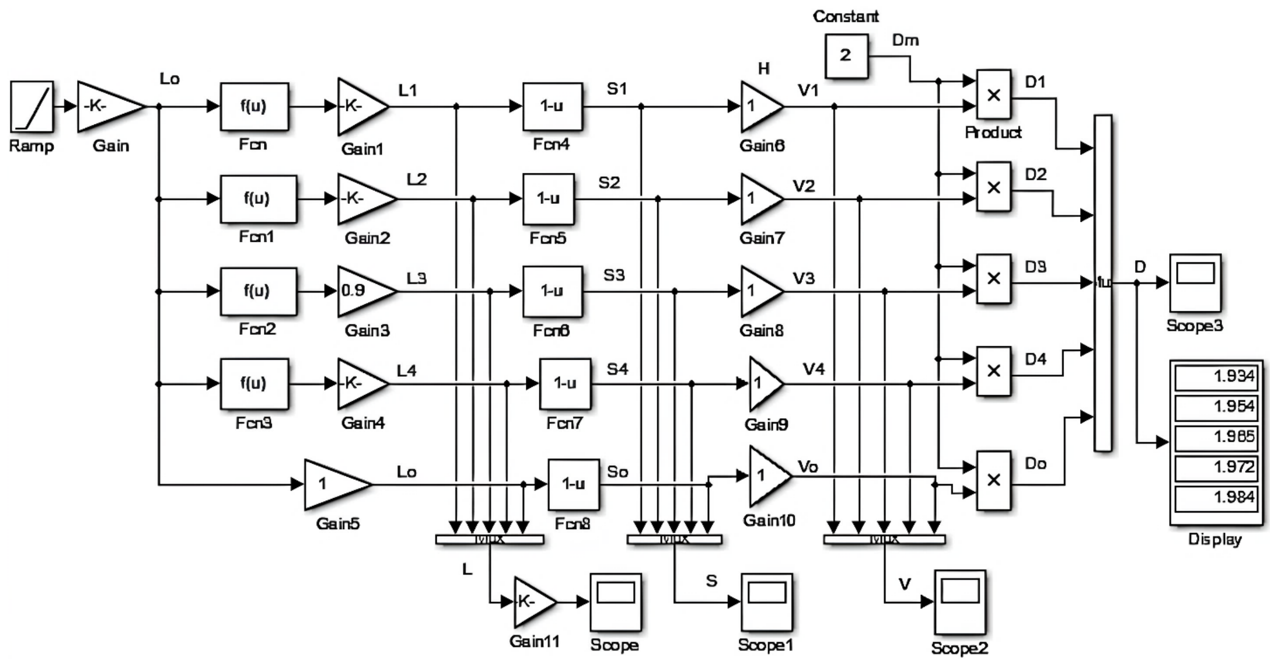

Fig. 1. Structural diagram of the model of the simulator of rasterization and determination of the coloring density

The Ramp block generates a linear scale L_0 which is scaled by the Gain block and fed to the inputs of the blocks of mathematical functions F_{cn}–F_{cn3}. The programs are written in the dialog windows of the blocks – expressions (2)–(4) of the normalized polynomial transformation for light tones. Their outputs are scaled to the limits $0 \leq L_i \leq 1$ and visualized by the Scope block. The obtained values are fed in parallel to the inputs of the blocks of mathematical functions F_{cn4}–F_{cn8} in the dialog windows of which the program is written – expressions $(1–U)$ of the raster transformation S_1–S_4, which are visualized by the Scope1 block. After that, they are fed in parallel to the inputs of the Gain blocks in which the relative thickness H of the paint layer is set. At their outputs, the amount of paint V on the surface of the raster elements is obtained, which is visualized by the Scope3 block. In the Product multiplication blocks, the raster optical density D_1–D_4 is determined by the expression (13) for the optical density of the plate D_m specified in the Constant block. The determined raster optical density is fed to the multiplexer input and visualized by the Scope2 block.

The mathematical function blocks were configured for the specified programs and parameters. In the interactive mode of the simulator, the following scales were determined: $M_1 = 0.65$, $M_2 = 0.75$, $M_3 = 0.9$, $M_4 = 1.15$. For the study, the relative thickness of the paint $H = 1.00$ and the optical density of the plate $D_m = 2.00$ were set.

The results of modeling the gradation characteristics of typical variants of the polynomial transformation are shown in Fig. 2.

For comparison, the gradation characteristic L_0 of the linear scale is presented below. The gradation characteristics are convex curves well stretched in the light and middle range compared to the power gamma transformation. At the beginning of the range, the gradation characteristics have a relatively small steepness, so the polynomial transformation of light tones eliminates posterization in dark areas, which is its advantage.

The results of the rasterization modeling of typical variants of the polynomial transformation of light tone images are presented in Fig. 3.

For comparison, the rasterization characteristic S_0 of the linear scale is shown above. The initial values of the relative areas are equal to unity, which corresponds to the dark tones of the image. The rasterization characteristics are concave curves well stretched in the middle and light ranges of the tone transmission, so the light details of the images are well distinguished.

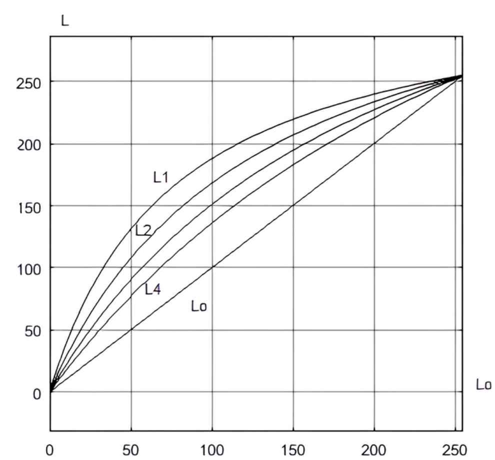

Fig. 2. Gradation characteristics of typical variants of polynomial transformation

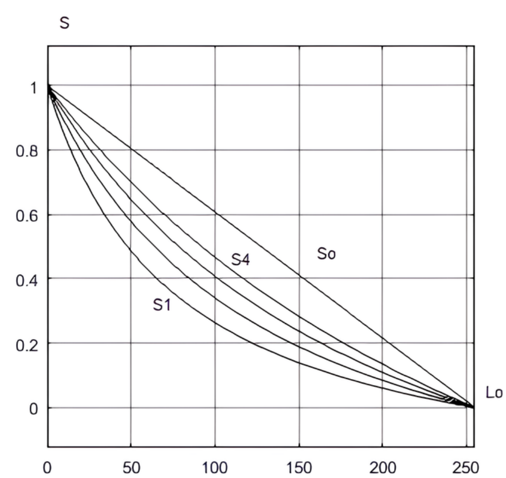

Fig. 3. Rasterization characteristics of typical variants of polynomial image transformation

The results of painting raster elements, which determine the amount of paint on the raster elements of typical variants of polynomial

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transformation for the polynomial thickness value $H = 1$ µm, are shown in Fig. 4.

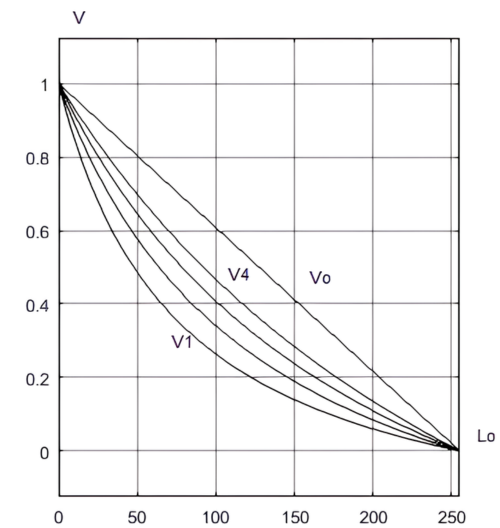

Fig. 4. Coloring raster elements graphs of typical variants of polynomial transformation

For comparison, the linear scale coloring characteristic V_0 is shown above. The initial values of coloring raster elements are units and do not depend on the variant of the polynomial transformation. The coloring characteristics are concave curves well stretched in the middle ranges, so the light details of the image are well distinguished. At a value of $L_0 = 50$, the coloring levels of raster elements are: 0.486; 0.577; 0.698; 0.804 units. It has been established that an increase in the thickness of the paint layer by 20 % of the nominal shifts the initial coloring values. In particular: at $V = 1.2$ the characteristics shift towards dark tones – the image darkens, and at $V = 0.8$ the coloring characteristics shift towards midtones – the image becomes lighter. Thus, the proposed coloring raster elements quantitatively and qualitatively describes the raster tone reproduction of printed images, which expands the functional capabilities when preparing them for printing.

The results of modeling the optical density of coloring raster elements of typical variants of polynomial transformation are presented in Fig. 5.

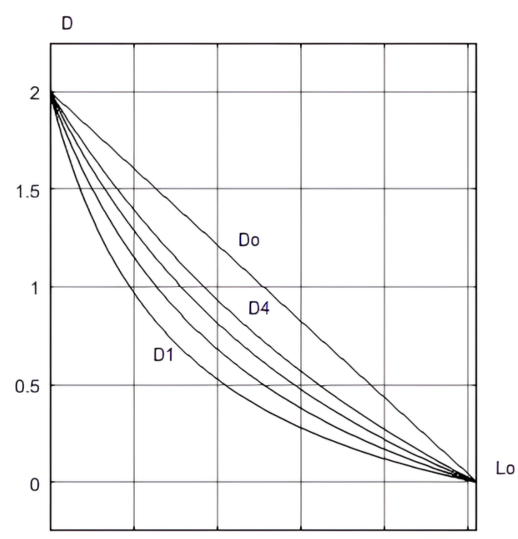

Fig. 5. Optical density graphs of coloring raster elements of typical variants of polynomial transformation

For comparison, the optical density $D = 2.0$ units is given above and does not depend on the transformation variant and decreases rapidly. At a value of $L_0 = 50$ levels, the optical density value is: 0.972; 1.155; 1.291; 1.397; 1.610 units. The optical density graphs of coloring raster elements are concave curves well stretched in the middle ranges, so the light details of the images are well distinguished and smoothly go to the zero final value.

Based on the obtained results of modeling raster-transformed digital images, it was established that at the beginning of the dark tone range, the optical density characteristics are significantly shifted towards dark tones. In this context, the raster optical density for different rasterization algorithms largely depends on the selected rasterization algorithm. Also, based on the results of modeling the raster transformation of typical algorithms for polynomial transformation of digital images, the following was established. The characteristics of the raster transformation of images for light tones are concave curves uniformly compressed towards small areas. In contrast, the characteristics of the raster transformation of images for dark tones are rather convex curves stretched towards dark tones.

Thus, the rasterization and coloring of the polynomial transformation of digital images, which consists in stretching its dynamic range of gray levels, is a way to improve the visual quality of the image by enhancing relative contrasts.

The developed coloring model for determining the amount of paint on the surface of the raster elements of the polynomial transformation of images includes: a simulator is developed for simulating the rasterization of the polynomial transformation of digital images; gradation characteristics and characteristics of the rasterization of the polynomial transformation of images are constructed; graphs of the coloring raster elements of the polynomial transformation of images and graphs of the optical density of the coloring raster elements of the polynomial transformation of images. Each of the components of the model is in a corresponding relationship with all the others, which is determined by the properties of the connections of the printing parameters in the process of preparing the image for printing, in particular in the process of coloring raster elements. Together, all the components of the model form a holistic structural and functional picture of the coloring model.

Thus, the developed coloring model for determining the amount of paint on the surface of raster elements of polynomial image transformation for light tones has significant advantages over existing methods of raster image transformation that are used in printing today. In particular, it expands the range of perception of the visual quality of images by enhancing relative contrasts. The use of polynomial image transformation eliminates the phenomenon of posterization, i. e. eliminates visually noticeable transitions in shadows. At the same time, the developed raster transformation makes it possible to obtain different algorithms for rasterizing digital images of light and dark tones for applying the optimal amount of paint to raster elements. This makes it possible to ensure optimal raster optical density of the image in order to obtain high quality prints, which is an advantage over existing rasterization and coloring models.

Based on the proposed developed model of coloring for determining the amount of paint (coloring) on the surface of raster elements of polynomial image transformation, the following have been developed:

- a simulator of the process of rasterization of polynomial transformation of digital images, which makes it possible to form typical variants of raster transformation of images;
- methods for calculating and constructing gradation characteristics of coloring raster elements, determining their optical density and analyzing their properties, which makes it possible to increase the efficiency of reproduction of technological and functional capabilities;
- a method for improving the visual quality of an image based on rasterization and coloring of polynomial transformation of digital images, which consists in stretching its dynamic range of gray levels.

The use of the proposed model significantly increases the accuracy of image correction when preparing it for printing; provides optimal application of paint to raster elements; provides high visual quality of the printed print in a wide range of tone reproduction.

Practical significance. The results of the conducted studies of raster tones are recommended to be used by operators at the stage of preparing digital images for rasterization in computer publishing systems.

Research limitations. The main limitations of this research are that the implementation of the proposed solutions requires an appropriate hardware base and special software. Providing appropriate mechanisms and tools makes it possible to calculate and build characteristics and dependencies of printing parameters for processing digital images in the process of preparing them for printing.

Prospects for further research. A promising direction for continuing research is the introduction of modern information technologies in pre-printing processes of image preparation. In particular, the use of neural networks in the process of processing digital images is extremely interesting. This can significantly increase the accuracy of analysis of complex image structures and, accordingly, improve the quality of tone reproduction.

4. Conclusions

A coloring model has been developed to determine the amount of paint on the surface of raster elements of polynomial image transformation. To analyze its properties, a simulator of coloring raster elements has been developed, which makes it possible to quantitatively determine its parameters and properties. In this context, a structural diagram of a model simulator of the rasterization process of polynomial transformation of digital images in the MATLAB:Simulink package has been developed. The simulator makes it possible to form typical variants of raster transformation, analyze, determine their differences and raster optical density.

According to the simulation results, it was found that polynomial image transformation eliminates posterization in dark areas of the image, which is its advantage over traditional power gamma transformation. The characteristics of the rasterization are concave curves in the middle and light range of the tone transmission, so the light details of the image are well distinguished.

The results of coloring determine the amount of paint on the raster elements, which are given by the coloring characteristics. The characteristics are concave curves well stretched in the middle ranges, and at the beginning of the range at $L_0=50$ levels of coloring raster elements are: 0.486; 0.577; 0.640; 0.698; 0.804 units. It is established that an increase in the thickness of the paint layer by 20 % of the nominal shifts the initial coloring values. In particular: at $V=1.2$ the characteristics shift towards dark tones – the image darkens, and at $V=0.8$ the coloring characteristics shift towards midtones – the image becomes lighter.

It is established that the initial values of the optical density are: 0.972; 1.155; 1.291; 1.397; 1.610 units, after which they smoothly approach zero. It is proven that the proposed coloring raster elements of polynomial transformation describes the raster tone reproduction of printed images, which expands the functionality when preparing them for printing.

The results of the conducted studies of raster tones can be applied at the stage of preparing digital images for rasterization in computer publishing systems.

Conflict of interest

The author declares that he has no conflict of interest in connection with this study, including financial, personal, authorial or any other interest that could affect the study and its results presented in this article.

Financing

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The author confirms that he did not use artificial intelligence technologies when creating this work.

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