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DEVELOPMENT OF AN ANALYTICAL MODEL OF THE CONTROLLED MOVEMENT OF GRAIN MATERIAL ON THE BULK SHELVES OF A LOADING GRAVITY-CASCADE UNIT

The object of research is the gravitational movement of grain along the transfer shelves and a cascade loading unit with two acceleration and two braking sections. The study of such movement is carried out to confirm theoretical studies on the development and justification of an analytical model of controlled gravitational movement of grain along transfer shelves.

When loading grain, it can be injured when falling from a considerable height and hitting the bottom and walls of the container. This problem requires the development and study of a technical solution that would provide regulation of the velocity of grain movement when loading it into the container.

Theoretical studies were carried out using the developed analytical model of grain movement and the proposed equations to find the relationships between the angles of inclination of the acceleration and braking shelves of the gravity-cascade unit. Based on the analytical model, an experimental unit was made of two acceleration and two braking shelves. The shelves can freely rotate on the axes to the required angle in the range from 0° to 90° relative to the horizontal plane. For the shelves of the acceleration sections, the angle of inclination α was chosen from the variable series of 45°, 50°, 60°. Based on the angle α, according to the model, the shelf of the first braking section was set at an angle of 20.43°, 20.48°, 20.32°, and the shelf of the second braking section was set at 38.46°, 35.28°, 29.32°.

Experimental studies have shown that the velocity of grain movement is indeed regulated by a combination of the ratios of the angles of the acceleration and braking shelves. In this case, the velocity of grain in the last braking section is close to the initial flow velocity at the beginning of the first acceleration shelf. The values of the absolute and relative errors of the experiments of the experimentally determined velocities and the theoretical value of the velocity indicate quite acceptable limits of deviations for this multifactorial experiment. The relative deviation of the experimental from the theoretical velocity of movement of the grain mass does not exceed 12.76%.

The results obtained and their analysis indicate that the presented analytical model and the designed gravity-cascade unit due to the braking and acceleration sections allow solving the problem of controlled movement of the velocity of grain for its loading into containers without injury, in particular into silo structures.

Keywords: *velocity of grain movement, acceleration and braking shelves, variable angles of inclination.*

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1. Introduction

For storage of large batches of dry and cleaned grain in elevators, metal silos with a flat concrete bottom are used. At these enterprises, all technological operations for transporting grain to silos are fully mechanized. Therefore, when moving grain material before loading it into silos, the grain repeatedly contacts the working bodies of transport machines and at the same time it is possible to damage it. Studies have shown that the working bodies of combines during threshing damage up to 20% of grains [1]. In turn, the amount of grain damaged by elevators can reach up to 12%, and by moving elements of conveyors up to 2.5% [2]. This means that when the grain load approaches the silo loading mouth, some part of the grain load already has a certain percentage of damaged grain.

During further loading into the silo, the grain is subjected to mechanical damage during its free fall and impact interaction with the silo elements. This is due to the fact that the grain gains velocity with increasing fall height, which leads to large impact forces. In addition, the grain flow velocity can exceed the terminal velocity of one kernel when the fall height exceeds 15 m [3]. The level of injury depends on the type of grain and its physical and mechanical properties, in particular, for wheat it is 0.4% lower than for soybeans and corn [3]. Grain injury is also affected by its moisture level. Thus, with grain moisture within 14–16%, the limiting impact value at which injury occurs is within 0.11–0.16 J [4]. It has been established that almost 80% of the total grain deformation occurs at the stage of initial loading into silos [5]. Injured grain has less resistance to storage, and its gravitational loading leads to significant compaction of grain material in the lower layers of the grain embankment [6, 7], which negatively affects its unloading. Therefore, mechanical and impact damage to grain negatively affects its quality. Damaged grain is less stable, it breathes more intensively,

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it releases more heat and moisture. All this leads to the emergence of foci of self-heating of the grain mass, which in turn leads to spoilage of the grain or its complete death. In addition, the impact interaction of the grain with the bottom and walls of the silo can also lead to damage to the kernel germ. Such damage also negatively affects the sowing quality of the grain. Thus, reducing damage to the grain load at the stage of loading it into silos is an important problem, the necessity of which is due to the losses of grain stored in silos. This problem requires finding ways to safely load silos with grain without damaging it. An option for overcoming the problem of loading high-altitude silos with grain without damaging it is to create a cascade loading device that allows to control the velocity of grain movement during its gravitational loading, and to mathematically substantiate its operation.

Some studies on the controlled movement of grain are described in [8, 9]. In [8], the dependences of the change in the velocity of movement of grain cargo in an inclined channel with three variable angles are given. This article describes a device consisting of straight sections made in the form of gutters, one of which is an acceleration section and the other two are braking sections. With the help of this device, it is possible to control the velocity of descent of grain material from significant heights. The proposed device cannot be used when loading cylindrical high-altitude containers. In [9], a peripheral open screw channel with acceleration and braking sections is presented, which is mounted on the inner side surface of the silo for gravitational loading of high structures with grain. In the acceleration screw section, the angle of inclination α is 30–40 degrees, in the braking section it is determined analytically depending on the angle α and is much smaller. This leads to a decrease in the velocity of grain movement, and therefore to a decrease in its injury, but the device is difficult to manufacture and install. The study of the movement of grain mass along a helical line is also described in [10], which provides the equation of motion of a material point along the surface of a spiral chute under the action of gravity. The case is separately considered when the lowest turn of the chute has a lift angle equal to the angle of friction of the grain on the surface. This model does not take into account the change in the angle of inclination of the chute and the ability to regulate the velocity of grain movement. In [11], to reduce injury and evenly distribute grain, it is proposed to use a toroidal plate installed in the lower part of the corrugated loading sleeve. The grain load, falling on the concave surface of the plate, flies out of the plate at a certain angle to the horizon, while extinguishing excess kinetic energy. The disadvantage of the device is the size of the plate and the complexity of its manufacture. In [12], loading devices mounted under the silo neck were analyzed for uniform distribution of grain. One of them is a centrifugal spreader, where the shaft of the device is driven into rotation by an electric motor. The disadvantage is the injury of the grain by inertia forces when the grain hits the walls of the container.

A separate problem is the gravitational loading of high-altitude silos with grains of non-uniform size, shape and mass, which leads to stratification of the grain material by density and the appearance of segregation and compaction of the grain mound [12, 13]. In this case, the grain caking occurs, and as a result, its nutritional and technical qualities deteriorate. Many studies indicate that the degree of uneven distribution of grain material along the entire perimeter of the container depends on the loading height. In particular, in [14] it was experimentally established that the coefficient of lateral pressure is less than the coefficient of active ground pressure, i. e. uncontrolled gravitational loading of high-rise silo structures does not ensure uniform distribution of grain material along the entire perimeter of the container. At the same time, compacted pressure is formed in the lower layers of the embankment. Studies have shown that the vertical load of the compacted embankment on the support depends on the height, shape and geometry of the silo structures [15]. The greatest load on the cylindrical container occurs at the initial moment of grain release and it increases in a galloping manner [16]. To eliminate these negative phenomena, researchers propose special loading devices [9, 11, 12] with which it is possible to reduce the height of free fall of grain, and therefore reduce its injury. At the same time, they are not convenient and structurally complex for loading high-rise silos.

In the process of designing loading devices, much attention is paid to the development of physical and mathematical models of grain movement along their working bodies. In particular, in works [17, 18], dependencies are given for determining the movement of grain, but the velocity of its movement along the working bodies of machines is not determined. In [19], the influence of loading velocity on the deformation of wheat grains is studied. In the study [20], a numerical model of the movement of gravitational flow of particles inside the conical outlet channel of the bunker is presented. In [21], the velocity of grain movement along a cylindrical sieve is described. Unfortunately, the considered models do not pay attention to controlling the velocity of gravitational movement of grain.

Therefore, despite the significant number of devices and studies devoted to the issues of uniform and shock-free loading of silos with grain material, the problem of loading high-altitude silo structures remains relevant and not fully resolved. Existing devices are unable to control and regulate the velocity of grain material movement during its gravitational loading. To solve the above problem, it is necessary to develop and implement technical solutions that ensure controlled grain movement when loading it into high-altitude silo structures. Devices that are intended to reduce and stabilize the loading velocity should not be complicated in their manufacture and operation. Therefore, an urgent problem is the need to develop a loading gravity-cascade unit that provides control of the grain movement velocity due to different angles of inclination of the acceleration and braking shelves.

The aim of research is to develop and substantiate a model of controlled grain movement velocity along the transfer shelves of the loading gravity-cascade unit, which will allow determining its main geometric parameters and experimentally verifying the adequacy of the presented model.

2. Materials and Methods

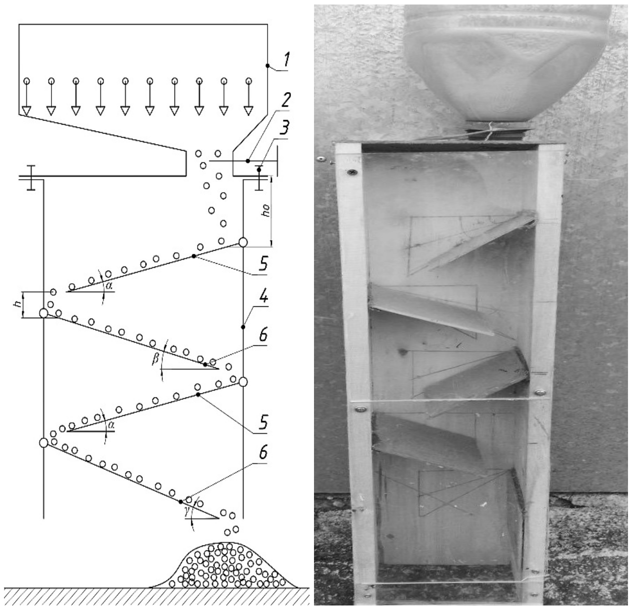
The object of research is the gravitational movement of grain along the transfer shelves and a cascade loading unit with two acceleration and two braking sections. The study of such movement is carried out to confirm theoretical research, to develop and substantiate an analytical model of controlled gravitational movement of grain along transfer shelves.

The material on the movement of grain material along transfer shelves inclined at different angles to the horizon was theoretical and practical research on the movement of grain during its loading into high-altitude silo structures. When conducting theoretical research, the law of conservation of energy, transformations and graphical definitions based on the use of the laws of theoretical mechanics were applied.

The processing of the results of grain movement along the transfer shelves of the gravity-cascade unit was carried out using the method of mathematical statistics and the method of empirical research. This made it possible to establish the optimal angles of inclination of the accelerating and braking transfer shelves of the unit that would provide a controlled velocity of grain movement.

To conduct experiments on the movement of grain material along transfer shelves, a schematic diagram of a gravity-cascade unit for controlled loading of silos with grain was proposed (Fig. 1, *a*).

The movement of grain along the transfer shelves of a gravity-cascade unit (Fig. 1, *a*) consisting of four transfer shelves is considered. Two shelves are accelerating shelves with length l_1 and inclination angles α. Two shelves are braking shelves with length l_2 and inclination angles β and γ, respectively. The shelves alternate sequentially.



a *b*

Fig. 1. Gravity-cascade unit with four transfer shelves: *a* – schematic diagram; *b* – prototype; 1 – hopper; 2 – damper; 3 – mounting; 4 – metal case in the form of a parallelepiped; 5 – acceleration shelves; 6 – braking shelves

During the movement of grain, internal friction occurs inside the grain flow and external friction occurs between the shelves and the outer layer of the grain mass. It is possible to assume that the grain mass is poured out of the hopper of the experimental unit and has a stationary flow. To reduce grain injury, it is necessary that the final velocity of the grain at the end of the movement along the second brake shelf be the same as the initial velocity of the flow at the beginning of the first acceleration shelf [8], but not less than it $V_{start} \leq V_{end}$. This condition will ensure careful loading of the grain mass, and will also facilitate the passage of grain without its accumulation on the sections of the brake shelves. To simplify further calculations, it is possible to assume that these velocities are equal to each other $V_{start} = V_{end} = V_0$. The height of the fall h between all 4 shelves is the same.

The initial velocity V_0 of the grain is acquired as a result of its fall from the hopper, which is located at a height ho from the edge of the first acceleration shelf. At the moment of falling of grain mass from a hole of a bunker on initial edge of the first acceleration section its potential energy transition to kinetic energy occurs, as a result of which grain acquires velocity V_0. Using the law of conservation of energy, there is an initial velocity of grain $V_0 = \sqrt{2gh_0}$ [8].

Continuing movement, grain flow increases its velocity to $V_{\max}$ passing path l_1 on the first acceleration section, located at an angle α to the horizon. Taking into account transition of part of potential energy of grain $mgl_1\sin\alpha$ into kinetic energy, as well as coefficient of friction μ between grain layer and surface of shelf [8, 9], it is possible to simplify equation describing transformation of energy of grain flow on first acceleration shelf

$$\frac{mV_0^2}{2}\sin\alpha + mgl_1\sin\alpha - \mu mgl_1\cos\alpha = \frac{mV_{\max}^2}{2}. \quad (1)$$

After passing the first acceleration shelf, the grain falls from a distance h, which is the average value of the grain fall trajectories (Fig. 1) to the first braking shelf. Taking into account the angles of inclination of the acceleration and braking section

$$\left(\frac{mV_{\max}^2}{2}\sin\alpha + mgh\right)\sin\beta. \quad (2)$$

The movement of the grain flow along the first braking shelf with a length of l_2, which is located at an angle β to the horizon, occurs due to the previously acquired maximum kinetic energy on the first acceleration shelf. At the same time, the additionally acquired kinetic energy of falling from a height h and the potential energy of the grain $\mu mgl_2\sin\beta$ are taken into account. At the same time, this process is counteracted by the work of the friction force $\mu mgl_2\cos\beta$, which, due to the smaller angle of inclination $\beta \leq \alpha$, is already much larger [8, 9, 16] and causes a decrease in the velocity of movement of the grain flow from $V_{\max}$ to $2V_{\max}/3$. Thus, after passing the first braking section, the model assumes a reduction in the maximum grain velocity by one third, which should prevent the accumulation and settling of grain. From the law of conservation of energy

$$\left(\frac{mV_{\max}^2}{2}\sin\alpha + mgh\right)\sin\beta + mgl_2\sin\beta - \mu mgl_2\cos\beta = \frac{m\left(\frac{2V_{\max}}{3}\right)^2}{2}. \quad (3)$$

After passing the first braking shelf, the grain again falls from a distance h to the second accelerating shelf. Using the law of conservation of energy and taking into account the angles of inclination of the accelerating and braking sections, there is $\left(\frac{2mV_{\max}^2}{9}\sin\beta + mgh\right)\sin\alpha$. Continuing to move along the shelf, the grain flow increases its velocity from to the velocity V, $\left(\frac{2V_{\max}}{3} \leq V \leq V_{\max}\right)$.

The movement of the grain flow along the second braking shelf, located at an angle to the horizon, occurs similarly due to the previously acquired kinetic and potential energy. This energy is counteracted by the work of the friction force, which, due to the smaller angle of inclination γ $(\gamma \leq \alpha)$, is already much larger and causes a decrease in the velocity of the grain flow from V to the initial V_0.

3. Results and Discussion

3.1. Analytical model of controlled grain movement

Combining the above dependencies that determine the movement of grain along the acceleration and braking sections, an analytical model of the gravitational movement of grain along the shelves of the gravity-cascade unit was obtained

$$\begin{cases} mgh_0 = \dfrac{mV_0^2}{2}, \\ \dfrac{mV_0^2}{2}\sin\alpha + mgl_1\sin\alpha - \mu mgl_1\cos\alpha = \dfrac{mV_{\max}^2}{2}, \\ \left(\dfrac{mV_{\max}^2}{2}\sin\alpha + mgh\right)\sin\beta + mgl_2\sin\beta - \mu mgl_2\cos\beta = \dfrac{2mV_{\max}^2}{9}, \\ \left(\dfrac{2mV_{\max}^2}{9}\sin\beta + mgh\right)\sin\alpha + mgl_1\sin\alpha - \mu mgl_1\cos\alpha = \dfrac{mV^2}{2}, \\ \left(\dfrac{mV^2}{2}\sin\alpha + mgh\right)\sin\beta + mgl_2\sin\gamma - \mu mgl_2\cos\gamma = \dfrac{mV_0^2}{2}. \end{cases} \quad (4)$$

To find the relations that will determine the dependence between the angles of the acceleration and deceleration sections, it is necessary to solve the system (4). From the first, second and third equations of the system (4) there is the equation that determines the angle β through the given angle α

$$\left((h_0 + l_1)\sin^2\alpha - \mu l_1\sin\alpha\cos\alpha + h + l_2\right)\sin\beta - \mu l_2\cos\beta = \frac{4}{9}\left(h_0\sin\alpha + l_1\sin\alpha - \mu l_1\cos\alpha\right). \quad (5)$$

By equating the second, fourth and fifth equations of the system (4) and simplifying the expression, the equation that determines γ through the angles α and β was obtained

$$l_2\sin\gamma-\mu l_2\cos\gamma=h_0-h\sin\beta-\frac{4}{9}\left(h_0+l_1\right)\sin^3\alpha\sin^2\beta+$$
$$+\frac{4}{9}\mu l_1\cos\alpha\sin^2\alpha\sin^2\beta-\left(h+l_1\right)\sin^2\alpha\sin\beta+\frac{1}{2}\mu l_1\sin2\alpha\sin\beta. \quad (6)$$

Equation (5) was solved through the appropriate substitutions. Given that the angles α and β lie in the range from 0 to $\pi/2$, the equation describing the dependence of the angle β on the angle α was obtained

$$\beta=2\mathrm{arctg}\left(\frac{-a+\sqrt{a^2+(\mu^2l_2^2-b^2)}}{\mu l_2-b}\right)+2\pi k,$$
$$a=\left(\left(h_0+l_1\right)\sin^2\alpha-\mu l_1\sin\alpha\cos\alpha+h+l_2\right),$$
$$b=\frac{4}{9}\left(h_0\sin\alpha+l_1\sin\alpha-\mu l_1\cos\alpha\right). \quad (7)$$

Similarly, equation (6) was solved, through the appropriate substitutions

$$\gamma=2\mathrm{arctg}\left(\frac{-c+\sqrt{c^2+(\mu^2l_2^2-d^2)}}{\mu l_2-d}\right)+2\pi k. \quad (8)$$

The resulting system of equations determines the angles β and γ through the given angle α

$$\begin{cases} a=\left(h_0+l_1\right)\sin^2\alpha-\mu l_1\sin\alpha\cos\alpha+h+l_2, \\ b=\frac{4}{9}\left(h_0\sin\alpha+l_1\sin\alpha-\mu l_1\cos\alpha\right), \\ \beta=2\mathrm{arctg}\left(\frac{-a+\sqrt{a^2+(\mu^2l_2^2-b^2)}}{\mu l_2-b}\right)+2\pi k,\ c=l_2, \\ d=h_0-h\sin\beta-\frac{4}{9}\left(h_0+l_1\right)\sin^3\alpha\sin^2\beta+ \\ +\frac{4}{9}\mu l_1\cos\alpha\sin^2\alpha\sin^2\beta-\left(h+l_1\right)\sin^2\alpha\sin\beta+ \\ +\frac{1}{2}\mu l_1\sin2\alpha\sin\beta, \\ \gamma=2\mathrm{arctg}\left(\frac{-c+\sqrt{c^2+(\mu^2l_2^2-d^2)}}{\mu l_2-d}\right)+2\pi k. \end{cases} \quad (9)$$

The resulting model, defined by the systems of equations (4) and (9), determines the dependence of the angles β and γ of the inclination of the braking shelves on the angle α of the inclination of the accelerating shelves. The model takes into account the initial height of the grain fall h_0 on the first accelerating section, the height of the grain fall h from one shelf to another, the lengths of the accelerating and braking sections l_1 and l_2, as well as the friction coefficient μ between the grain layer and the shelf material.

3.2. Determination of the main geometric parameters of the loading gravity-cascade unit and their discussion

The study of the nature of the change in the parameters of the gravity-cascade unit was carried out under the following conditions: the height of the fall $h=0.2$ m; the length of all shelves $l_1=l_2=l$; the initial height of the fall $h_0=0.3$ m; the angles of inclination of the shelves $\beta\leq\alpha$, $\gamma\leq\alpha$. The angle of inclination α of the shelves of the accelerating sections and the length of the shelves l were taken as variable factors. In this case, the angle α varied in the range from 40° to 75°, and the length of the shelves was taken from the range of 0.2–0.6 m.

The determining parameters of the theoretical studies were the angle β of the slope of the first shelf of the braking section and the angle γ of the slope of the second shelf of the braking section. In this case, the analysis was subjected to those results where both angles were less than or equal to the angle α of inclination of the acceleration shelves.

The calculation of the determining angles β and γ according to equations (6) was carried out in several stages, in each of which one of the variable factors took a constant value, and the other varied within the limits accepted for it.

Thus, Fig. 2 shows the results of determining the angles β and γ when changing the length of the shelves l in a variable series of constant values of the angle of inclination α of the acceleration sections: 40°, 50°, 60° and 75°. The graphs show a different nature of the formation of angles β and γ both in terms of trend and numerical range of their change when varying the shelf length l.

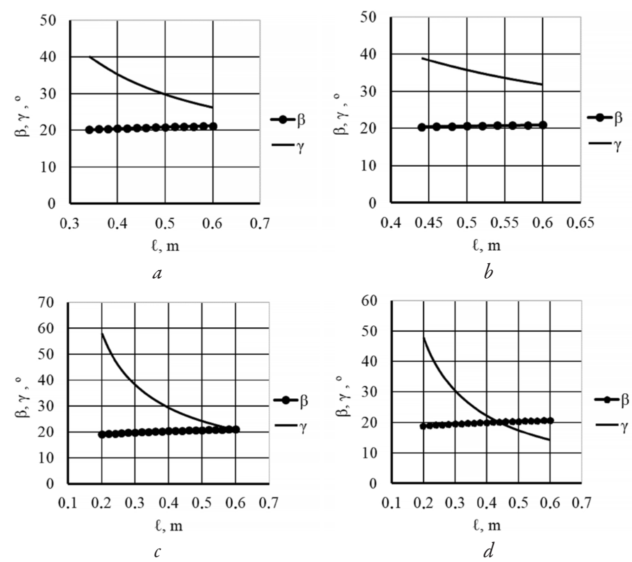


Fig. 2. Dependence of angles β and γ on l and α: *a* – $\alpha=40°$; *b* – $\alpha=50°$; *c* – $\alpha=60°$; *d* – $\alpha=75°$

The angle of inclination β of the first braking section with increasing shelf length l for each constant angle α has a linear growth character. In this case, the range of angle β change is from 18.9° to 21° for all variants of angle α.

Unlike angle β, angle γ of the second braking section is characterized by its nonlinear change, similar to a hyperbolic form, and decreases with increasing length l. It should be noted that at certain angles α the angle γ is formed only in a partial range of change l. In particular, at $\alpha=40°$ the angle γ is within 31.79–38.9° when changing length l from 0.44 m to 0.6 m. This is due to the fact that when l decreases to its lower limit of 0.2 m, the value of angle γ becomes greater than α. This violates the outlined conditions of the relationship between these angles. This restriction on angle γ also slightly corrects the limits of change of angle β. Thus, its minimum value will be 20.41° instead of 18.89°.

A similar nature of the restrictions for the angles β and γ is observed at $\alpha=50°$: the angle γ varies only within the range of 26.22–40.29° at $l=0.34$–0.6 m, and the range for β is 20.18–21.17°.

For other angles α, the angle γ can be found for the entire accepted interval l from 0.2 m to 0.6 m. For example, at $\alpha=60°$ the angle γ varies from 20.84° to 58.02°, and at $\alpha=75°$ the angle $\gamma=14.18$–$47.90°$.

It should also be noted that at $\alpha=75°$ the characteristics of the angles β and γ have an intersection point corresponding to the angle 20° at the length $l=0.43$ m. This allows to install shelves of different braking sections at the same angle to the horizontal plane.

Fig. 3 shows the results of determining the angles β and γ when changing the angle of inclination α of the acceleration sections in a variable series of constant values of the shelf length *l*: 0.2, 0.3, 0.4 and 0.6 m.

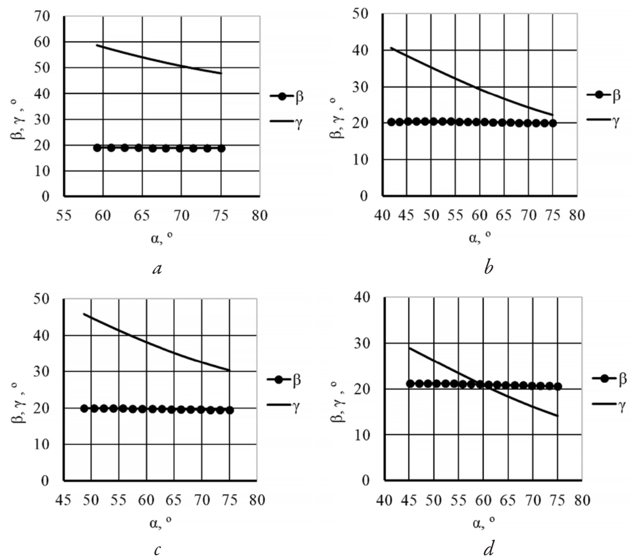


Fig. 3. Dependence of angles β and γ on α and *l*: *a* – $l=0.2$ m; *b* – $l=0.3$ m; *c* – $l=0.4$ m; *d* – $l=0.6$ m

For both angles β and γ, an increase in angle α leads to their decrease. At the same time, the nature of their change is as close as possible to linear dependences, which is most clearly observed for angle γ. In turn, the dependence of angle β on α at different lengths *l* is an almost flat section of a straight line, at the boundary of which the angle β decreases by a small amount – 0.14–0.32°. The absolute value of angle β, depending on the influencing factors, varies from 18.75° to 20.93°, which is significantly less than any accepted angle α and fully complies with the condition of the ratio between these angles. In the situation with angle γ, the accepted condition of the ratio with angle α is fulfilled only in a certain range of α change. Thus, at $l=0.2$ m the required ratio is achieved in the range of α change from 59.25° to 75°, when γ decreases from 58.66° to 47.90°. With increasing shelf length, the limits of variation of angles γ and α are expanded, subject to the imposed restrictions. For $l=0.3$ m, the permissible decrease in γ occurs in a wider range of 45.83–30.38° with an increase in angle α from 48.75° to 75°. The complete correspondence of angle γ to angle α throughout the entire range of its change is observed only at $l=0.6$ m. In this case, γ decreases from 31.79° to 14.18°. It should be noted that at $l=0.6$ m, the graphical characteristics of angles β and γ have a common intersection point at $\alpha=60.5°$, in which both angles take the same value of 21°. This allows the shelves of both acceleration sections to be installed at the same angle to the horizontal plane.

Taking into account the formed and analyzed characteristics of the change in the inclination angles of the shelves of the gravity-cascade unit from the influencing factors, it is possible to state that it is theoretically possible to comply with the key condition for using this unit, namely the equality of the velocity of grain movement at the entrance and exit from it, that is, controlled grain movement.

3.3. Experimental studies of the adequacy of the model and their analysis

Based on the theoretical studies conducted, a prototype of a gravity-cascade unit with four transfer shelves (Fig. 1, *b*) was proposed. The shelves are of equal length, which can freely rotate on axes fixed in the device body to the required angle inside the device relative to the horizontal plane. The angle can be set in the range from 0° to 90°. The angle was controlled using УН-127 Vernier goniometer. The horizontal plane was set with a self-leveling laser level.

For the manufacture of shelves, sheet material (fiberboard) was selected, cut into parts of equal length. The length of each shelf was 0.4 m. The coefficient of friction of the grain material when moving along the shelves was taken equal to $\mu=0.35$, based on the results of previously conducted tribological studies.

On the proposed gravity-cascade unit, experimental studies of the movement of wheat grain of standard humidity along four shelves with several variants of their spatial orientation were carried out. Each shelf was located one under the other on opposite sides of the gravity-cascade unit body in such a way that the grain falling from the edge of one shelf fell to the beginning of the other. In this case, the height of the grain fall from each shelf was 0.2 m. Above the unit was a bunker with grain, the outlet neck of which was at a distance of 0.3 m from the upper edge of the first shelf. For the shelves of the acceleration sections, the angle of inclination α was selected from the variable series 45°, 50°, 60°. Based on the angle α, according to the analytical model, the shelf of the first braking section was installed at an angle of 20.43°, 20.48°, 20.32°, and the shelf of the second braking section was installed at an angle of 38.46°, 35.28°, 29.32°.

Tracking the velocity of grain movement at the exit of the gravity-cascade device was carried out by fixing the time of grain movement between control sections, which were at a distance of 0.1 m from each other. The velocity was determined in a similar way at the entrance to the device, but the control distance was the height of the location of the output neck of the grain bunker. The moment of grain appearance in a certain section was determined using capacitive presence sensors.

The research was carried out in three stages. At each stage, the angular location of the shelves changed in accordance with the formed variable series of inclination angles and the input and output velocity of grain movement were determined and compared. The studies were conducted with five-fold repeatability for each stage. The results of the experimental studies and their statistical processing are given in Table 1.

From the static analysis of the obtained experimental research results, it is clear that for each stage the initial V_{start} and final V_{end} grain velocity do not differ significantly. The maximum deviation between them is 0.46 m/s, and their relative deviation from the theoretically achievable velocity $V_{theor}=2.43$ m/s does not exceed 12.76%.

Table 1

Results of experiments on determining the velocity of grain movement at the entrance and exit of the gravity-cascade unit

Stage No.	Angles of inclination			Results of the experiment with five-fold repeatability of each stage							Theoretical velocity	Absolute/relative deviation from theoretical velocity	
	α	β	γ	Average value			Maximum deviation between values		Relative deviation from the average				
				V_{start}	V_{end}	Δ	V_{start}	V_{end}	V_{start}	V_{end}	V_{theor}	V_{start}	V_{end}
1	45	20.43	38.46	2.64	2.18	0.46	0.21	0.18	7.95	8.26	2.43	0.21/8.82	0.25/10.29
2	50	20.48	35.28	2.54	2.62	–0.08	0.32	0.34	12.6	12.98		0.11/4.53	0.19/7.82
3	60	20.32	29.32	2.74	2.30	0.44	0.20	0.21	7.30	9.13		0.31/12.76	0.13/5.35

In addition, it should be noted that the velocity indicators in each of the five experiments of the selected research stage are quite close. Thus, for the initial velocity V_{start}, the maximum deviation between the velocity values is observed at the second stage and is 0.32 m/s, which in relative terms is 12.6% of the average value at this stage. For the final velocity V_{end}, the maximum discrepancy is also observed at the second stage, when the absolute deviation is 0.34 m/s with a relative equivalent to the average value of 12.98%. For other stages, the discrepancies are somewhat lower and do not exceed 0.21 m/s in absolute values and 9.13% in relative values. This indicates an appropriate level of quality and repeatability of the conducted complex of experimental studies.

The values of absolute and relative errors of experiments of experimentally determined velocities and the theoretical value of the velocity indicate quite acceptable limits of deviations for this multifactorial experiment. Taking into account the existing complexity of the full-scale implementation of the experiment, everything indicates an acceptable level of accuracy of the presented analytical model.

The presented model and the gravity-cascade unit, due to the found ratios of the angles of the accelerating and braking shelves, allow solving the problem of controlled movement of grain velocity for its loading into silos without injury. The results obtained are better than previous studies by the authors [8, 9], regarding the solution of the problem of possible grain settling during its loading. In particular, the presented unit has four transfer shelves instead of an open inclined channel with variable angles of inclination, where in the areas of changing the inclination of the chute sometimes there was a settling of the grain mass. Also, due to the reduction of the free fall height and the controlled velocity of grain movement, the proposed unit does not cause segregation effects [13] and is structurally much simpler and less metal-intensive than, for example, the use of a toroidal plate with a variable angle of inclination [11].

3.4. Limitations of the results and prospects for the development of the research

A certain limitation of the proposed unit is the limits of the inclination angles α for the acceleration sections and the angles β and γ for the braking shelves. This ratio, according to (6), is selected in such a way that the angle β is not significantly less than the angle of natural slope ξ of the grain, and the angle γ is less than the angle α, that is, within $200 \leq \beta \leq \gamma \leq \alpha$.

The war significantly affected scientific research within the country. Among the main factors of influence: the destruction or damage to universities, laboratories, equipment, materials and libraries significantly complicates research; the greatest attention is paid to scientific developments for the needs of the army; scientists in Ukraine work under constant stress.

The prospects for further research are to study the feasibility of developing units with a different number of acceleration and braking transfer shelves with the possibility of adjusting their inclination relative to the horizon made of different materials depending on their frictional properties.

4. Conclusions

The presented analytical model of controlled grain movement along the transfer shelves of the loading gravity-cascade unit takes into account all the necessary parameters: the initial height of grain fall, the distance between the shelves, their length, as well as the friction coefficient. The model determines the dependence of the two angles of inclination of the braking shelves on the angle of inclination of the accelerating shelves, which ensures the passage of grain without its subsidence in any section and at the same time prevents injury and segregation of the grain mass due to a controlled decrease in the final velocity.

Using the analytical model, theoretical studies of the relationships between the angles of inclination of the accelerating and braking sections were carried out, which made it possible to substantiate the values of the recommended angles α, β and γ within $200 \leq \beta \leq \gamma \leq \alpha$. Based on these studies, a prototype of a gravity-cascade unit was proposed, on which experimental studies of grain movement along four transfer shelves with three variants of their spatial orientation were carried out.

Analysis of theoretical and experimental data has shown the adequacy and acceptable accuracy of the proposed model. The relative deviation of the experimental grain velocity from the theoretical does not exceed 12.76%. Thus, the designed gravity-cascade unit due to the braking and acceleration sections allows solving the problem of controlled grain velocity movement along the transfer shelves. This allows further use of the proposed model for the development of industrial versions of such units for loading grain mass into high-altitude silo structures without injury.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, including financial, personal, authorship or other, which could affect the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Derev'ianko, D., Sukmaniuk, O., Sarana, V., Derev'ianko, O. (2020). Justification of influence of the working bodies of combine harvesters on damage and quality of seed. *Visnyk Agrarnoi Nauky, 98 (2),* 64–71. https://doi.org/10.31073/agrovisnyk202002-10
2. Nurmagambetov, A., Kurmanov, A., Ryspayev, K., Bekmyrza, Z., Keklis, A. (2024). Analysis of Grain Damage by the Bucket Elevator during Loading/Unloading. *Communications – Scientific Letters of the University of Zilina, 26 (1),* B54–B62. https://doi.org/10.26552/com.c.2024.013
3. Chen, Z., Wassgren, C., Ambrose, K. (2020). A Review of Grain Kernel Damage: Mechanisms, Modeling, and Testing Procedures. *Transactions of the ASABE, 63 (2),* 455–475. https://doi.org/10.13031/trans.13643
4. Derev'ianko, D. A., Melnik, V. I., Derev'ianko, O. D. (2015). Influence workspace bucket carrier injury and quality cereal seeds. *Tekhnika, energetika, transport APK, 3 (92),* 73–78.
5. Zeng, C., Wang, Y. (2019). Compressive behaviour of wheat from confined uniaxial compression tests. *International Agrophysics, 33 (3),* 347–354. https://doi.org/10.31545/intagr/110809
6. Gao, M., Cheng, X., Hu, M., Du, X. (2019). Simulation of static stress distribution of wheat piles in silos by the modified Cam-clay model. *International Agrophysics, 33 (1),* 11–19. https://doi.org/10.31545/intagr/103749
7. Tverdokhlib, I. V. (2017). Dynamics of particle movement in cutting grain environment. *Vibratsiia v tekhnitsi ta tekhnolohiiakh, 3 (86),* 128–135.
8. Antonets, A. V., Flegantov, L. O., Ivanov, O. M., Arendarenko, V. M., Koshova, O. P. (2021). Investigating controlled grain gravitational movement in sloping channel with three variable angles. *Bulletin of Poltava State Agrarian Academy, 3,* 265–273. https://doi.org/10.31210/visnyk2021.03.33
9. Arendarenko, V., Antonets, A., Ivanov, O., Dudnikov, I., Samoylenko, T. (2021). Building an analytical model of the gravitational grain movement in an open screw channel with variable inclination angles. *Eastern-European Journal of Enterprise Technologies, 3 (7 (111)),* 100–112. https://doi.org/10.15587/1729-4061.2021.235451

10. Pylypaka, S., Nesvidomin, V., Zaharova, T., Pavlenko, O., Klendiy, M. (2019). The Investigation of Particle Movement on a Helical Surface. *Advances in Design, Simulation and Manufacturing II,* 671–681. https://doi.org/10.1007/978-3-030-22365-6_67
11. Arendarenko, V., Samoilenko, T., Ivanov, O., Ryzhkova, T. (2023). Results of experimental research on the distribution of a falling grain from a toro-shaped plate on a flat surface. *Scientific Progress & Innovations, 26 (1),* 96–101. https://doi.org/10.31210/spi2023.26.01.15
12. Melnyk, V. I., Samoilenko, T. V. (2018). Analysis of directions for improving the design of devices for loading silos. *Engineering of nature management, 1 (9),* 83–90.
13. Narendran, R. B., Jian, F., Jayas, D. S., Fields, P. G., White, N. D. G. (2019). Segregation of canola, kidney bean, and soybean in wheat bulks during bin loading. *Powder Technology, 344,* 307–313. https://doi.org/10.1016/j.powtec.2018.12.042
14. Han, Y., Li, D., Chen, J., Jing, H., Duan, J. (2018). Experimental study on boundary pressure and wall friction under static grain storage in silo. *Transactions of the Chinese Society of Agricultural Engineering, 34 (13),* 296–302. https://doi.org/10.11975/j.issn.1002-6819.2018.13.036
15. Abdelbarr, M. H., Ramadan, O. M. O., Hilal, A., Sanad, A. M., Abdalla, H. A. (2024). Retracted Article: Current design of rectangular steel silos: limitations and improvement. *Journal of Engineering and Applied Science, 71 (1).* https://doi.org/10.1186/s44147-024-00401-1
16. Samojlenko, T. V., Arendarenko, V. N., Melnik, V. I. (2019). Theoretical simulation of the process of gravitational loading of a silo with grain through an open screw channel. *Engineering of nature management, 2 (12),* 73–78.
17. Morozov, I. V., Dudin, O. V. (2003). Model of grain trajectory on the surfaces of agricultural machines. *Bulletin of Kharkiv State Technical University of Agriculture "Mechanization of Agricultural Production", 21,* 124–131.
18. Gevko, B. M. (2012). Mathematical model of grain movement on moving surfaces of sowing machines. *Collection of scientific works of Vinnytsia National Agrarian University. Technical Sciences, 11 (1),* 113–118.
19. Omarov, A., Müller, P., Tomas, J. (2013). Influence of Loading Rate on the Deformation and Fracture Behavior of Wheat Grains. *Chemie Ingenieur Technik, 85 (6),* 907–913. https://doi.org/10.1002/cite.201200054
20. Fan, J., Wang, H., Luu, L.-H., Philippe, P., Wang, L., Wei, Z., Yu, J. (2023). Numerical study of granular discharge flows through centred and off-centred rectangular hoppers using discrete element simulations. *Powder Technology, 429,* 118964. https://doi.org/10.1016/j.powtec.2023.118964
21. Naumenko, M. M., Sokol, S. P., Filipenko, D. V., Guridova, V. O. (2017). Mathematical model of grain mixture motion in a cylindrical sieve rotating around an axis. *Geotechnical Mechanics, 133,* 250–256.

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