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ANALYSIS OF SINUSOIDAL TRANSFORMATION MODEL OF DARK TONE DIGITAL IMAGES

The object of the study is the technological process of sinusoidal transformation of dark tones into a digital image, used at the stage of preparation for printing. One of the most problematic areas is posterization, which occurs with traditional power-law gamma transformation, creating noticeable bands on the image that distort its quality and limit the capabilities of the operator, technologist, and printer.

The study employed mathematical modeling and quantization of gradation characteristics to eliminate these shortcomings. A mathematical model of sinusoidal transformation was developed, describing the brightness of the image in the range of $0 \leq L \leq 255$ levels. A structural scheme of the simulator model was also created in MATLAB: Simulink, allowing for the calculation and plotting of gradation characteristics, optical density, and contrast sensitivity for different transformation frequencies.

As a result of the simulation, it was found that the sinusoidal transformation has significantly smaller initial quantization shifts (0.5–2 units) and first step lengths (1–2 levels) compared to the traditional gamma transformation (11–31 units and 10–15 levels, respectively). This eliminates posterization. The contrast sensitivity of the sinusoidal transformation increases up to 2.2, exceeding the constant value of 1 in the linear scale, which ensures improved tone perception. Thus, the proposed method demonstrates higher efficiency in reproducing images in both dark and light areas.

The results obtained demonstrate the absence of posterization in the sinusoidal transformation of dark tones. This is due to the proposed approach having several features, including a steeper gradation characteristic at the beginning of the range, which eliminates posterization of dark tones without losses in highlights. This ensures the ability to obtain high-quality images with improved gradation characteristics.

Compared to similar known methods, this provides advantages in the form of improved image quality and elimination of posterization, which is crucial for the quality preparation of images for printing. The results of the research and simulation modeling can be used to select optimal reproduction characteristics, ensuring improved perception of the printed image by the human visual system. This allows achieving high print quality without losses in detail and contrast, which is a significant advantage in the printing industry.

Keywords: *image, modeling, sinusoid, simulator, gradation, optics, density, contrast, sensitivity, quality.*

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1. Introduction

Modern digital images undergoing prepress preparation require high accuracy in conveying gradations of dark tones. Existing methods, particularly power-law gamma transformation, often do not provide sufficient smoothness in the dark range, leading to posterization – image distortion that is especially critical in artistic, advertising, and packaging printing. This limits the effectiveness of digital technologies amid increasing demands for print quality. Despite the widespread use of such transformations, the issues of reducing artifacts and ensuring precise tone control remain relevant. Therefore, there is a need to develop an alternative approach that would eliminate these shortcomings and consider the psychophysical aspects of image perception.

For image processing, simple linear transformations, logarithmic, and power-law gamma transformations are often used. In the literature, little attention is paid to analyzing the properties of logarithmic and power-law gamma transformations, particularly when reproducing dark-toned images at boundary tone values and the occurrence of posterization [1, 2]. Insufficient attention is given to the connection of image transformations with standard classical parameters for printing, such as optical density, contrast, and their provision at the image processing stage.

In computer graphics programs, various methods are used to correct and enhance the input image. The most popular image correction method is the Curves tool, where the operator uses a computer mouse to adjust the image observed on the monitor screen. In most cases, the digital image is corrected in the absence of the original, and the operator adjusts the image by eye and personal judgment, so the image quality cannot be optimal [3]. In computer graphics programs, there is no program for building gradation characteristics, which significantly limits the operator's, technologist's, and printer's ability to evaluate the image and adjust the printing system's ink apparatus for a given print run [4].

Thus, the development of models for sinusoidal transformation of digital images of dark tones is a relevant task. This allows for the formation of gradation characteristics that eliminate posterization in dark areas of the image, enabling the determination of optical density and contrast sensitivity. This ensures the necessary quality indicators and reduces sensitivity to posterization. Such models improve the perception of the printed image by the human visual system. They can be used at the stage of preparing images for printing.

The problem of improving image quality during preparation for printing is reduced to forming a gradation characteristic that, according to the tone, places characteristics on the tone transfer interval, involving "stretching" and "compressing", which is a contradictory task [5, 6]. In the traditional method, characteristic correction is performed by selecting the exponent of the power function. If the input image is described by gray levels [0...255], where 0 corresponds to black and 255 to white, within which gray images of different tonalities are located, traditional power-law gamma transformation is performed according to the known expression [7, 8]

$$L = kL_0^r \cdot L_{max}, \text{ if } 0 \le L_0 \le 255, \quad (1)$$

where L_0 – the continuous input image (linear scale), the transformed (corrected) output image $L_{max} = 255$, and $k = 1 / 255$ – the scale coefficient, and r – the exponent used for image correction. Usually, the power transformation (1) is described by a continuous expression to simplify analysis and correction procedures. If the exponent is greater than one, the image darkens. At the beginning of the range, striped transitions appear in dark areas – posterization of dark areas, which distorts the image and limits the functional capabilities for correcting dark tones [9].

Power-law gamma transformation is used in scanners for correcting scanned images [10] and for correcting and calibrating monitors [11], which is effective only within certain limits – until posterization appears in dark areas or losses occur in highlights. In the literature, little attention is paid to analyzing the properties of gamma transformation, the phenomenon of posterization, its causes, and the limits of existence, which can be used to analyze its properties and eliminate posterization.

In the author's publication [12], a new model of sinusoidal transformation of digital images was developed, in which posterization and noticeable transitions in dark areas of the image are absent, and there are no losses in highlights. Typical variants of gradation characteristics, optical density, and contrast sensitivity graphs were built and their properties and perception by the human visual system were analyzed.

In modern gradation editors such as Photoshop, Corel, and others, oriented for use in computer publishing systems for preparing images for printing, there is no program for building gradation characteristics and analyzing their properties, which significantly limits the operator's and technologist's capabilities for quality image preparation for printing and adjusting the ink apparatus for a given print run [13].

Therefore, *the aim of research* is to develop a sinusoidal transformation of digital images of dark tones that does not have posterization. This will allow forming specified gradation characteristics by changing the transformation frequency and improve the perception of the printed image by the human visual system.

2. Materials and Methods

The object of research is the technological process of correcting digital images of dark tones based on sinusoidal transformation at the stage of preparation for printing.

For the experimental part of research, a sinusoidal transformation simulator was created in the MATLAB: Simulink environment. The simulator implements the generation of a linear brightness scale (Ramp block), which was fed to mathematical function blocks implementing three typical variants of sinusoidal transformation with different frequencies. Each transformation was normalized to 255 levels using scale coefficients $M1 = 0.96$; $M2 = 1.00$; $M3 = 0.983$. Graphs of gradation characteristics, optical density, and contrast sensitivity were constructed.

For each variant, quantization of the output gradation characteristic was performed using the Quantizer block with analysis of the zero discrete shift and the length of the first steps. The obtained characteristics were also compared with the linear scale and traditional gamma transformation, the results of which were input into the simulator for visual and numerical analysis. Indicators were evaluated for the presence / absence of posterization, curve shape, and numerical parameter values: optical density, contrast sensitivity, stepwise nature, and tone smoothness.

The selected frequency values ν – 0.0016; 0.0019; 0.0022 – were chosen based on experimental tests to cover the range from low to medium frequencies of sinusoidal transformation, which demonstrate noticeable changes in gradation characteristics without creating excessive oscillations that could degrade image quality. These values allow demonstrating the impact of frequency on the steepness of the gradation curve and the absence of posterization in dark tones.

Similarly, scale coefficients $M1 = 0.96$; $M2 = 1.00$; $M3 = 0.983$ were chosen to ensure normalization of the output signal to the maximum level of 255, as well as to reflect brightness variations within nominal values without distortion. Initial shifts Z were determined empirically based on the analysis of gradation characteristic quantization and correspond to the limits where visible posterization is absent.

Overall, these parameters ensure a balance between the clarity of detail transmission in dark areas and the preservation of smooth transitions, which is critically important for printing preparation.

The following scientific methods were used in the research:

– the theory of digital image processing in the development of sinusoidal transformation, formation of typical gradation characteristics during preparation for printing to improve image quality;

– object-oriented programming in the MATLAB: Simulink package for creating mathematical and simulation models of sinusoidal transformation and simulators consisting of graphical symbols of operators and functional blocks with specified properties and corresponding parameters found in a number of Simulink libraries and visualization tools for modeling results;

– a structural scheme of the sinusoidal transformation simulator model was developed, with the main blocks of the scheme being operational blocks of mathematical functions F_{cn}. In the dialog windows, program expressions for calculating typical variants of sinusoidal transformation gradation characteristics, optical density, and contrast sensitivity expressions are recorded, modeling results are presented, and their analysis is performed.

3. Results and Discussion

Based on the inversion of sinusoidal transformation and the addition of a linear component, a model of a digital image of dark tones was developed

$$L = \left[2L_0 - 255\left(\sin \cdot \nu \cdot pi \cdot L_0\right)\right] \cdot M, \text{ if } 0 \le L_0 \le 255, \quad (2)$$

where 255 – the specified nominal amplitude, ν – the frequency, L_0 – the linear component (scale), and M – the transformation scale to the nominal level $L_{max} = 255$.

The initial zero level of the digital image corresponds to black, and 255 to white, within which gray images of different brightness are located, over which various transformations were performed. For the study, three typical variants of sinusoidal transformation of digital images of dark tones were formed by changing the frequency:

$$L1 = \left[2L_0 - 255(\sin(0.0016 \cdot pi \cdot L_0))\right] \cdot M1, \quad (3)$$

$$L2 = \left[2L_0 - 255(\sin(0.0019 \cdot pi \cdot L_0))\right] \cdot M2, \quad (4)$$

$$L3 = \left[2L_0 - 255\left(\sin(0.0022 \cdot pi \cdot L_0)\right)\right] \cdot M3, \text{ if } 0 \le L_0 \le 255, \quad (5)$$

where $M1$, $M2$, $M3$ – scale coefficients for transformation to 255 levels. To determine the optical density of sinusoidal transformation, the known formula is applied [14, 15]

$$D = \log 10 \frac{255}{L+1}, \quad (6)$$

unit input for coordinate origin shift.

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To quantitatively assess the quality of sinusoidal transformation of digital images of dark tones in terms of tone perception by the human visual system, their contrast sensitivity is determined for different frequencies [3]

$$C = \frac{dL}{dL_0}, \text{ if } 0 \le L_0 \le 255. \qquad (7)$$

Based on the above and expressions (3)–(7), gradation characteristics of sinusoidal transformation of dark tones of images, optical density, and contrast sensitivity can be calculated. To simplify the solution of the tasks, simulation modeling in the MATLAB: Simulink package was applied. Based on object-oriented programming and functional blocks of Simulink libraries, a structural scheme of the sinusoidal transformation simulator model for dark tones of images was developed, the scheme of which is shown in Fig. 1.

The main blocks of the simulator model are operational blocks of mathematical functions F_{cn}. The Ramp block generates a linear scale L_0, which is fed to the inputs of the operational blocks of mathematical functions F_{cn1}–F_{cn2} of the first row, where program expressions (3)–(5) for calculating typical variants of sinusoidal transformation of the image are recorded. These are multiplied by M in the Gain blocks for nominal values $L = 255$, which are fed to the multiplexer and visualized by Scope and Display blocks. They are then fed to the inputs of the operational blocks of mathematical functions F_{cn3}–F_{cn5} for calculating optical density according to expression (6), which are visualized by Scope1 and Display1 blocks. To determine the contrast sensitivity of sinusoidal transformation, the Derivative block is applied, which is visualized by Scope2 and Display2 blocks. For quantizing gradation characteristics, the Quantizer block is applied, which is visualized by the Scope3 block. For comparison of results, a linear scale L_0 is additionally introduced into the simulator, and its optical density is determined.

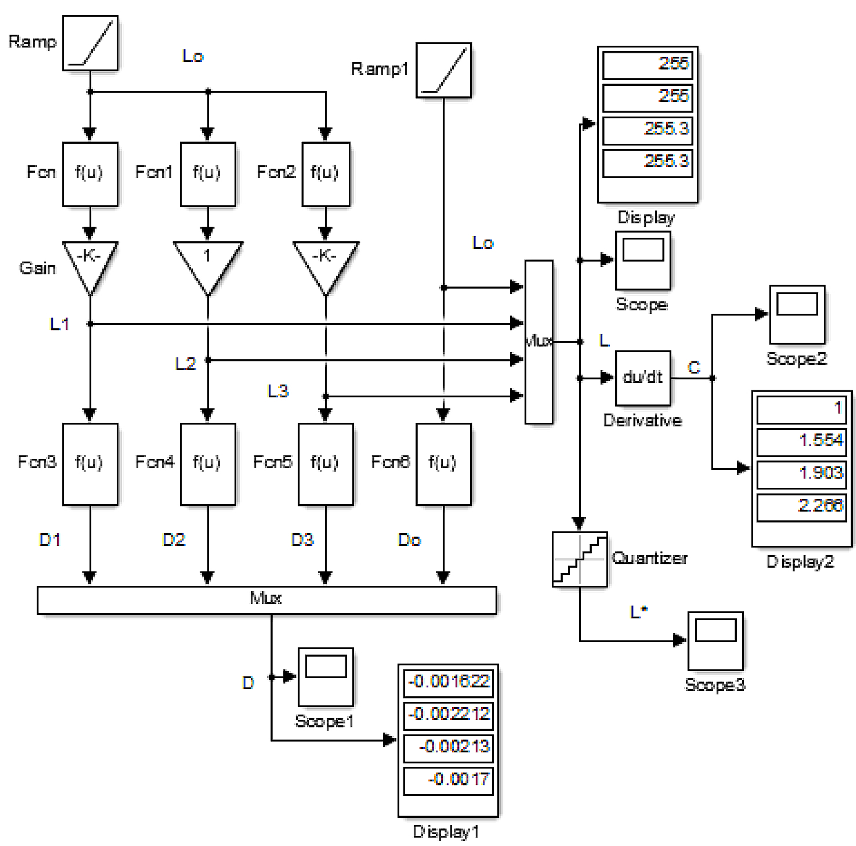

Fig. 1. Structural scheme of the sinusoidal transformation simulator model

The simulator was configured. In the dialog windows of the mathematical function blocks, programs were recorded, and corresponding parameters were set. In the interactive mode of the simulator, using Gain blocks, the outputs Li were adjusted to nominal values of 255 levels, and scales $M1 = 0.96$; $M2 = 1.00$; $M3 = 0.983$ were determined. The results of modeling typical variants of gradation characteristics of sinusoidal transformation for dark tones are presented in Fig. 2.

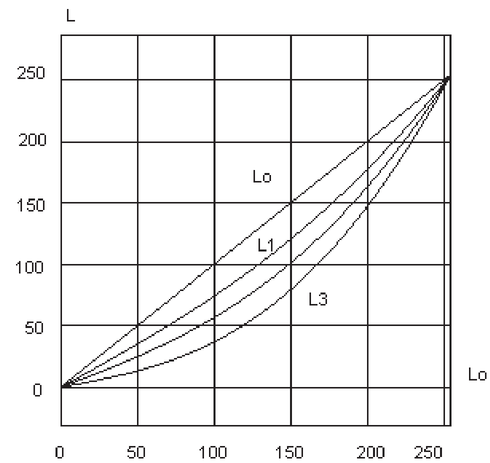

Fig. 2. Gradation characteristics of typical variants of sinusoidal transformation of dark tones

For comparison, the linear characteristic L_0 of image reproduction with uniform tone distribution on the tone transfer interval is presented at the top. The following gradation characteristics are concave curves evenly stretched towards dark tones, having sufficient steepness at the beginning of the range, so details in dark areas of the image are well reproduced. In the light range, gradation characteristics have significantly greater steepness, so light images are well reproduced. Thus, sinusoidal transformation (3)–(5) well reproduces dark tones and simultaneously ensures quality reproduction of light images. Increasing the frequency of sinusoidal transformation shifts the characteristics down and to the right, darkening the image. Based on the above, we conclude that the developed typical variants of sinusoidal transformation for dark tones ensure better tone reproduction of the image over the entire tone transfer interval, which is an advantage of sinusoidal transformation over power-law gamma transformation.

For a complete analysis, gradation characteristics were discretized using the Quantizer block, the results of which are presented in an enlarged plan in Fig. 3 for different transformation frequencies.

The quantized gradation characteristics are stepwise. The zero discretes have initial shifts Z: 0.5; 0.7; 2.0 units, and the lengths of the first steps are: 0.5; 2.0; 1.0, so they are not noticed by the human visual system (absence of posterization), which is an advantage of the developed sinusoidal transformation over gamma transformation.

It was found that the graphs of quantized gradation characteristics of power-law gamma transformation of dark images have initial shifts Z: 11; 12; 31 levels, and their lengths are: 12; 10; 15 levels. Large initial shifts and their lengths create noticeable bands in dark areas, leading to posterization of the image, which distorts the image perceived by the human visual system.

The results of modeling the optical density of sinusoidal transformation of dark tones of the image are presented in Fig. 4.

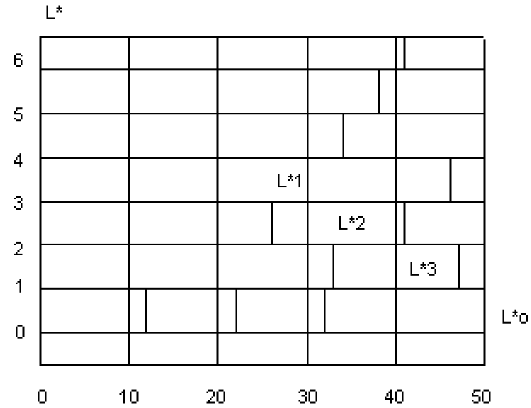

Fig. 3. Graphs of quantized gradation characteristics of sinusoidal transformation

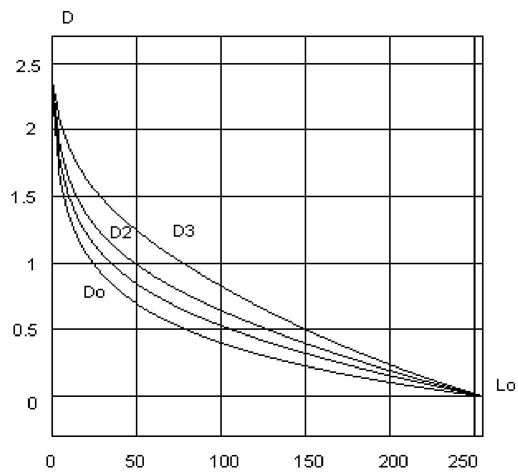

Fig. 4. Graphs of optical density of sinusoidal transformation of dark tones of the image

Regardless of the transformation frequency, the initial values of optical density are 2.406 units. The first characteristic from the bottom D_0 corresponds to the optical density of the linear scale. All characteristics are concave curves. Increasing the transformation frequency shifts the characteristic downward, resulting in a decrease in optical density and a lightening of the image. At the beginning of the range, the optical density decreases rapidly and at L_0 = 50 levels, it is: 0.8488; 0.9912; 1.248; 0.699 units. After this, the rate of density decrease slows down and eventually approaches zero.

The results of modeling the contrast sensitivity of sinusoidal transformation of dark tones at different frequencies are presented in Fig. 5.

The graphs of contrast sensitivity of sinusoidal transformation are concave curves, and their initial values depend on the frequency, being C: 0.23; 0.48; 0.68 units. They gradually increase and around L_0 = 150 gray levels, they intersect the unit line and quickly reach final values of C: 1.55; 1.90; 2.26 units. Note that the contrast sensitivity of the linear scale is constant and equals one over the entire tone transfer interval. Contrast sensitivity quantitatively and qualitatively evaluates the response of the visual system to tone perception of the image, which corresponds to the psychophysical Weber-Fechner law [1, 3]. Images are better perceived in the medium and light range, which should be considered when preparing the image for printing.

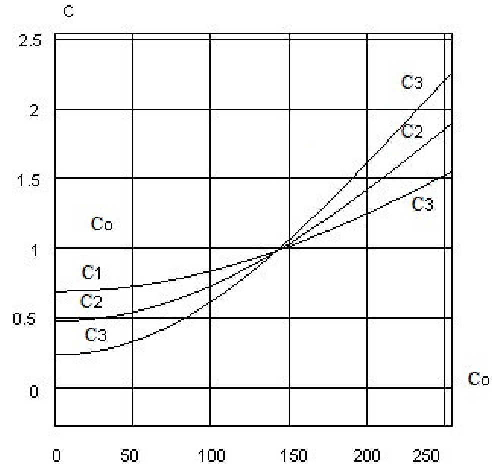

Fig. 5. Graphs of contrast sensitivity of sinusoidal transformation

Unlike traditional gamma transformation, the results of sinusoidal transformation demonstrate low initial shift values (0.5–2 units) and first step lengths (1–2 levels), which significantly reduce the likelihood of posterization. Due to the features of the sinus function with adaptive frequency, a higher steepness of the gradation characteristic in dark areas is ensured without losses in highlights. This makes the proposed model particularly suitable for images with a large amount of detail in shadows.

Compared to other known works [1, 7, 10], where the main focus is on logarithmic or power-law correction functions, the proposed approach demonstrates better detail preservation, better tone uniformity, and more natural visual perception, confirmed by contrast sensitivity > 2.2, which exceeds the linear approach.

The limitations of the study include the fact that sinusoidal transformation has not yet been implemented in ready-made software packages for prepress preparation, and its practical application requires integration into existing software systems or the creation of specialized plugins.

In the future, it is advisable to further develop this approach towards adaptive control of the sinusoid frequency depending on the local content of the image, as well as to explore its application in color processing, taking into account the interconnection of RGB channels or Lab/CMYK models for comprehensive improvement of printed image quality.

4. Conclusions

A new model of digital image of dark tones based on the inversion of sinusoidal transformation and the addition of a linear component has been developed. This model allows for the formation and correction of specified gradation characteristics of the image by changing the transformation frequency. It eliminates posterization in dark areas and prevents losses in light areas. The optical density and contrast sensitivity of the image, which quantitatively assess their properties, have been determined. In the MATLAB: Simulink package, a structural scheme of the sinusoidal transformation simulator model has been developed, enabling parallel calculation and construction of gradation characteristics, optical density graphs, contrast sensitivity, and analysis of their properties.

The results of simulation modeling of gradation characteristics, which are concave curves with increasing curvature as the transformation frequency increases ν = 0.0016; 0.0019; 0.0020 are presented. It has been established that at the beginning of the range, the steepness of the gradation characteristics increases, thus better reproducing details in dark areas of images while simultaneously ensuring quality reproduction of light images, which is an advantage of the developed sinusoidal transformation.

It has been found that the quantized gradation characteristics of zero discretes have initial shifts of 0.5; 0.7; 2.0 units, and the lengths of

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the first steps are 1, 2, 4 levels, which are an order of magnitude less than in gamma transformation, thus the sinusoidal transformation of dark tones of the image lacks posterization, which is its advantage. The optical density graphs are concave, with initial values of $D = 2.406$ units, which quickly decrease and at $L_0 = 50$ levels are: 0.848; 0.912; 1.248 units, after which the rate of decrease slows and approaches zero.

The contrast sensitivity graphs are concave curves, and their initial values depend on the transformation frequency, being C: 1.544; 1.903; 2.766, gradually increasing and around $L_0 = 155$ levels intersecting the unit line and quickly reaching final values of C: 1.544; 1.903; 2.766 units. Contrast sensitivity quantitatively evaluates the human visual system's response to changes in brightness, corresponding to the psychophysical Weber-Fechner law.

The results of the study expand the informativeness of sinusoidal transformation and can be used to select optimal gradation characteristics when preparing digital images for printing.

Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship, or any other interest that could have influenced the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that no artificial intelligence technologies were used in the creation of this work.

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