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Sviatoslav Kavyn

DEVELOPMENT OF A MODEL OF POWER-LINEAR CONVERSION OF DIGITAL IMAGES FOR DARK TONES

The object of research is the technological process of digital image processing using power transformation in pre-printing processes.

A significant problem in preparing an image for printing is the phenomenon of posterization, which distorts the image and limits the possibilities of power transformation for correcting dark areas of the image. This is a disadvantage of power transformation, which is that at power indicators (r<0.45) and (r>1.5) power transformation is too sensitive to changes in black levels.

The mathematical model of power-linear transformation of images for dark tones has been improved, which, unlike the known ones, involves the summation of power and linear transformation and includes a simulator of power-linear transformation of images. Taking into account the improved model, gradation characteristics, optical density dependences and contrast sensitivity were obtained, which quantitatively assess the perception of images by the human visual system.

The validity of the improved model was verified by mathematical modeling using object-oriented programming and the MATLAB:Simulink software package.

The results of mathematical modeling indicate that the development of the mathematical model allowed to further expand the possibilities of image correction. This is due to the fact that the length of the discrete gradation characteristics is 3–4 levels, which are not noticed by the human visual system (posterization is eliminated).

The proposed model has significant advantages over image conversion methods used in printing. In particular, it expands the range of visual perception of images, eliminates the phenomenon of posterization, provides the ability to change (stretch and compress) contrast within wide limits. At the same time, it expands the functionality of power-law image conversion, and accordingly provides an increase in image quality when preparing it for printing.

The results of the conducted research are recommended to be used at the stage of preparing images for printing and in workflows by operators and technologists.

Keywords: *power-linear transformation, simulator, gradation characteristics, optical density, contrast sensitivity, posterization.*

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1. Introduction

In printing and other branches of science and technology, scanned digital images of originals of different quality and tone are widely used. Scanner manufacturers add various scanner control programs, for example, setting the white and black point, which can be set automatically or manually, determining the minimum and maximum optical density, etc. [1–3]. The main operation during scanning is image tone adjustment. In professional scanners, scanning is used with a batch of images of different tones, respectively, such image scans are ineffective for direct use in the printing process. Therefore, after entering the scanned image into the computer, it is necessary to perform image correction in computer graphics programs, for example, Photoshop, which performs the image conversion function [4, 5].

When preparing images for printing, its scanned digital version almost always requires tone adjustment. Scanned images are mostly too dark and, accordingly, with unclear tone separation. To ensure the required image quality, it is necessary to provide it with adequate brightness and appropriate contrast [6, 7].

Therefore, the task of developing a model of power-linear transformation of digital images specifically for dark tones is relevant.

Existing methods allow various transformations of the input image to improve its quality. In printing, image processing in the spatial domain is most often used, which consists in stretching or compressing its dynamic range of gray levels to the maximum using a simple power transformation, which is actually a standard for printing.

In [8], an information technology for tone reproduction in short ink printing systems of a sequential structure is proposed. However, in [8] the task of developing a model for converting digital images is not considered.

In [9], a computer program for synthesizing an image on a printout is proposed for new information and traditional color printing technologies. However, in [9] no attention is paid to digital images for dark tones.

In [10], the use of modern methods for correcting scanned images in the process of preparing them for printing is considered. The methods analyzed in [10] have limitations regarding the visual perception of the image, and the issue of expanding the range of visual perception of the printed print is not considered.

In [11], mathematical methods of digital image processing are substantiated, in particular, spatial transformation, Fourier transformation, linear and nonlinear image filtering, affine image transformations, morphological operations, etc. They can be selected and applied in accordance with the purpose and task of the transformation. But in [11] the possibilities of directly applying image transformation methods in printing in order to eliminate the phenomenon of posterization in dark areas of the image are not considered.

In [12], the corresponding methods and methods of image transformation in the process of preparation for reproduction are considered, in particular, histogram analysis, gradation correction, nonlinear correction, frequency correction, image filtering, contrast enhancement. But in [12] the correction of tone generation in order to eliminate posterization in dark areas of the image is not considered.

In the fundamental work on digital image processing [13], existing methods of image processing are thoroughly described. However, in [13] there is no description of the application of the method of correction of digital images specifically in printing, taking into account the properties of printing parameters such as optical density, contrast sensitivity and gradation characteristics. As in [13], in [14] modern approaches to digital image processing are described in detail. However, as in [13], in [14] no attention was paid to the application of existing modern methods of image correction, taking into account the specifics of purely printing processes such as gradation correction and rasterization.

In [15], the use of power (gamma) transformation for image correction in the process of preparing it for printing is analyzed, which is considered a standard in printing when using prepress processes. However, in [15], the problem of eliminating the posterization phenomenon caused by power transformation due to its sensitivity to the tone transmission of dark tones is not considered.

In [16], the printing prepress processes are thoroughly described, in particular, the process of preparing images for printing in the context of the interrelation of the properties of purely printing image parameters, in particular optical density, contrast sensitivity, gradation characteristics, rasterization characteristics, etc. However, in [16], how to eliminate visually noticeable transitions in shadows that distort the image is not considered.

In particular, in the process of prepress preparation of an image, the power gamma transformation function is used, which brightens or darkens the image, which is actually a standard in computer graphics. In this context, to obtain a high-quality image, gradation characteristics of different shapes are used, in particular: convex tonal curves lighten the image, while concave ones darken it. However, as the image lightens or darkens, the phenomenon of posterization occurs – visually noticeable transitions in dark areas of the image, which distorts the image and limits the capabilities of the power gamma transformation to correct dark areas of the image. This is a disadvantage of the power transformation, which is that at small power indices ($r<0.45$) and large ones ($r>1.5$), the power transformation is too sensitive to changes in black levels, which causes the phenomenon of posterization – the appearance of noticeable transitions (bands) in dark areas of the image, which limits its functional capabilities for reproducing dark tones [15].

In available sources, little attention is paid to the properties of the power transformation and the connection with the classic parameters for printing with gradation characteristics, optical density, contrast, etc. In printing, various tests and scales are widely used to assess the quality of the reproduced image. Based on densometric measurements of the optical density of the ink layer on the print plane, the print contrast is calculated [16]:

$$K=\frac{D_f-D_{0.8}}{D_0}, \quad (1)$$

where D_f – the optical density of the plate; $D_{0.8}$ – the raster density of the printed sample, for which the area of the raster elements is 0.8 ($S=0.8$).

The print contrast evaluates reproduction only in shadows. Therefore, the print contrast limits the quantitative assessment of the image.

Therefore, it is relevant to develop a model of power-linear transformation of digital images for dark tones, which would eliminate the phenomenon of posterization.

The aim of research is to develop a mathematical model of power-linear transformation of digital images for dark tones to improve the quality of perception of printed images by the human visual system.

2. Materials and Methods

The object of research is the technological process of digital image processing in the system of pre-print preparation of digital images for printing using power transformation.

The following scientific methods were used in the research:

– methods of mathematical modeling in constructing mathematical models of power-linear transformation of digital images for calculating and constructing gradation characteristics, optical density and contrast sensitivity and analyzing their properties;

– object-oriented programming in developing software tools – simulators for simulating power-linear transformation, in order to ensure high-quality tone reproduction of images;

– theory of image processing in analyzing, synthesizing and correcting digital images of different tonalities in the process of preparing them for printing in order to improve the visual quality of the print.

The modeling was carried out in the interactive MATLAB software environment using the Simulink package. Ready-made operational blocks and functions of the Simulink library were used in the modeling process. In particular, the main blocks are the operational blocks of mathematical functions that calculate the gradation characteristic, optical density and contrast sensitivity of the power-linear transformation of images for dark tones. A quantization block was used to analyze the posterization of the power-linear transformation. A scaling block and a multiplexer were used to visualize the simulation results. The Curves software tool was used to correct the scanned images. In this context, a detailed description of the simulator of the power-linear transformation of images for dark tones based on the operators of the MATLAB application is presented: Simulink.

3. Results and Discussion

In printing, when preparing images for printing and correcting, the power gamma transformation is widely used. However, little attention is paid to its properties in relation to the classical parameters for printing, gradation transformations, optical density, contrast sensitivity, etc. It is known that at low power values, posterization occurs in dark areas of the image – visually noticeable transitions from one level of black to another, which distorts the image and limits its capabilities. To eliminate posterization and expand the functional capabilities of the power transformation, a power-linear transformation of images of dark tones has been developed. It is based on the principle of adding images, which is carried out on the basis of the sum of the power and linear transformations, which in general terms is given by the expression:

$$L=L_p-L_l, \quad (2)$$

where L_p – gradation characteristic of the power transformation; L_l – linear transformation.

With the addition of images, the limit value $L>255$ of the levels increases, as a result of which restrictions and distortions occur. To eliminate this phenomenon, let's limit the linear component and scale the sum of the images. To simplify the solution of the problem, let's use the relative form of image transformation and the following scaling and formed a power-linear transformation:

$$L=\left[0.5\cdot L_0-L_0^r\right]\cdot M,\ \text{if}\ 0\le L_0\le 1,\ \text{and}\ 0\le L\le 255, \tag{3}$$

where L_0 – the linear scale; r – the power exponent; M – the scale.

By setting different values of $r>1$, it is possible to obtain a set of power-linear transformations. Based on the above, three typical options for power-linear transformation of dark tone images were developed for research and analysis:

$$L_1=[0.5\cdot L_0-L_0^4]\cdot 255\cdot M_1, \tag{4}$$

$$L_2=[0.5\cdot L_0-L_0^{2.5}]\cdot 255\cdot M_2, \tag{5}$$

$$L_3=[0.5\cdot L_0-L_0^{1.5}]\cdot 255\cdot M_3,\ \text{if}\ 0\le L_0\le 1,\ \text{and}\ 0\le L_i\le 255. \tag{6}$$

The scales were determined: $M_1=0.67$; $M_2=0.66$; $M_3=0.66$. For the study, the optical density of the power-linear transformation of images was determined by the expression:

$$D=\log 10\left[\frac{255}{L+1}\right]. \tag{7}$$

To quantitatively assess the quality of the power-linear transformation, their contrast sensitivity was determined for typical transformation options:

$$C=\frac{dL_i}{dL_0}. \tag{8}$$

Based on the above and expressions (4)–(8), it is possible to calculate typical options for the gradation characteristics of the power-linear transformation of images, the optical density and contract sensitivity, and analyze its properties. To simplify the solution of the tasks set, object-oriented programming was used in the MATLAB:Simulink package. Ready-made operational blocks and functions of the Simulink library were used, and a structural diagram of the simulator model of the power-linear transformation of dark tones of the image was developed, which is presented in Fig. 1.

The main blocks of the simulator are the operational blocks of mathematical functions F_{cn1}–F_{cn6}, which calculate the gradation characteristics and optical densities of images. The Ramp block generates a linear scale L_0 which is scaled by the Gain block, and is fed in parallel to the inputs of the operational blocks of mathematical functions F_{cn}–F_{cn2}. In the dialog windows of the blocks, the programs of expressions (4)–(7) are written for calculating the gradation characteristics L_1, L_2, L_3, which in the Gain blocks are scaled to the nominal values of 255 levels. The obtained values are fed in parallel to the Mux multiplexer and visualized by the Step block, after which they are fed to the input of the operational blocks of mathematical functions F_{cn4}–F_{cn6}. In the dialog boxes of the blocks, a program is written – expression (7) for determining the optical density D of the power-linear transformation, which is visualized by the Step1 block. In the differentiation block Derevative, the contrast sensitivity C of the power-linear transformation is determined and visualized. To analyze the posterization of the power-linear transformation it is possible to use the Quantizer quantization block of gradation characteristics L^*, which are visualized.

The blocks of mathematical functions of the simulator were adjusted to typical variants of the power-linear transformation. In the interactive mode of the simulator, using the Gain blocks, the gradation characteristics L_i were adjusted to the nominal values of 255 gray levels, determining the scales: $M_1=0.67$; $M_2=0.66$; $M_3=0.66$.

The results of modeling the gradation characteristics of typical variants of the power-linear transformation of dark tones of the image are shown in Fig. 2.

For comparison, the characteristic of the linear scale L_0 is shown above. Gradation characteristics are concave curves stretched into dark tones. The lower characteristic L_3 has the greatest stretch in the middle range and sufficient steepness at the beginning of the range. This eliminates posterization of the power-linear transformation and a sufficiently large steepness at the end range, which ensures high-quality reproduction of details of light images. The following characteristics L_1 and L_2 located above have a greater steepness at the beginning of the range, which eliminates posterization and high-quality reproduction of images in the worlds.

The results of modeling the optical density of typical options for power-linear reproduction of dark tone images are shown in Fig. 3.

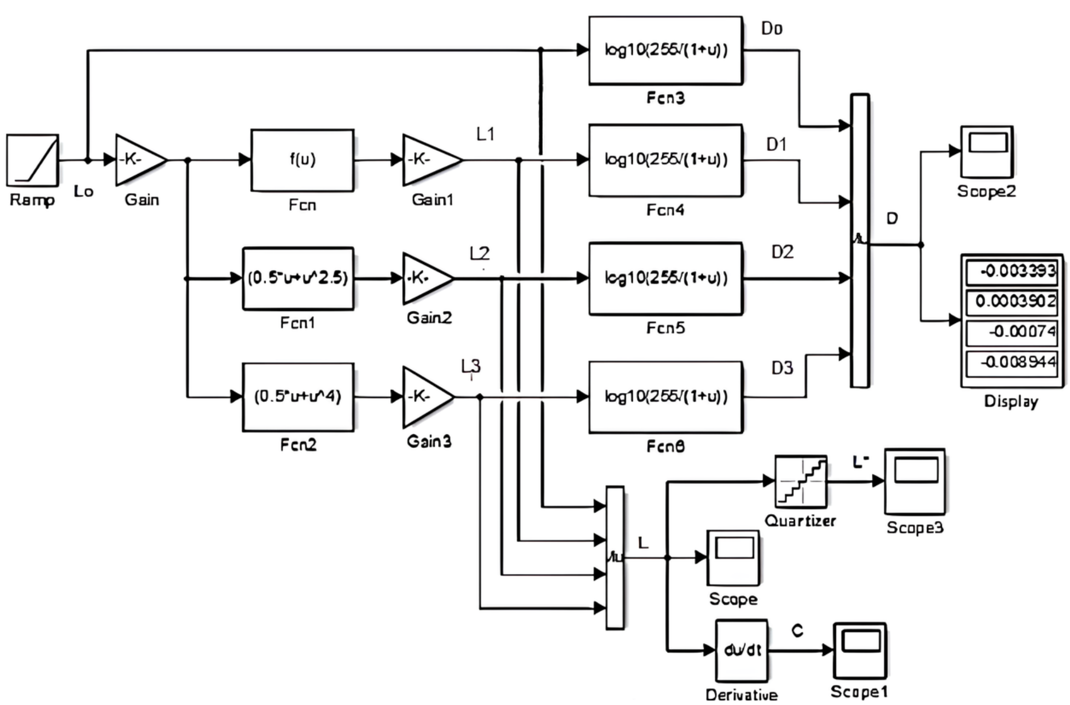

Fig. 1. Structural diagram of the power-linear transformation simulator model

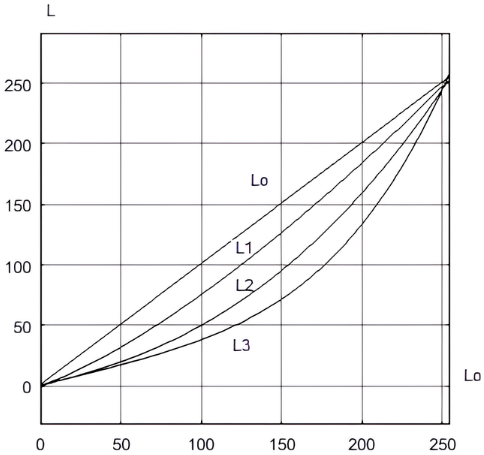

Fig. 2. Gradation characteristics of the power-linear transformation of dark tones of the image

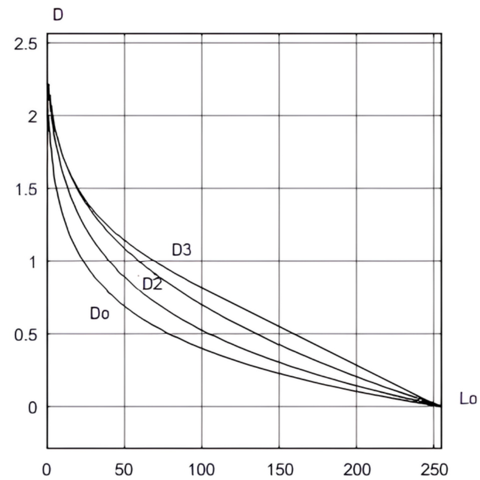

Fig. 3. Optical density graphs of typical power-linear transformations of dark tone images

The initial optical density values are 2.5 units. Below is the optical density characteristic D_0 of the scale, which is a rather slightly concave curve. The optical density characteristics are slightly concave curves shifted towards dark tones. The optical density values at the beginning of the range in the vicinity of $L_0=50$ levels are: 0.691; 0.889; 1.083; 1.145 units. Therefore, the power-linear transformation darkens the image well.

The results of modeling the contrast sensitivity of typical power-linear transformations of dark images are presented in Fig. 4.

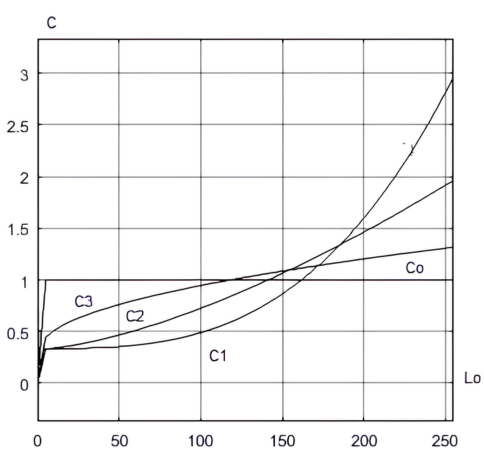

Fig. 4. Contrast sensitivity graphs of typical power-linear transformations of dark tone images

The initial contrast sensitivity of images is close to zero, crosses the unit line and reaches the final values: $C_1=2.9$; $C_2=2.0$; $C_3=1.4$ units. Note that the contrast sensitivity of the normalized linear scale is small and equals unity over the entire tone range. In printing, black is the main color and serves to reproduce text and images on a white background of paper and is characterized by a powerful effect on the visual system. The relative change in contrast sensitivity on dark and light tones is approximately the same. Therefore, power-linear transformation is characterized by good contrast, which quantitatively assesses the reaction of the human visual system and the change in brightness over a limited tone range.

To analyze the phenomenon of posterization of the power-linear transformation of dark tone images, quantization of gradation characteristics at the beginning of the dark range was performed.

The results of modeling the discretized gradation characteristics in close-up are presented in Fig. 5.

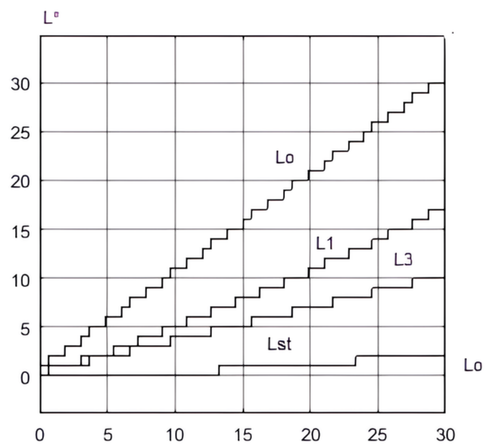

Fig. 5. Graphs of discretized gradation characteristics

For comparison, the figure additionally shows the discretized gradation characteristic L_{st} of the power transformation of dark tone images with the power index $r=2.0$, which is located in the lower part of the figure. At the beginning of the range, the zero discrete has a length of 12 levels, and the next single discrete has a length of 17 levels. The large length of the discretes (degrees) when transitioning from the zero to the first discrete in dark areas causes the phenomenon of posterization – visually noticeable transitions in dark areas of images. At the top are the discretized gradation characteristics L_3 and L_1, which have a length of the first degree of 4 and 3 levels that the human visual system does not notice. Thus, through mathematical modeling, the appearance of posterization of the traditional power transformation and the absence of posterization in the power-linear transformation in dark areas of the image were established. It was established that at the beginning of the range, the gradation characteristics of dark images have sufficient steepness, which eliminates posterization. Based on the conducted research and modeling results, it was established that the presented model expands the functionality of the power-law transformation. Accordingly, it provides an increase in image quality when preparing it for printing.

Thus, the developed new power-linear image transformation for dark tones has significant advantages over image transformation methods that are currently used in printing. In particular, it expands the range of visual perception of images. Based on the summation of gradation characteristics (power-law and linear components), it eliminates the phenomenon of posterization, that is, it eliminates visually noticeable transitions in shadows. At the same time, it makes it possible to change (stretch and compress) the contrast within wide limits and, accordingly, is a method of improving the visual quality of images by enhancing contrasts.

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Practical significance. The research results significantly expand the informativeness of image transformation processes. Therefore, they are recommended to operators and technologists of computer publishing systems for selecting optimal gradation characteristics and rasterization characteristics at the stage of preparing digital images for printing.

Research limitations. The main limitations of this research arise from the dependence of the proposed solutions. In particular, they consist in the fact that the presented algorithms and concepts for improving the quality of the printed print require the operator to have additional knowledge in the field of digital image processing. This is an important component, as it ensures the professional determination by the operator and technologist of the optimal printing parameters in the process of preparing the image for printing.

Prospects for further research. Further research can be focused on improving the developed models to increase their adaptability in pre-printing processes of image preparation. This approach takes into account the natural features of the relationship between the printing parameters of digital image processing in the process of preparing the image for printing. This will make it possible to significantly expand the range of image correction and, accordingly, ensure an increase in the quality of the print.

4. Conclusions

A mathematical model of power-linear transformation of dark tone images has been developed based on the addition of images, which is carried out on the basis of the summation of power-linear transformation. The obtained value is scaled to 255 levels. Using the power exponent r, three typical variants of power-linear transformation of dark tone images have been formed and their properties have been analyzed.

A structural diagram of a simulator model of power-linear transformation of dark tone images in the MATLAB:Simulink package has been constructed. The simulator makes it possible to calculate and build gradation characteristics, optical density and contrast sensitivity, as well as analyze their properties.

It has been established that gradation characteristics are concave curves stretched into dark tones and have the greatest stretch in the middle range and sufficient steepness in the final range. This ensures high-quality reproduction of details in the image worlds. It has been established that optical density characteristics are weakly concave curves shifted towards dark tones. And the optical density at the beginning of the range in the vicinity of $L_0=50$ levels is: 0.691; 0.889; 1.088; 1.145 units and darkens the image well.

Contrast sensitivity graphs of typical variants of power-linear transformation of dark tones have been constructed. Their initial values are close to zero, they are concave curves, intersect a single line and reach the final values: $C_1=2.9$; $C_2=2.0$; $C_3=1.4$ units. It has been established that power-linear transformation is characterized by good contrast sensitivity, which quantitatively and qualitatively assesses the reaction of the human visual system to changes in brightness in a limited range of tone transmission.

Discretized gradation characteristics of power-linear transformation of dark tones with the power index $r=2.0$ have been constructed. It has been found that in the initial range the zero discrete has a length of 12 levels, and the next single discrete has a length of 17 levels. In this context, it has been noted that the large length of discretes in dark areas of the image causes the phenomenon of posterization. In other words, visually noticeable transitions in dark areas that distort the image. It has been found that the discretized gradation characteristics of the power-linear transformation at the beginning of the range have a length of 3–4 levels that are not noticed by the human visual system. Accordingly, through mathematical modeling, the appearance of posterization of the gradational power transformation and the absence of posterization in the power-linear transformation in dark areas of the image have been established.

The results of the conducted research are recommended to be used at the stage of preparing images for printing and used by operators and technologists in work processes.

Conflict of interest

The author declares that he has no conflict of interest in connection with this research, including financial, personal, authorial or any other interest that could influence the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The author confirms that he did not use artificial intelligence technologies when creating this work.

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Sviatoslav Kavyn, *PhD Student, Department of Computer Technologies in Publishing and Printing Processes, Lviv Polytechnic National University, Lviv, Ukraine, e-mail: kavinsviatoslav@gmail.com, ORCID: https://orcid.org/0000-0002-6189-3848*