UDC 336.761:330.131
JEL Classification: C51, C58
DOI: 10.15587/2706-5448.2025.329243

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QUANTIFYING ASSET PRICE VOLATILITY WITH FRACTIONAL BROWNIAN MOTION

The object of the study is the behavior of stock market volatility in response to sudden shocks and crisis-driven fluctuations, with a specific focus on capturing its complex temporal structure and memory effects. One of the biggest challenges in this domain lies in the inherent stochastic nature of volatility: it evolves irregularly over time, cannot be directly observed, and must be estimated from indirect indicators. Conventional models, particularly those grounded in classical Brownian motion, often fall short in accurately representing such dynamics, as they neglect the long-range dependence – or "market memory" – commonly observed in real financial time series. This oversight can lead to significant errors in volatility estimation, especially during phases of market turbulence such as financial crises or global events.

A fractional diffusion framework was used during the study to model asset price dynamics, incorporating a time-dependent and initially unknown volatility function. This approach relies on fractional Brownian motion, whose non-Markovian properties enable the model to effectively account for long-term correlations in market behavior. To estimate the volatility, it is possible to employ statistical tools based on p-variations, which allowed to compute the Hurst index and reconstruct the underlying path of realized volatility with high sensitivity to structural market changes.

It is possible to obtain that this method significantly improves the accuracy of volatility tracking, particularly under stress conditions, such as those observed during the 2020 COVID-19 crisis. It is connected to the fact that the suggested method has a number of features, in particular its ability to incorporate memory effects and to respond adaptively to high-frequency data variations. Thanks to that, let's manage to capture abrupt volatility spikes and sustained market uncertainty more precisely. Compared to the standard models, it is possible to achieve the following advantages: enhanced responsiveness to market dynamics, improved reliability of volatility forecasts during crisis periods, and a more realistic reflection of financial market complexity.

***Keywords**: stock market, fractional Brownian motion, parameter estimation, markets with memory, volatility.*

Received: 06.02.2025
Received in revised form: 02.04.2025
Accepted: 27.04.2025
Published: 12.05.2025

How to cite
Iurchenko, M., Šaltyte-Vaisiauske, L., Babenko, V. (2025). Quantifying asset price volatility with fractional Brownian motion. Technology Audit and Production Reserves, 3 (4 (83)), 34–41. https://doi.org/10.15587/2706-5448.2025.329243

1. Introduction

The stock market is a critical player in the modern market economy, providing a mechanism for investment engagement in both private businesses and the state budget. The market's importance has resulted in a significant growth in interest in new types of risky assets and, therefore, an increased demand for mathematically sound ways to maximize profits from such investments. It is evident that effective investment portfolio management is dependent on the estimation of the *volatility* of prices for financial assets. In this context, the term "*volatility*" should be understood as the measure of fluctuation in the price of a financial asset over time, which can be attributed to a variety of factors including market supply and demand, economic and political events as well as changes in interest rates, company news, and other events that may affect investor behavior. The accurate modeling of volatility is crucial not only for the optimization of portfolio performance but also for risk management, pricing derivatives, and maintaining market stability. However, the unpredictable behavior of financial instruments makes the quantification of volatility a challenging task, especially during shock states on the market.

The literature on mathematical finance highlights a significant limitation of the classical market models in capturing changes in volatility behavior. The Black-Scholes-Merton-Samuelson model (e. g. [1–3]), for instance, assumes that volatility remains deterministic and constant, which fails to capture the nontrivial and statistically significant dependence of the implied volatility on the strike prices of options, commonly known as "volatility smiles" or "volatility skews" [4, 5]. One of the ways to address this issue lies in replacing the constant volatility parameter with a random process; e. g. a diffusion driven by Brownian motion. This approach turns out to be relatively successful in reproducing volatility smiles [6–8] and, additionally, is consistent with a number of other empirical stylized facts (for more details, e. g. [9–11]). However, the "correct" choice of a stochastic model for volatility is not a trivial task: for example, Brownian diffusion approaches such as CEV [12] and Heston [13] models do not generate implied volatility surfaces that align well with the actual data [4].

One of the main reasons for that outlined in the literature is the fact that Markovian models fail to capture the *market memory effect*, i. e. nontrivial dependence of future asset prices on the entire history of the latter [13–19]. A widely used technique for modeling markets with memory is to substitute the standard Brownian motion in stochastic models with the fractional Brownian motion $B^H = \{B^H(t), t \geq 0\}$ [20–24] which is essentially the only Gaussian process enjoying self-similarity, i. e.

$$B^H(at) = |a|^H B^H(t), \quad t \geq 0, \quad a \geq 0, \tag{1}$$

together with the stationarity of increments

$$B^H(t) - B^H(s) \sim B^H(t-s), \quad 0 \leq s \leq t. \tag{2}$$

The fractional Brownian motion offers several advantages over the standard Wiener process, which enhance the reliability of the models. Firstly, it addresses the market's "memory effect", which is achieved through the correlation of the fractional Brownian motion increments. Secondly, the stationarity of the increments in the fractional Brownian motion makes it feasible to employ it as a randomness generator in financial models. However, it should be noted that the fractional Brownian motion is generally not a semimartingale, except in the case of $H = 1/2$. This implies that the standard L_2 integration theory cannot be employed to define stochastic integrals. In order to overcome this limitation, it has been proposed [25] to use fractional derivatives and integrals to determine the pathwise integral with respect to fractional Brownian motion. An additional problem associated with fractional models is statistical estimation of their parameters. While a significant number of works have focused on estimating parameters for diffusions driven by the standard Wiener process (among multiple examples of research on this topic, it is possible to mention [26–28]), relatively few studies (e. g. [29–32]) have examined models based on fractional Brownian motion.

In this paper, let's utilize fractional Brownian motion as a driver for the dynamic model of price and employ statistical methods based on *p-variations* to analyze the volatility of multiple assets, which take into account volatility changes over time. The efficacy of the p-variation method for volatility estimation is demonstrated through empirical analysis of high-frequency financial data from 2020. Specifically, let's showcase the accuracy of the method in capturing the volatility explosion that occurred during the COVID-19 pandemic.

The key scientific novelty of the present paper lies in the development and application of a new method for quantifying market volatility within the fractional diffusion framework. Building on the foundational works [30, 31], who introduced the p-variation technique for parameter estimation in fractional Brownian motion, let's extend this methodology not only to estimate the Hurst parameter but also to reconstruct the time-dependent *spot* volatility function with a novel application of the moving average smoothing technique. This innovation enables a data-driven, nonparametric analysis of market uncertainty that is highly sensitive to temporal changes, including sharp increases in volatility during crisis periods. Crucially, our method does not require additional input beyond asset prices, yet it captures complex market behaviors – such as long-range dependence and volatility clustering – that are typically inaccessible through classical models. By advancing the theory and extending the application of the p-variation method, our work provides a novel, powerful tool for volatility modeling that enhances the descriptive and predictive capabilities of fractional market models.

To summarize, *the aim of this research* is to propose a method for quantifying the shock state effect for market volatility using statistical estimation techniques through high-frequency observation of fractional diffusion.

The necessity and relevance of this research are driven by the rapidly increasing instability of financial markets and the growing frequency of so-called "shock" states. Contemporary volatility estimation models often fail to adequately describe price dynamics under conditions of sharp market fluctuations. At the same time, effective risk management and the construction of resilient investment portfolios require accurate and timely assessments of market volatility. The use of fractional Brownian motion opens new perspectives in modeling market memory and capturing long-range dependencies, making it particularly valuable under high uncertainty. Applying methods based on p-variations improves the precision of volatility estimation using high-frequency data, thereby contributing to a deeper understanding of market behavior. Thus, this study aims to address an important scientific and practical challenge that is highly relevant to both the academic community and financial market professionals.

2. Materials and Methods

2.1. Market model with fractional diffusion price

In this study, it is possible to assume that the asset price $S = \{S(t), t \geq 0\}$ is given by

$$S(t) = e^{X(t)}, \tag{3}$$

where $X = \{X(t), t \geq 0\}$ is a fractional diffusion of the form

$$X(t) = X(0) + \int_0^t \beta(s)ds + \int_0^t \sigma(s)dB^H(s), \tag{4}$$

with $B^H = \{B^H(t), t \geq 0\}$ denoting a fractional Brownian with unknown Hurst index $H \in (0,1)$, i. e. a centered Gaussian process with the covariance function

$$E[B^H(s)B^H(t)] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right).$$

Unlike Brownian motion, fractional Brownian motion is not Markovian, which enables to capture the "*market memory*" effect. The Hurst index H is responsible for the type of it: if $H > 1/2$, the increments of B^H are positively correlated (which corresponds to the so-called "long memory" case) whereas, if $H < 1/2$, the increments are negatively correlated (leading to the so-called "short memory" effect). The case $H = 1/2$ complies with independent increments and, consequently, the absence of memory.

The function $\sigma = \sigma(t)$ that is interpreted as the volatility of our model is assumed to be an (unknown) stochastic process with trajectories that are locally Hölder continuous of order $\alpha > 1 - H$, i. e. for any $T > 0$ and $s, t \in [0,T]$

$$|\sigma(t) - \sigma(s)| = C_T|t-s|^{\alpha}, \tag{5}$$

where C_T is some positive random constant that possibly depends on T. Let's note that there are no assumptions on the distribution of σ and condition (5) is purely technical: it ensures that the integral w. r. t. the fractional Brownian motion in (4) is well-defined. Finally, let's assume that the process $\beta = \{\beta(t), t \in [0,T]\}$ belongs to the space $L_p(\Omega \times [0,T])$ for some $p \geq 2$.

It is important to note that the pricing model presented in (3) is not a semimartingale, and consequently, exhibits arbitrage under the assumption of a frictionless market with no transaction costs [33, 34].

However, it is important to recognize that such an assumption is not reflective of real-world market conditions. In recent years, various studies have emphasized the importance of considering transaction costs or realistic constraints on transaction times, which ensure the absence of arbitrage opportunities in financial markets (e. g. [35, 36]).

2.2. Estimation of the integrated volatility based on p-variations

Our goal is to estimate the unknown Hurst index H and the volatility $\sigma = \sigma(t)$ and, in order to do that, let's employ the technique of p-variations as described in [30]. Let $\{X_{i-1}, i \geq 1\}$ be a sequence of random variables. For $k \geq 1$, define the difference of the k-th order as

$$\Delta_k X_{i-1} = \Delta_{k-1} X_i - \Delta_{k-1} X_{i-1}, \quad \Delta_1 X_{i-1} = \Delta X_{i-1} = X_i - X_{i-1},$$

i. e.

$$\Delta_k X_{i-1} := \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} X_{i+j-1}.$$

 ISSN-L 2664-9969; E-ISSN 2706-5448

Definition. It is possible to assume that $T>0$, $p>0$ and denote by

$$\tau^n:=\{i/n\,|\,i=0,1,...,[nT],\quad n\in\mathbb{N}\},$$

a partition of $[0,T]$. Then p-variation of the k-th order of random process $Z=\{Z(t),\ t\in[0,T]\}$ w. r. t. τ^n is a random process $V_{k,p}^n(Z)_t$, $t\in[0,T]$, of the form

$$V_{k,p}^n(Z)_t:=\sum_{i=1}^{[nt]-k+1}|\Delta_k Z_{(i-1)/n}|^p=\sum_{i=1}^{[nt]-k+1}\left|\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}Z_{(i+j-1)/n}\right|^p,$$

with an agreement that the sum is equal to zero, if $[nt]-k+1<1$.

Let $X=\{X(t),\ t\geq 0\}$ be the stochastic process defined by (4). Denote

$$c_{k,p}:=E[|\Delta_k B_n^H|^p],\tag{6}$$

and observe that, by [30],

$$c_{k,p}=\frac{2^{p/2}\Gamma((p+1)/2)}{\Gamma(1/2)}(\rho_{k,H}(0))^{p/2},$$

where

$$\rho_{k,H}(0):=\frac{1}{2}\sum_{i=-k}^{k}(-1)^{1-i}\binom{2k}{k-i}|i|^{2H}.$$

In particular

$$c_{1,2}=1,\quad c_{2,2}=4-2^{2H}.$$

Define

$$Z_t=\int_0^t\sigma(s)dB^H(s),$$

and note that, by [30],

$$\frac{1}{n^{1-pH}}V_{k,p}^n(Z)_t\to c_{k,p}\int_0^t|\sigma(s)|^p\,ds,\quad n\to\infty,\tag{7}$$

where the convergence is understood as uniform convergence in probability on compact sets, i. e. for any $T>0$

$$P\left(\sup_{t\in[0,T]}\left|\frac{1}{c_{k,p}n^{1-pH}}V_{k,p}^n(X)_t-\int_0^t|\sigma(s)|^p\,ds\right|>\varepsilon\right)\to 0,\quad n\to\infty.$$

Next, note that

$$n^{pH-1}V_{1,p}^n\left(\int_0^T\beta(s)ds\right)_T=n^{pH-1}\sum_{i=1}^{[nT]}\left|\int_{(i-1)/n}^{i/n}\beta(s)ds\right|^p\leq$$

$$\leq n^{-p(1-H)}\|\beta\|_p^p\to 0,\quad n\to\infty.$$

Moreover, from the definition of $V_{k,p}^n(Z)_t$ and the Hölder inequality, it is trivial to deduce that

$$n^{pH-1}V_{k,p}^n\left(\int_0^T\beta(s)ds\right)_T\leq n^{pH-1}\sum_{i=1}^{[nt]-k+1}2^{p-1}\left(\left|\Delta_{k-1}\int_0^{i/n}\beta(s)ds\right|^p+\right.$$

$$\left.+\left|\Delta_{k-1}\int_0^{(i-1)/n}\beta(s)ds\right|^p\right).$$

Hence, by induction, for all $p>0$ and $k=1,2,...$

$$\frac{1}{n^{1-pH}}V_{k,p}^n\left(\int_0^{\cdot}\beta(s)ds\right)_T\to 0,\quad n\to\infty.\tag{8}$$

In this way, summarizing (7) and (8), it is possible to obtain

$$\frac{1}{n^{1-pH}}V_{k,p}^n(X)_T\to c_{k,p}\int_0^T|\sigma(s)|^p\,ds,\quad n\to\infty.\tag{9}$$

2.3. Estimation of the Hurst exponent

The convergence (9) can be used to get a (weakly) consistent estimator of the integrated volatility $\int_0^T|\sigma(s)|^p\,ds$, *if the Hurst index were known*. However, in our setting it is not the case and it is possible to estimate H first. In order to do that, let's use the approach suggested in [31]. Namely, let's observe that

$$\frac{1}{(2n)^{1-pH}}V_{k,p}^{2n}(X)_T\to c_{k,p}\int_0^T|\sigma(s)|^p\,ds,\quad n\to\infty,\tag{10}$$

where $V_{k,p}^{2n}(X)_T$ is the p-variation of k-th order w. r. t. the partition

$$\tau^n:=\{i/(2n)\,|\,i=0,1,...,[2nT],\quad n\in\mathbb{N}\},$$

and the convergence is understood as the uniform convergence in probability on compact sets. Dividing (9) and (10), let's obtain that

$$2^{1-pH}\frac{V_{k,p}^n(X)_T}{V_{k,p}^{2n}(X)_T}\to 1,\quad n\to\infty.$$

Therefore,

$$(1-pH)\log 2+\log\left(\frac{V_{k,p}^n(X)_T}{V_{k,p}^{2n}(X)_T}\right)\to 0,\quad n\to\infty,$$

in probability and hence let's obtain a (weakly) consistent estimator of the Hurst index $H\in(0,1)$

$$\widehat{H}=\frac{1}{p}-\frac{\log V_{k,p}^{2n}(X)_T-\log V_{k,p}^n(X)_T}{p\log 2}.\tag{11}$$

Then it is possible to plug $\widehat{H}$ from (11) to (10) and use

$$\hat{I}_\sigma(t):=\frac{1}{c_{k,p}n^{1-p\hat{H}}}V_{k,p}^n(X)_t,\tag{12}$$

as an estimator of the integrated volatility

$$I_\sigma(t):=\int_0^t|\sigma(s)|^p\,ds.$$

2.4. Estimation of the spot volatility based on the moving average smoothing

Finally, in order to retrieve σ from $\hat{I}_\sigma$ in (12), let's use a combination of numerical differentiation and moving average smoothing techniques.

Namely, for $m,n\in\mathbb{N}$, where n denotes the number of points in the partition, put

$$\tilde{\sigma}^p(t) := \frac{\hat{I}_\sigma(t+1/n) - \hat{I}_\sigma(t)}{1/n},$$

and

$$\hat{\sigma}^p(t) := \frac{1}{2m}\sum_{k:k/n\in\mathcal{I}_{m,n}(t)} \tilde{\sigma}^p(k/n). \qquad (13)$$

The estimator (13) can then be regarded as an estimator for σ^p provided that $m,n\to\infty$ and $m/n\to 0$.

3. Results and Discussion

3.1. Volatility estimation on empirical data

To showcase the efficacy of our estimation methodology, let's employ it to analyze the high-frequency (minute-by-minute) data obtained from the global financial markets. The aforementioned estimations were conducted on diverse financial instruments, such as ordinary stocks of Apple (31837 observations), Dow Jones industrial average index (33701 observations), ordinary stocks of Amazon (24330 observations), and the exchange rate of BitCoin cryptocurrency to USD (165299 observations), over the period of January 1 to May 1, 2020 (refer to Fig. 1).

Let's first estimate the Hurst index for the instruments listed above using the estimator (11). The results are given in Table 1.

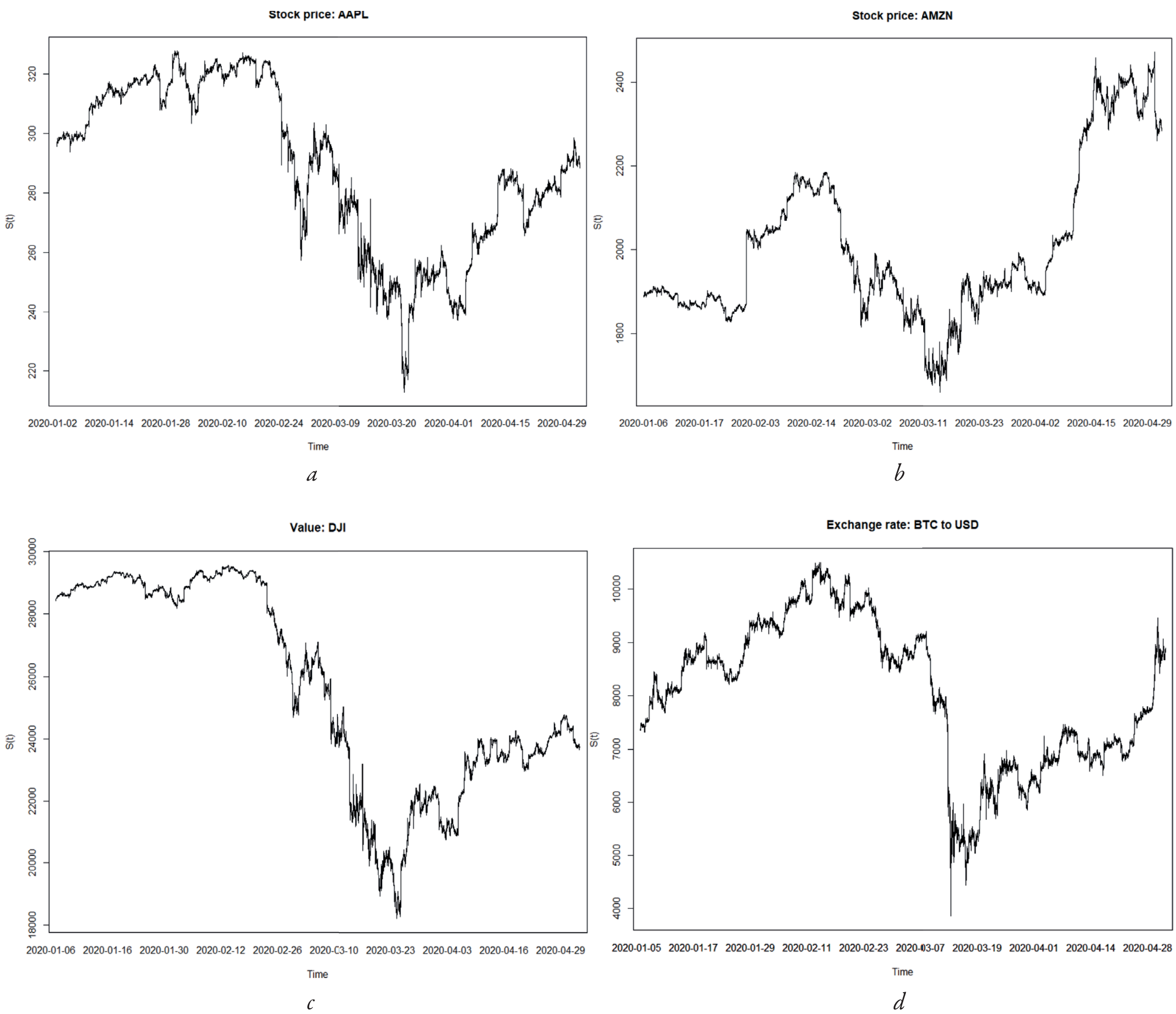


Fig. 1. Prices of financial instruments under consideration: *a* – Apple (AAPL); *b* – Amazon (AMZN); *c* – Dow Jones Index (DJI); *d* – BTC to USD

Table 1

Estimated Hurst index for high-frequency financial time series

Instrument	AAPL	AMZN	BTC/USD	DJI
Hurst index	0.4636085	0.5172341	0.3820844	0.1903221

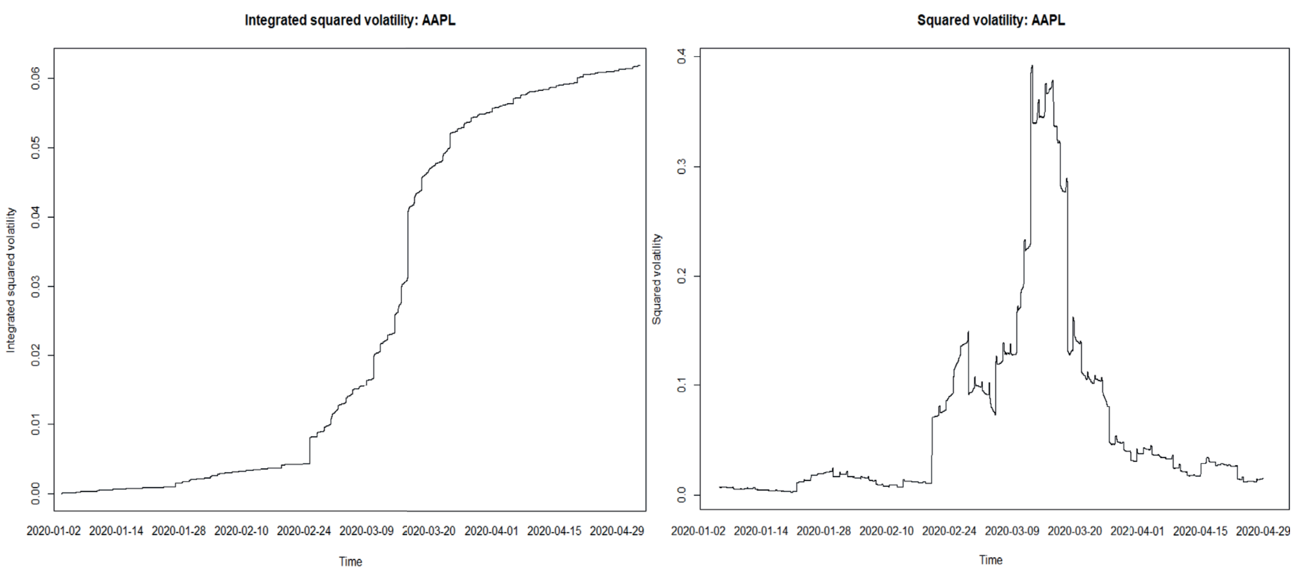
The estimations of integrated (12) and point (13) volatilities are represented in Fig. 2–5. In all cases, $k=1$, $p=2$.

Let's note that, by definition, the integrated volatility (12) is a non-decreasing function, whose slope reflects the level of market uncertainty on a given date: the steeper the slope, the more prone the asset price was to fluctuations on that day. As shown in Fig. 2, *a*–5, *a*, the slope of the integrated volatility sharply increases in the first quarter of 2020, corresponding precisely to the onset of the COVID-19 pandemic.

The spot volatility, depicted in Fig. 2, *b*–5, *b*, represents the estimated value of this slope. A higher value indicates a greater magnitude of randomness in the market. As evident from the figures, there is a pronounced surge in volatility in March 2020, which aligns with the initial phase of the COVID-19 crisis and supports the intuitive understanding of market behavior under extreme conditions. In other words, the model proposed in Section 2 efficiently captures the volatility explosion with high temporal resolution, demonstrating its sensitivity to abrupt structural breaks in market behavior. Additionally, for the AMZN stock (Fig. 4, *b*), a distinct volatility surge occurs in late January 2020. This spike coincides with the release of Amazon's annual financial report, which likely led to intensified trading activity and short-term price turbulence.

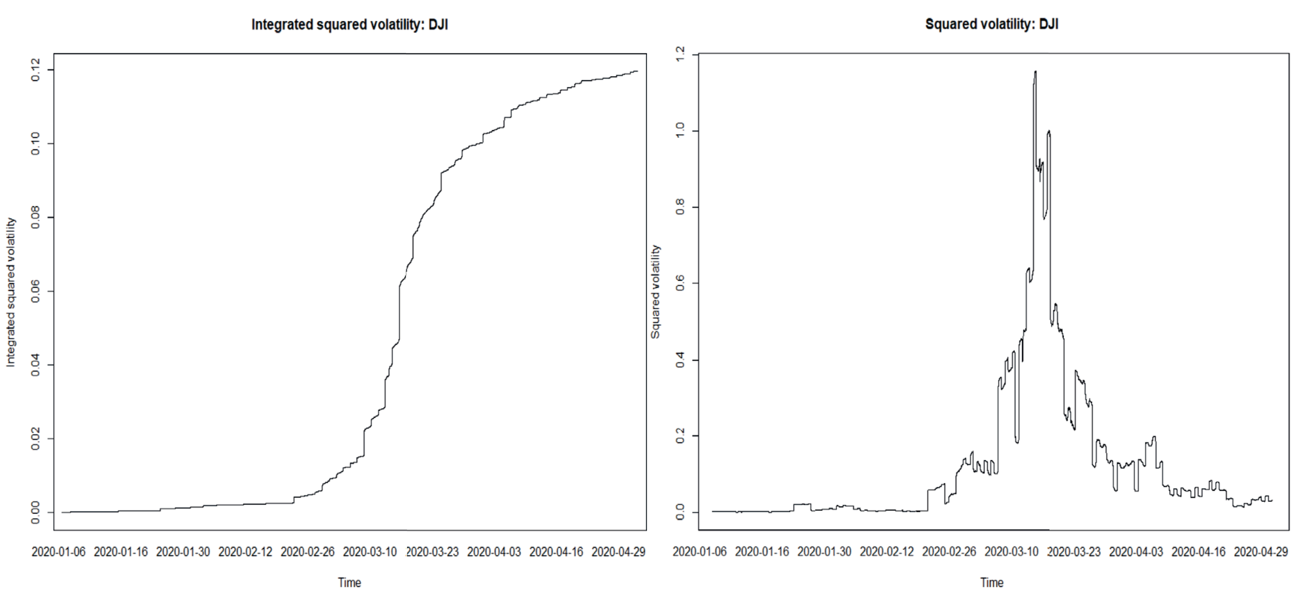
Table 1 further contextualizes the volatility dynamics by summarizing the estimated Hurst indices for the assets under consideration. In all cases except for Amazon, the Hurst index is found to be less than 0.5.

This result indicates a "short memory" regime characterized by negative autocorrelation in price increments – an expected feature during crisis periods, where market movements become more erratic and mean-reverting behavior dominates. The exception of Amazon, with a Hurst index exceeding 0.5, suggests relatively more persistent volatility, possibly due to firm-specific factors and investor sentiment that were less directly tied to the broader macroeconomic stress.



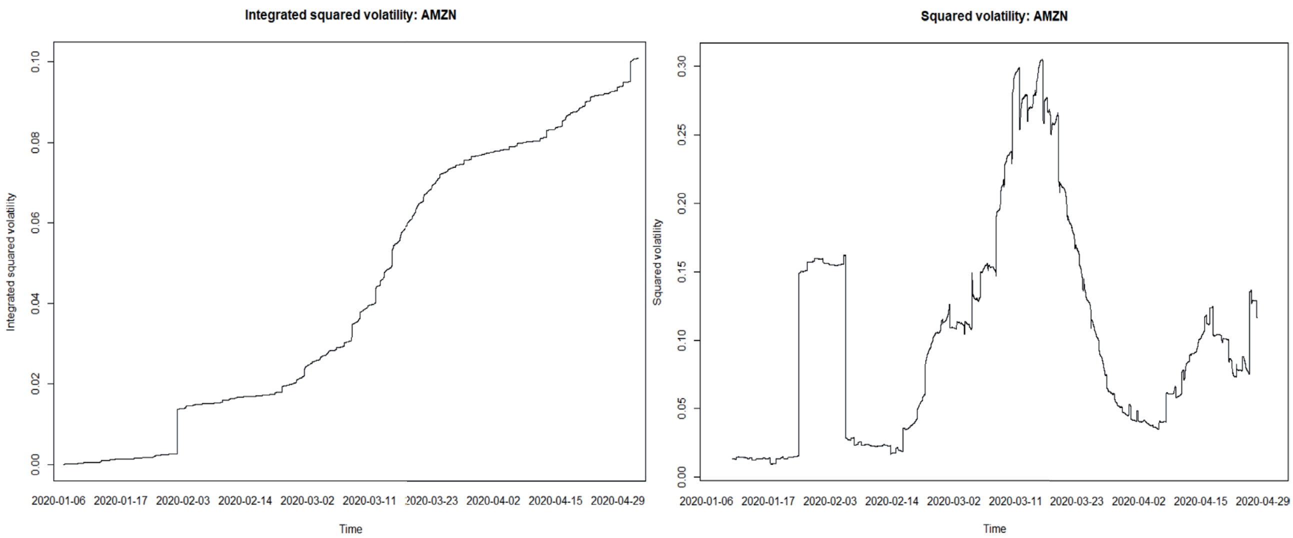
a *b*

Fig. 2. Volatility of Apple stock: *a* – estimated integrated volatility; *b* – estimated (squared) volatility $\sigma^2(t)$



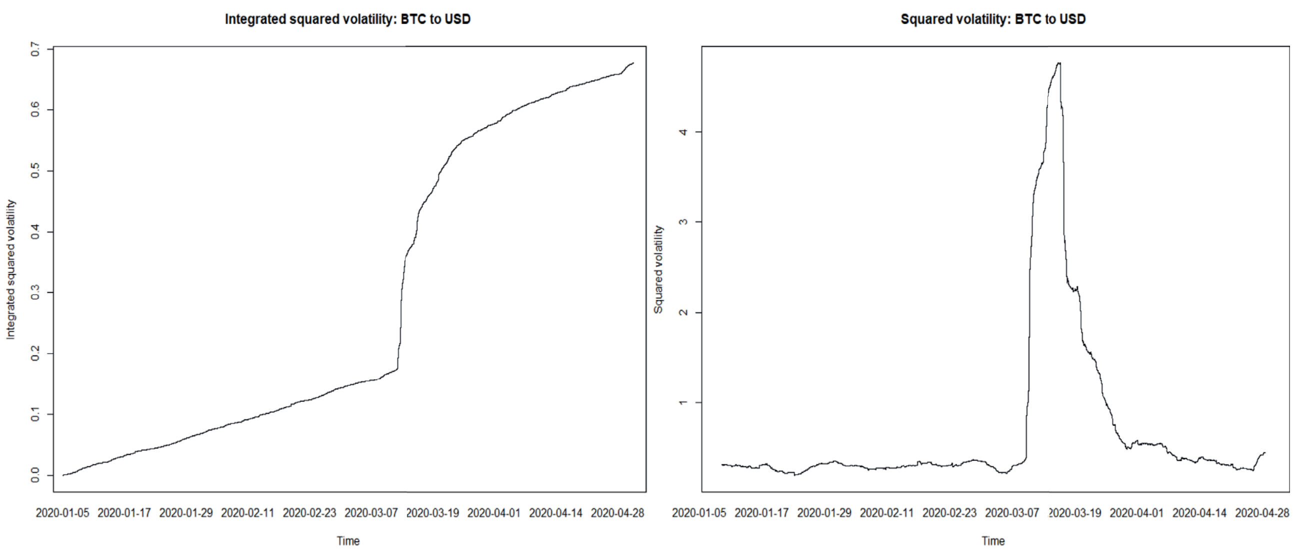
a *b*

Fig. 3. Volatility of DJI index: *a* – estimated integrated volatility; *b* – estimated (squared) volatility $\sigma^2(t)$



a *b*

Fig. 4. Volatility of Amazon stock: *a* – estimated integrated volatility; *b* – estimated (squared) volatility $\sigma^2(t)$



a *b*

Fig. 5. Volatility of BTC/USD exchange rate: *a* – estimated integrated volatility; *b* – estimated (squared) volatility $\sigma^2(t)$

Fig. 2–5 demonstrate that the uncertainty quantification methodology developed in Section 2 is consistent with the observed data and effectively captures the dynamics of time-dependent volatility under varying market conditions. Notably, with no input beyond the price data itself, the method enables clear identification of distinct market regimes and the timing of volatility surges.

This, in turn, allows for the delineation of crisis periods and an assessment of their magnitude. An added advantage of the approach lies in the incorporation of the Hurst exponent, which introduces an additional degree of freedom. The non-Markovian framework facilitates the modeling of dependence structures and complex correlations between price increments, capturing features that standard models often overlook.

Overall, the proposed methodology represents a significant improvement over the traditional constant volatility assumption, offering a more realistic and flexible tool for analyzing financial market behavior.

3.2. Discussion

Applications. The proposed p-variation-based method provides a practical and effective tool for estimating historical volatility under realistic assumptions, particularly in environments marked by long-range dependence and structural market shifts. Given that volatility remains the central measure of risk in financial theory and practice, these findings have direct implications for a wide array of applications. In particular, the reconstructed volatility paths can inform more accurate pricing of derivatives, dynamic hedging strategies, and portfolio optimization models, especially in periods of heightened uncertainty where traditional models may fail.

Limitations of research. The modeling framework assumes a continuous diffusion process, without accounting for discontinuities such as jumps or the presence of microstructure noise in high-frequency data. While the exclusion of these effects allows for a more tractable analysis, it may limit the precision of volatility estimates in settings where abrupt price changes or market frictions play a significant role.

ISSN-L 2664-9969; E-ISSN 2706-5448

Incorporating such factors could enhance the robustness of the model but would require more sophisticated statistical machinery and potentially more granular data.

Further research. The methodology introduced in this study opens the door to further investigations into the dynamics of financial volatility during periods of systemic disruption. A particularly promising direction is to apply the same framework to analyze the impact of the full-scale Russian invasion of Ukraine on global and regional financial markets. Given the method's demonstrated ability to detect and quantify volatility responses to geopolitical and economic shocks, such an extension would provide valuable insights into the persistence, memory, and structural shifts induced by war-related uncertainty.

4. Conclusions

A novel methodology for estimating volatility in financial time series using a fractional diffusion model is proposed in this research. The model, which is derived from the Black-Scholes-Merton-Samuelson framework, accounts for both the market memory effect through fractional Brownian motion and the temporal changes in volatility. The Hurst index and volatility functions are estimated using the p-variations of k-order method and subsequent numerical differentiation accompanied by moving average smoothing. Our analysis reveals an unprecedented surge in volatility during March 2020, which can be attributed to the onset of the COVID-19 pandemic and its impact on the global stock markets. Remarkably, the proposed model is able to accurately capture this phenomenon, highlighting its superior performance in comparison to the standard Black-Scholes-Merton-Samuelson model.

Furthermore, our method operates without requiring exogenous inputs or parametric volatility assumptions, relying solely on observed asset prices. This makes it not only broadly applicable but also particularly effective during periods of high market stress, as evidenced by its performance during the COVID-19 volatility spike.

In addition to its empirical accuracy, the methodology offers practical utility in various domains of quantitative finance. The ability to estimate historical volatility with high resolution and minimal assumptions opens the door to improved derivative pricing, more effective risk management strategies, and optimized portfolio allocation, particularly in volatile environments.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, including financial, personal, authorship or other, which could affect the research and its results presented in this article.

Financing

The research was performed without financial support.

Data availability

The data used in this manuscript can be provided upon reasonable request.

Use of artificial intelligence

The authors conform that artificial intelligence technologies were not used in this research.

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