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EXPRESS METHOD FOR DETERMINING POWER OF EQUIVALENT DOSE IN RADIATION-CONTAMINATED TERRITORIES OF RADIOACTIVE TAILINGS STORAGE FACILITIES

Operation of radiation-hazardous facilities, such as tailings facilities of the former uranium production of the Prydniprovsky Chemical Plant (PCP, Ukraine), with buildings, structures, observation points, communications, technological equipment, etc. located on their territory, is impossible without a system of physical protection and radiation monitoring. Operation of such facilities in peacetime allows for fairly rapid data collection in the operating mode at the radiation-hazardous facility itself using the method of walking gamma imaging on the perimeter of the tailings storage facility. In conditions of martial law and under certain restrictive circumstances, it is not possible to go directly to the industrial site and conduct full-scale measurements. For this, express methods of mathematical forecasting can be used. Based on the conducted research, the dynamics of observations is calculated, and the predictive model allows determining the regulated radiation parameters (RRP), one of which is the equivalent dose rate, without using radiation control devices with specialists who will conduct measurements.

For ten years, the actual values of radiation doses to personnel at the tailings storage facilities of the former uranium production of the PCP were determined. The article presents the developed universal mathematical model for determining the equivalent dose rate of gamma radiation for personnel conducting one-time measurements at a radiation-hazardous facility. The developed mathematical model for measuring the equivalent dose rate values is used for 2D modeling in places where dusty particles with radionuclides settled from the leeward side in the summer in places where the tailings mirror surface decreases. This makes it possible to predict the further radiation situation that will occur in the coming years and improve the system for calculating the total effective dose to a person.

Keywords: *mathematical model, equivalent dose rate, tailings storage facility, radiation-hazardous facility, γ-radiation.*

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1. Introduction

The operation of radiation-hazardous facilities (RHF), such as the tailings storage facilities of the former uranium production of the Prydniprovsky Chemical Plant (PCP, Ukraine) poses a threat of radiation exposure to personnel. All RHFs located on their territory: buildings, structures, observation points, communications, technological equipment, cannot be serviced without a physical protection system, a radiation surveillance system, and systematic monitoring of tailings storage facilities and adjacent territories. Their operation even in peacetime is a rather difficult task, not to mention operation under martial law. The most appropriate tool for predicting the situation in radiation-contaminated areas of an industrial site or a separate tailings storage facility is the development of a mathematical express model. Mathematical express models for determining the basic indicator of external radiation of personnel in conditions of uncertainty of the martial law situation or for modeling prognostic situations in the future are quite convenient and applicable.

In the 2007 ICRP recommendations, instead of [1], which was issued in 1991, the Commission issued recommendations [2], which became a further development of the scientific and methodological basis for calculating radiation doses, risk modeling and damage assessment. The ICRP recommendations [2] made changes to the procedure for calculating the effective dose, expanded the list of organs and tissues included in determining the effective dose, partially changed the values of weighting coefficients for organs and tissues and types of radiation, and also regulated reference voxel phantoms of an adult conditional man. Previously, the ICRP did not regulate a specific phantom for calculations. In practice, various mathematical models have been used, such as hermaphroditic MIRD phantoms or phantoms of different ages for newborns, children under 5 years of age, adolescents and adults.

In 2008, the ICRP recommendations [3] were issued for workers working directly with radionuclides, which presented updated data on radiation characteristics for more than 1200 radionuclides. The text of the recommendations [3] contains all the necessary information on radionuclides for calculating dose characteristics in tabular form. The table includes half-life, type of decay, energy and intensity of radiation, daughter products, branching factors. It is believed that this fully meets the current needs of research in the field of radiation safety of nuclear fuel cycle facilities, protection of personnel of radiation facilities from radiation and radiation hygiene issues.

A detailed description of the recommended voxel phantoms for the "adult" age category appeared in 2009 [4], which allowed them to be used from that moment on in the calculations of dose coefficients for external and internal radiation.

Thus, the authors in [5] proposed approaches to improve the gamma-neutron shielding efficiency of polyvinylidene fluoride for high-power applications in the nuclear industry by incorporating tungsten compounds into polyvinylidene fluoride (PVDF). The authors conducted simulations using the Monte Carlo method in the MCNP5 software environment. The models confirmed that PVDF-xWC reduces the gamma-ray dose rate more effectively than pure PVDF, highlighting its potential for improved radiation protection in nuclear programs. The proposed approaches are related to improving efficiency in the nuclear industry, while the studies presented in the article concern industrial sites with radioactive waste in natural formations. However, these studies can be applied to the installation of protective screens around tailings of former uranium production, which will solve the issue of reducing the total radiation dose to personnel [5].

In [6], the process of Monte Carlo modeling in the field of calculating the radiation dose to organs for patients undergoing medical diagnostic procedures is presented.

Monte Carlo modeling in the study of individual variation in radiation doses to organs during photon external irradiation was used in [7]. The use of the Monte Carlo method has found its application in studies of modeling processes in medicine and in personnel irradiation [8] showed that in determining exposure during seed treatment with 125 I using a thermoluminescent dosimeter based on a computational phantom. This methodological approach was applied in [9] when calculating isotopes of a projectile from therapeutic beams of carbon and helium ions in various materials. An option for further research may be systematic monitoring studies of the impact of increased radiation doses on personnel at industrial sites. However, it should be noted that the applied method was used for medical and agricultural research, while the article is devoted specifically to modeling processes in the conditions of an operating industrial site.

Of greatest interest, from the point of view of the exposure of PCP personnel are the publications [10–13], which contain new data on the internal exposure of personnel.

The ICRP publication [10] became the basis for the development of national standards for radiation protection of medical personnel working with interventional X-ray equipment.

In ICRP publication [11], the Commission provides guidance on the proper use of X-ray equipment in non-medical exposure situations, with the aim of minimizing the health risks to individuals exposed to ionizing radiation, and has become an important source for regulating non-medical radiation use practices, influencing the development of national standards and policies in the field of radiation safety.

ICRP publication [12] deals with internal intakes of RN in workers and is the third part of the series of documents "Occupational Intakes of Radionuclides", which is devoted to the methodology for estimating the doses that workers may receive from internal exposure due to the ingestion of radionuclides. It provides recommendations for the regular monitoring of workers who may be exposed to radionuclides in order to detect elevated levels of internal exposure in a timely manner.

The ICRP recommendations [13] contain information on individual chemical elements and their radioisotopes found in workplaces, a list of the main radioisotopes and their physical half-lives, the values of the parameters of reference biokinetic models and information on the methods of monitoring radioisotopes that are most common in workplaces. Information is presented for most elements on inhalation, ingestion and biokinetics, such as: beryllium, fluorine. magnesium, silicon, chlorine, potassium, scandium, titanium, manganese, nickel, copper, arsenic, selenium, bromine, rubidium, radium, palladium, hafnium, tantalum, tungsten, rhenium, and others. Additional dosimetric information on exposure when in a cloud is provided for inert gases: neon, argon, krypton, xenon.

According to [14] for storage (reservoirs) – the total mass of hazardous substances must comply with the developed working project, taking into account the requirements of regulatory legal acts of Ukraine, so the Rules [15] establish general requirements for limiting exposure doses to persons who do not belong to category A and B personnel in production conditions, from technogenically enhanced sources of natural origin (TESNO). The norms [16] define interference in human activities in the event of uncontrolled and unforeseen exposure in situations of emergency, chronic or temporary exposure of personnel or the population. The resolution of the Cabinet of Ministers [17] defines a list of critical infrastructure sectors, identifies critical infrastructure facilities of their sectors (subsectors), determines the facility code, the degree of its stability and the recovery time. For administrative and industrial buildings/structures located on the territory of the RHF or industrial site [18], permissible RRP levels are established.

The recommendations [19] provide dose coefficients for assessing internal exposure to RNs that enter the body of adults together with air, water, food and form the external component and total effective dose of radiation. The ICRP publication [20] describes the construction of reference computational phantoms of the adult mesh type (MRCP), which are analogues for modeling reference computational phantoms of the voxel type in the publication [4], include all source and target areas necessary for determining the total effective dose of radiation in the skin, respiratory organs, digestive tract, etc., assimilating additional models.

In [21], the authors developed a mathematical model using a structured population dynamics approach to study the effects of ionizing radiation of direct and indirect action. The effect of ionizing radiation is mathematically described in the model using ordinary differential equations (ODEs), and the simulation results are reduced to a linear-quadratic formula (LQ) to obtain the ratio for alpha/beta. The work also evaluates the parameters of the model using experimental data of human colon cancer cells using MATLAB programming. The calculated parameter values can explain the biological significance, which can confirm the result of the experimental design. The proposed mathematical approach has a sum of squared errors (SSE) of 0.0019 between the obtained simulation data and the experimental data. The model is able to explain the dynamics of the direct and indirect effects of ionizing radiation on the cell population. A free radical carries an unpaired electron in the outer shell, which is very reactive and becomes toxic to DNA. The direct action of a free radical affects the DNA cell directly, while the indirect action consists in the interaction between ionizing radiation and a water molecule, which subsequently produces a free radical. These studies and a number of others are relevant to the medical field and can be used in hospitals and clinics for specialized research. The authors of [22] used a mathematical approach using the Lagrangian Particle Scattering Model (LPDM), which is a common tool for modeling the scattering of tracers in the atmosphere. In such models, real tracers are represented by numerical particles that propagate in space and time relative to the flow and turbulence field, forming noticeable trajectories for each numerical particle. The radiation caused by the RN release into the atmosphere from nuclear facilities is difficult to obtain experimentally. In this paper, a local sensitivity analysis of the input parameters considered to be most influential on the dispersion of the exhaust flow, such as diffusion category (DC), path length, zero plane offset and source height above ground level (AGL), is performed. The authors focus on the dispersion properties of ARTM, using a general modeling setup without orography or obstacles, but considering the concentration and size of particles at ground level. Therefore, Lagrangian models such as MSS/PMSS, FLEXPART, HYSPLIT, PALM or NAME are more realistic compared to Gaussian models. LPDMs are increasingly used for regulation, safety or hazard authorization, especially in the RN release from nuclear power plants.

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The authors of [23] conducted environmental radiation monitoring, with the construction of a three-dimensional 3D model, in the Ulsan region, which has a high density of nuclear power plants (NPPs). Sampling points were selected to confirm the impact of nuclear facilities, taking into account not only the NPPs around Ulsan, but also non-destructive testing facilities or hospitals using radioactive isotopes. Water at the selected sampling point was sampled and analyzed for total beta and gamma isotope values. The results of the analysis confirmed that there was no impact of artificial radionuclides. According to the monitoring results, it was established that the dose rate of gamma radiation did not exceed the permissible level. The authors practically confirmed that the radiation level in the environment of Ulsan, which has the highest density of NPPs, is not higher than in other areas of South Korea where there are no NPPs.

Express methods in mathematical calculations have also been used in works [24], where new models for modeling dust in "glove boxes" are presented. The calculated doses are compared in different configurations to determine whether homogeneous models systematically lead to an overestimation of the received doses, which would be a good property for radiation protection calculations. Variants of the obtained configurations allow to take into account more accurate geometry of dust in modeling. Recent advances in stochastic geometry [25] in Monte Carlo simulation models, such as TRIPOLI-4® [26], allow to consider stochastic modeling of dust accumulations by generation and placement in space. That is, their use can be adapted for practical and theoretical studies of the movement of dust particles on the surface of tailings facilities.

The aim of research is to develop a mathematical express method for determining one of the basic RRPs – equivalent dose rate (EDR) to create data for the base of initial values of analytical or program forecasting, with subsequent theoretical substantiation of the data planned to be obtained in 2023–2026, based on actual values for the period 2009–2018.

2. Materials and Methods

The object of research is the industrial site "Base-S", on which two tailings facilities are located, namely: Base S – a former uranium raw material warehouse with trenches measuring 30 × 200 × 2.5 meters and Blast Furnace No. 6.

The research was aimed at monitoring the values of gamma radiation EDR in three stages, for which the following tasks were set:

– to conduct a classic collection of radiation parameter data, by monitoring the tailings storage facility in the period from 2009 to 2018 (under the conditions of access to the RHF presented in Fig. 1, using the example of the first year of data collection);

– to conduct analytical studies of the obtained data and, using the extrapolation (interpolation) method, to determine possible values for the next five years, which are implemented in the developed express model;

– to develop a mathematical model MMRC $f(x, y)$ for determining the EDR by the express method with subsequent comparison of the actually obtained data and theoretically calculated radiation parameters to ensure radiation safety at the RHF with adaptation of the mathematical model to other RHFs and industrial sites with sources of ionizing radiation.

Systematic measurements of the beta particle flux density, absorbed dose rate and equivalent dose rate were carried out on the perimeter of the RHF for ten years from 2009 to 2021.

3. Results and Discussions

Measurement of radiation parameters at the nuclear fuel cycle (NFC) of Ukraine, forecasting of radiation and sanitary and hygienic regulations, process modeling using integrated and mathematical models (table method, triangle method, square method, finite element method, methods of building models using software and software packages such as; ANSYS, PYTON, MCNP, Geant 4, MATLAB. As a result, the authors conducted an analysis of research, empirical and analytical research methods on the tailings of the PCP former uranium production. This work determined the calculated, predicted and actual EDR values of personnel exposure at the Sukhachivske tailings storage facility I section, in the period from 2010 to 2023. The first task was implemented by conducting annual systematic measurements of the values of the absorbed dose rate (ADR), EDR and flux density (FD) on the RHF perimeter for ten years during the period 2009–2018 (Table 1 and Fig. 1).

Table 1

Determination of the γ-radiation equivalent dose rate in 2009

Point No.	γ-radiation EDR in 2009, μSv/h	Point No.	γ-radiation EDR in 2009, μSv/h	Point No.	γ-radiation EDR in 2009, μSv/h	Point No.	γ-radiation EDR in 2009, μSv/h
Entry	0.289	31	0.188	62	0.182	93	0.369
1	0.264	32	0.125	63	0.378	94	0.411
2	0.269	33	0.196	64	0.181	95	0.438
3	0.183	34	0.199	65	0.139	96	0.486
4	0.164	35	0.162	66	0.204	97	0.407
5	0.183	36	0.196	67	0.187	98	0.292
6	0.170	37	0.202	68	0.191	99	0.284
7	0.163	38	0.286	69	0.175	100	0.263
8	0.158	39	0.279	70	0.171	101	0.271
9	0.121	40	0.264	71	0.158	102	0.264
10	0.126	41	0.288	72	0.166	103	0.287
11	0.111	42	0.262	73	0.174	104	0.294
12	0.130	43	0.198	74	0.174	105	0.426
13	0.254	44	0.204	75	0.204	106	0.486
14	0.214	45	0.214	76	0.193	107	0.490
15	0.232	46	0.268	77	0.163	108	0.417
16	0.368	47	0.247	78	0.158	109	0.389
gate	0.375	48	0.240	79	0.187	110	0.208
17	0.268	49	0.266	80	0.173	111	0.276
18	0.224	50	0.256	81	0.143	112	0.273
19	0.193	51	0.381	82	0.158	113	0.268
20	0.136	52	0.402	83	0.198	114	0.260
21	0.124	53	0.318	84	0.196	115	0.290
22	0.126	54	0.281	85	0.226	116	0.291
24	0.136	55	0.440	86	0.238	117	0.268
25	0.148	56	0.268	87	0.380	118	0.273
26	0.199	57	0.326	88	0.294	119	0.284
27	0.196	58	0.273	89	0.287	120	0.261
28	0.196	59	0.268	90	0.276	121	0.289
29	0.193	60	0.253	91	0.316	122	0.278
30	0.149	61	0.194	92	0.308	Check point	0.396

2009 (122 points - 20 metre pitch)

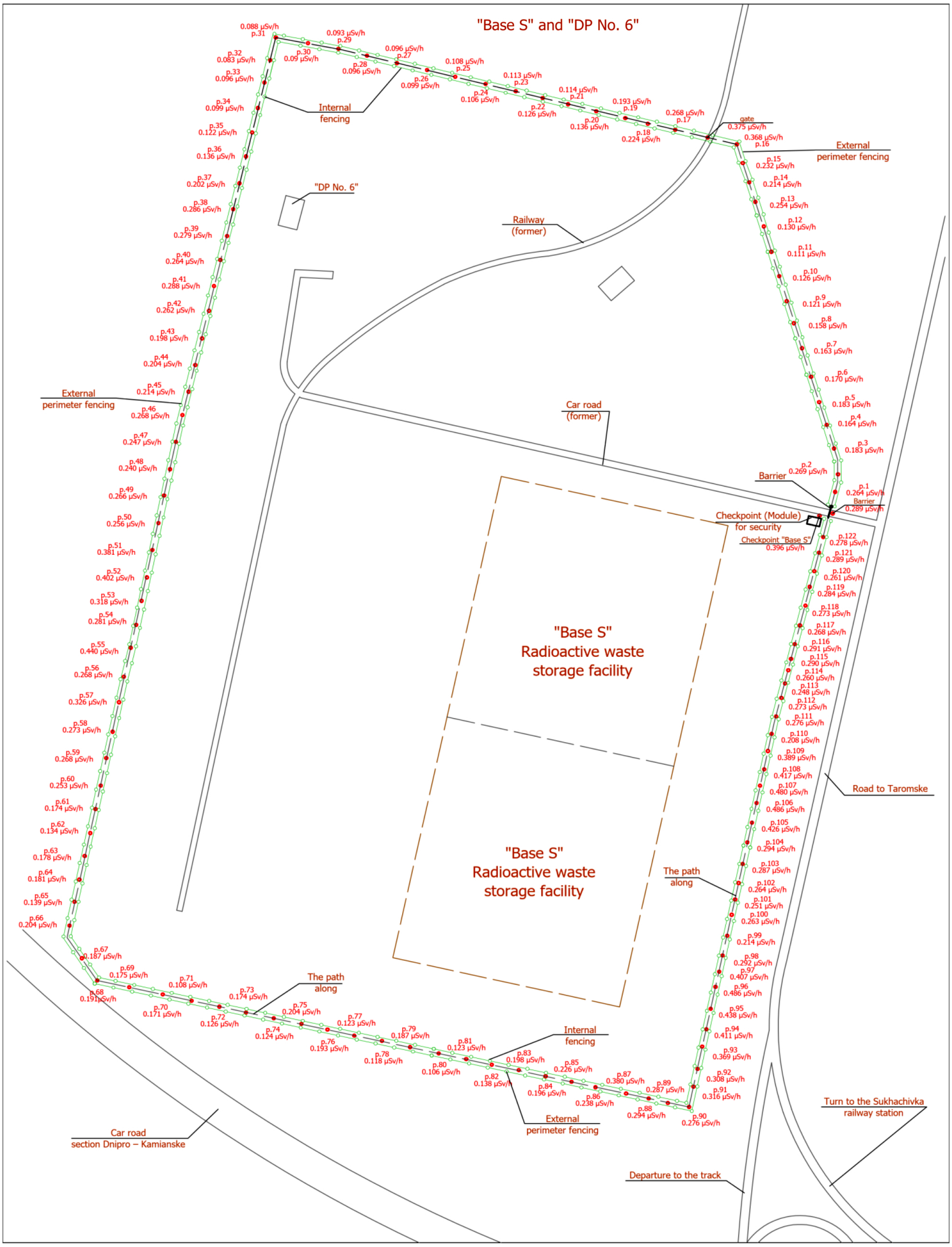

Fig. 1. Variants of the forms of the tailings storage facility reduced to a flat type for determining the γ-radiation EDR values

To solve the second problem, it was necessary to build 2D models of the distribution of radiation contamination over the territory of the tailings storage facility, presented in Fig. 2.

To solve the third problem, it is necessary to determine the initial data and the scope of application.

To build a mathematical model MMRC $f(x, y)$ to determine the value of the gamma radiation EDR at a certain point on the perimeter of the industrial site or a separate tailings storage facility, the following data must be taken into account: the presence of PRN, its activity, half-life, atmospheric stability category, temperature, presence of precipitation, pressure, humidity and type of tailings storage facility.

For modeling the process, the worst-case scenario for weather conditions and flat tailings facilities with a small height difference of up to 1–1.5 meters was adopted (Fig. 3).

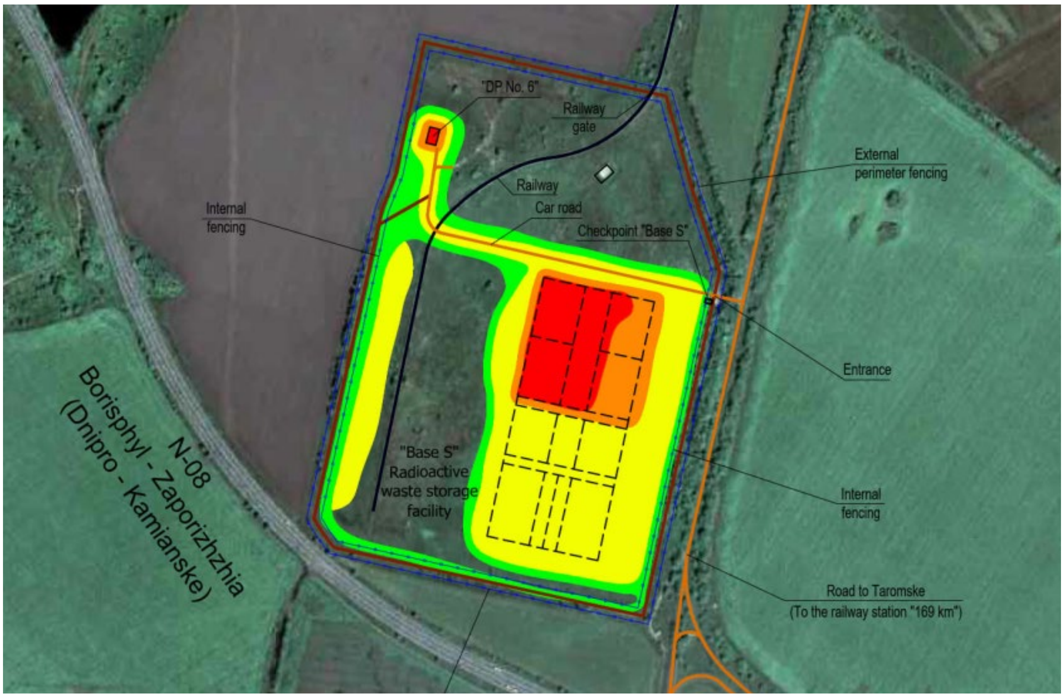

Fig. 2. Construction of a 2D map of radiation contamination of the industrial site "Base-S" by the equivalent dose rate of γ-radiation in 2009

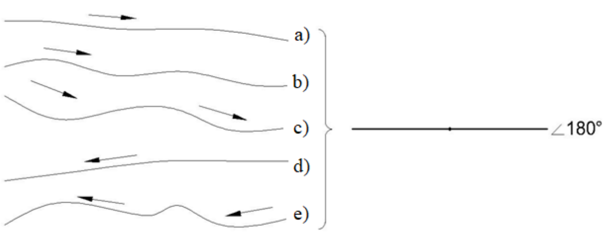

Fig. 3. Variants of the forms of the tailings storage facility reduced to a flat type for determining the γ-radiation EDR values

The general initial data for developing the mathematical model have the form (1)–(4), respectively, for determining the normative values (1) determining the influence of individual parameters on the total radiation dose (2) and for determining the EDR (3), (4) using the finite element method and the extrapolation method, for personnel.

$$H_{add}^{\Sigma}=\begin{cases} H_{ef_{norm}}=f\left(\sum RRP\right)\leq 1\ \text{mS}\nu-\text{category C}, \\ H_{ef_{norm}}=f\left(\sum RRP\right)\leq 2\ \text{mS}\nu-\text{category B}, \\ H_{ef_{norm}}=f\left(\sum RRP\right)\leq 20\ \text{mS}\nu-\text{category A}, \end{cases} \quad (1)$$

$$H_{ef_{calc}}=f\begin{pmatrix} A_{spec}\ RN, EDR, ADR, H_{red}, typeRN, m_{rn}, \\ OK, EEVA\ X_{antr}, K_{reg} \end{pmatrix}, \quad (2)$$

for example, determining the EDR at sites 1 and 2 for the Sukhachivske tailings storage facility I section in 2019:

– for 2019, site 1

$$y=0.184+0.049x-0.081x^2+0.005x^3; \quad (3)$$

– for 2019, site 2

$$y=0.203+0.082x+0.003x^2-0.0004x^3. \quad (4)$$

Physical statement of the problem. Around the tailings storage facility (the perimeter of the tailings storage facility with the accompanying trail) along the contour in certain places (this can be 20–150 measurement points of places), depending on the measurement step (measurement step from 10 to 100 meters) and the length of the tailings storage facility perimeter, measurements of the equivalent dose rate (EDR) were carried out. Measurements at each place (point) were carried out several times (2–5 times) with an exposure of 1–5 minutes. Measurements were carried out once a month (12 times a year). It is necessary to determine the level of EDR at any place on the perimeter of the tailings storage facility at any time of the year.

Mathematical statement. There is a certain area (two-dimensional or three-dimensional) in which the ADRs that are SIR are located. In total, the points of the contour receive certain EDRs, which are measured in real time several times in a few minutes (2–3 minutes). Measurements are carried out once a month (12 times a year). It is necessary to mathematically describe the EDR level at any point on the tailings storage facility contour at any time of the year, that is, it is necessary to construct a function $f(x,y,t)$ that depends on time t and on plane coordinates (x,y) and would make it possible to determine the RI at any point of the contour (not only at the measurement points) with the greatest accuracy (the smallest error).

For numerical calculation for the given constraints, a simulation of the EDR determination at each point of the tailings storage facility perimeter was carried out depending on time. The points $M_j(x_j,y_j)$ of the contour of the area with coordinates (x_j,y_j) $(j=1,2,\ldots,n)$, where n – the number of points, were obtained. At each point $M_j(x_j,y_j)$, over a short period of time, the values of a certain function are known, let's call it $P_j(x_j,y_j,t_k)$, t_k $(k=1,2,\ldots,l_j)$, k – the measurement number, l_j – the number of measurements at the point $M_j(x_j,y_j)$ (t_k means at what time the measurement was made). From the statement of the problem, it follows that the duration of measurements at each point took several minutes. Such measurements were made m in a year, where M_j – the designation of the j-th point on the contour at which the measurements were made; t_{ji} – the time (month, hour, minute) when the RI measurements were made at the j-th point of the contour at the i-th measurement; (x_j,y_j) – the coordinates of the point M_j; $M_j=M_j(x_j,y_j)$; l_j – the number of measurements at this point for a short period of time (2–3 minutes).

It is necessary to construct a EDR function $f_j(x_j,y_j,t)$ for each point $M_j(x_j,y_j)$, which would depend on t and approximate the actual values $P_j(x_j, y_j, t_i)$ at each point $M_j(x_j, y_j)$, obtained for all measurements i, where $i=1,\ldots,m$.

M_j – the EDR measurement number at the point $(i=1,2,\ldots,m$; m – the number of measurements at each point m, the same for all points; in reality, natural measurements were carried out every month at each point, i. e. in reality $m=12$).

j – number of the point of the tailings storage facility contour, in which the RI measurements are carried out; $(j=1,2,\ldots,n$; n – number of measurement points on the tailings storage facility, in reality $n=50\div150$).

The following methodology is proposed to solve this problem: as the actual value for the i-th number of EDR measurement at a given point $M_j(x_j, y_j)$ for a short period of time, the average value $P_j(x_j, y_j, t_i)$ from l_j of the actual measurements $P_j(x_j, y_j, t_k)$ is taken, which has the form

$$P_j\left(x_j, y_j, t_i\right)=\frac{1}{l_j}\sum_{k=1}^{l_j}P_j\left(x_j, y_j, t_{ik}\right). \quad (5)$$

It is assumed that at each point $M_j(x_j, y_j)$ is the same number of averaged measurements, denote their number by m with the numbering $i=1,\ldots,m$. For each measurement number, the actual value EDR $P_j(x_j, y_j, t_i)$ is known.

Restrictions for the function. If the number of measurements at different points m is different, then let's take as the smallest number, which was carried out at points $M_j(x_j, y_j)$. At those points where the number of measurements was greater m, let's take for consideration some measurements with the largest EDR values, based on the principle of choosing the worst case.

The m discrete values $P_j(x_j, y_j, t_i)$ $(i=1,\ldots,m)$ at a given point $M_j(x_j, y_j)$ $(j=1,2,\ldots,n)$ are approximated as a function of time by an analytical continuous function of t in the form of a polynomial of the $(m-1)$-th degree in the form

$$\begin{aligned}&P_j\left(x_j, y_j, t\right)\approx f_j\left(x_j, y_j, t\right)=\\&=a_{j,0}\left(x_j, y_j\right)+a_{j,1}\left(x_j, y_j\right)t+a_{j,2}\left(x_j, y_j\right)t^2+\ldots\\&\ldots+a_{j,m-1}\left(x_j, y_j\right)t^{m-1}=\sum_{i=1}^{m}a_{j,i-1}\left(x_j, y_j\right)t^{i-1},\end{aligned} \quad (6)$$

where $a_{j,k}\left(k=0,1,\ldots,m-1; j=1,2,\ldots,n\right)$ – unknown constant coefficients that depend on the coordinates of the point $M_j(x_j, y_j)$ at which the measurements are made:

These coefficients $a_{j,k}=a_{j,k}(x_j, y_j)$ are found from the condition that the approximating function depends on the fixed point (coordinates (x_j, y_j)) and is a function of time t. It coincides with the discrete known (from field measurements) RI function $P_j(x_j, y_j, t)$ for all measurement numbers $i=1,2,\ldots,m$ at the point $M_j(x_j, y_j)$, that is, at $t=t_i\left(i=1,2,\ldots,m\right)$.

Therefore, to determine the m unknown constant coefficients $a_{j,k}\left(k=0,1,\ldots,m-1; j=1,2,\ldots,n\right)$ of the function $f_j(x_j, y_j, t)$ at each j-th point there is the following system of linear algebraic equations (LAE) with respect to the m unknown coefficients $a_{j,k}=a_{j,k}(x_j, y_j)$ $(k=0,1,\ldots,m-1)$ (in the system below, each coefficient $a_{j,k}$ depends on the coordinates of the point $M_j(x_j, y_j)$) and will have the form

$$\begin{cases}f_j\left(x_j, y_j, t=t_{j,1}\right)=a_{j,0}+a_{j,1}t_{j,1}+a_{j,2}t_{j,1}^2+\ldots+a_{j,m-1}t_{j,1}^{m-1}=P_j\left(x_j, y_j, t=t_{j,1}\right);\\f_j\left(x_j, y_j, t=t_{j,2}\right)=a_{j,0}+a_{j,1}t_{j,2}+a_{j,2}t_{j,2}^2+\ldots+a_{j,m-1}t_{j,2}^{m-1}=P_j\left(x_j, y_j, t=t_{j,2}\right);\\\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots;\\f_j\left(x_j, y_j, t=t_{j,i}\right)=a_{j,0}+a_{j,1}t_{j,i}+a_{j,2}t_{j,i}^2+\ldots+a_{j,m-1}t_{j,i}^{m-1}=P_j\left(x_j, y_j, t=t_{j,i}\right);\\\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots;\\f_j\left(x_j, y_j, t=t_{j,m}\right)=a_{j,0}+a_{j,1}t_{j,m}+a_{j,2}t_{j,m}^2+\ldots+a_{j,m-1}t_{j,m}^{m-1}=P_j\left(x_j, y_j, t=t_{j,m}\right),\end{cases}$$

or in matrix form

$$A_jT_j=P_j\left(j=1,2,\ldots,n\right). \quad (7)$$

In the LAE system, the function (7) is known as a matrix T_j and P_j for each point $M_j(x_j, y_j)$ will have the form of the function

$$T_j=\begin{pmatrix}1\ t_{j,1}\ t_{j,1}^2\ldots t_{j,1}^{m-1}\\1\ t_{j,2}\ t_{j,2}^2\ldots t_{j,2}^{m-1}\\\ldots\ldots\ldots\\1\ t_{j,i}\ t_{j,i}^2\ldots t_{j,i}^{m-1}\\\ldots\ldots\ldots\\1\ t_{j,m}\ t_{j,m}^2\ldots t_{j,m}^{m-1}\end{pmatrix}; P_j=\begin{pmatrix}P_j\left(x_j, y_j, t=t_{j,1}\right)\\P_j\left(x_j, y_j, t=t_{j,2}\right)\\\ldots\ldots\ldots\\P_j\left(x_j, y_j, t=t_{j,i}\right)\\\ldots\ldots\ldots\\P_j\left(x_j, y_j, t=t_{j,m}\right)\end{pmatrix}. \quad (8)$$

The desired matrix of coefficients will take the form A_j

$$A_j=\begin{pmatrix}a_{j,0}\\a_{j,1}\\\ldots\ldots\\a_{j,i}\\\ldots\ldots\\a_{j,m-1}\end{pmatrix}. \quad (9)$$

Linear algebraic system (7) with a known matrix T_j and unknowns with a matrix A_j and a known column of the right-hand side of the system of equations (7) – matrix P_j is solved numerically using the appropriate subroutines or complexes.

It should be noted that the number of linear algebraic systems of type (7) is equal to the number of points at which RI measurements are made, i. e. n systems.

After finding the coefficients $a_{j,k}\left(k=0,1,\ldots,m-1; j=1,2,\ldots,n\right)$, the EDR values $P_j(x_j, y_j, t)$ are determined as a continuous function $f_j(x_j, y_j, t)$ at each point of the contour of the region depending on time t and will take the form

$$\begin{aligned}&P_j\left(x_j, y_j, t\right)\approx f_j\left(x_j, y_j, t\right)=\\&=a_{j,0}\left(x_j, y_j\right)+a_{j,1}\left(x_j, y_j\right)t+a_{j,2}\left(x_j, y_j\right)t^2+\ldots\\&\ldots+a_{j,m-1}\left(x_j, y_j\right)t^{m-1}=\sum_{i=1}^{m}a_{j,i-1}\left(x_j, y_j\right)t^{i-1},\end{aligned} \quad (10)$$

where $a_{j,k}(x_j, y_j)$ are the numbers for each point.

The functions $f_j(x_j, y_j, t)$ exactly coincide with the EDR values $P_j(x_j, y_j, t)$ at each point $M_j(x_j, y_j)$ at all times $t_i\left(i=1,\ldots,m\right)$ of the EDR measurement and are approximated $P_j(x_j, y_j, t)$ by other values of time t for a given (each) point $M_j(x_j, y_j)$.

In previous works, the state of radiation contamination for the period 2008–2024 was analyzed, set out in scientific and practical studies carried out at tailings storage facilities, to collect and accumulate a practical database of the values of the EDR, ADR, and FD values [27–29]; using remote devices and radiation monitoring devices [30, 31].

In the future, it is proposed to develop mathematical models of MMRC $f(t)$ and MMRC $f(N)$, based on the developed mathematical model for determining the radiation dose of personnel from the location of personnel in certain locations of the MMRC perimeter $f(x, y)$.

In the future, it is necessary to continue improving research methods on tailings storage facilities of the former uranium production of the PCP. The results of these studies can be used for other flat or gully-type tailings storage facilities [28], where land reclamation or earthworks were carried out to level the surface of the tailings storage facility.

4. Conclusions

Based on theoretical and instrumental studies, a scientific and practical justification was provided for determining the predicted EDR and ADR values on the perimeter of the *i*-th tailings storage facility using the express method.

The actual values of γ-radiation EDR of flat tailings storage facilities were analyzed and determined, using the example of the Baza-S industrial site. This, in turn, makes it possible to predict the further radiation situation on flat industrial sites in the coming years and improve the systems for calculating the total effective dose of radiation, both for the personnel of a radiation-hazardous facility and for the population living near the industrial site.

A mathematical model for measuring EDR values was developed, which can be used for 2D modeling [32] and used to form graphic materials. As a result of the studies, individual elevated levels were identified that were tied to a specific location relative to the "body" of the tailings storage facility. Moreover, this trend began to be observed after 2015, in places where dust particles with radionuclides settled from the leeward side in the summer and had annual confirmations in further forecasting and measurements.

The proposed mathematical model allows determining the gamma radiation EDR at any point in the coordinate system of the surface of a flat research object. However, the developed mathematical model requires certain clarifications regarding a sufficient number of Ni measurements, the application of correction factors (K_{IR}, K_{ef}, K_{reg}, K_{org}), the type of tailings storage facility and the anti-radiation technical measures applied to it.

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Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship or other, which could affect the research and its results presented in this scientific article.

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Data availability

Data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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