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DEVELOPMENT OF A METHOD FOR STATE ESTIMATION AND OPTIMISATION OF MULTIFACTOR SEMI-MARKOV SYSTEMS

The object of this research is a method for solving problems of analysis and optimization of semi-Markov systems. The importance of this topic is determined by the following circumstances. First, traditional, standard theoretical and practical problems of stochastic system research are solved analytically only for Markov systems for which the laws of distribution of the duration of stay in each state before leaving are exponential. Clearly, this strict requirement is not met for real systems. Second, a general method of analytical study does not exist for many probabilistic systems. Third, only numerical methods for solving such problems are available and feasible. Moreover, in each case, a solution can only be obtained for the specific system under study, operating under specific conditions. Clearly, such a solution is uninformative and practically useless for optimization problems of multifactor systems. In this regard, the study aims to develop a universal method for solving analysis and optimization problems, suitable for any semi-Markov systems. The proposed method for solving the formulated problem solves it in two stages. In the first stage, a matrix of distribution densities is found by processing experimental data, representing the duration of the system's stay in each state before transitioning to another state. It is crucial that the densities be in the Erlang distribution class of some order. These densities are found using the least-squares method, using histograms obtained by processing the experimental data. In the second stage, the resulting distribution densities are used to construct a system of differential equations for the probabilities of the system's stay in each possible state. This constructively utilizes the unique property of Erlang distributions: any Erlang flow is a sifted simplest Poisson flow. Sequentially completing these two stages yields a solution to the problem of studying any probabilistic (semi-Markov) systems. Thus, the method proposed in this paper for solving problems of analysis and optimization of semi-Markov systems is universal.

Keywords: *semi-Markov systems, system analysis and optimization, Erlang distribution, probabilistic modeling.*

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1. Introduction

The theory and practice of solving a variety of real-world problems divides this set into two classes: analysis (diagnostics) of the state of systems and their optimization for improved efficiency.

The difficulty and specificity of accurately assessing the state of the environment in which systems operate, as well as the numerical values of their parameters, impart a probabilistic nature to the study of these systems.

During their operation, systems randomly transition from one state to another. Probabilistic systems can have the following two properties. First, the system's operation after exiting a specific possible state depends only on that state, not on how the system arrived at that state. Second, the distribution law for the duration of stay in each state before exiting it is exponential. Such systems are called Markov systems. If a system possesses only the first property but not the second, it is called semi-Markov. It should be noted that real systems almost always possess the first property, while the second is extremely rare.

The mathematical theory for studying Markov systems is well-developed. Its application always leads to the desired result. For semi-Markov systems, the situation is different. This is why the problem of developing a method for studying semi-Markov systems is of fundamental practical interest. While there is no general analytical method for solving this problem, a numerical solution is possible. A simple example is considered.

It is assumed that a semi-Markov process is defined if the following are given:

1) the set E of possible process states;
2) the matrix of conditional distribution functions of the duration of the process's stay $Q_{ij}(t)$ in the i-th state before transitioning to the j-th state, $i \in E$, $j \in E$, obtained through statistical analysis of the initial data;
3) the initial state of the process at time $t = 0$.

Further, if t_{ij} – the random duration of stay in i before transitioning to j, then

$$Q_{ij}(t) = P(t_{ij} < t). \quad (1)$$

Moreover, the probability $P_{ij}(t)$ of a transition from state i to state j in time t is the probability that no transition to some other state occurs during this time interval. The probability of a transition from i to j in the vicinity of time t is equal to $dQ_{ij}(t)$. Then

$$P_{ij}(t) = \int_0^t \prod_{k \neq j} \left(1 - Q_{ik}(\tau)\right) \mathrm{d}Q_{ij}(\tau). \quad (2)$$

The set of functions $P_{ij}(t)$ uniquely defines a semi-Markov process. Furthermore, the probability of a transition from i to j in an unlimited time is equal to

$$P = Q_{ij}(\infty) = \int_0^{\infty} \prod_{k \neq j} \left(1 - Q_{ik}(\tau)\right) \mathrm{d}Q_{ij}(\tau).$$

The matrix $p = (P_{ij})$ defines a Markov chain embedded in the semi-Markov process.

To analyze the dynamics of the states of a semi-Markov process, a matrix $F_{ij}(t)$ of conditional distribution functions for the duration of stay in *i* before transitioning to *j* is introduced. By definition

$$F_{ij}(t) = P\left(t_{ij} < t / \xi(0) = i, \xi(t) = j\right). \quad (3)$$

Moreover

$$P_{ij}(t) = F_{ij}(t) P_{ij}.$$

The corresponding (3) density function for the duration of the process's stay in *i* before transitioning to *j* is equal to

$$f_{ij}(t) = \frac{dF_{ij}(t)}{dt}.$$

Now the main problem of studying the process can be formulated: to find the probability distribution of its stay on the set of possible states *E* at any time *t*. Introduced $\Phi_{ij}(t)$ is the conditional probability that at time *t* the analyzed system is in state *j* if at time $t = 0$ it was in state *i*. This probability is called the interval transition probability [1]. The corresponding events can occur as follows. First, if $i = j$, then the system may remain in state i throughout the entire interval *t*. Second, it may exit this state and then, at time *t*, return to it. Accordingly, the interval transition probability is described by the relation

$$\phi_{ij}(t) = \psi_i(t) + \sum_{\substack{k \in E \\ k \neq i}} P_{ik} \int_0^t f_{ik}(\tau) \phi_{kj}(t - \tau) \mathrm{d}\tau. \quad (4)$$

Third, if $j \neq i$, the system may end up in *j*, occupying some intermediate state *k* at time $\tau < t$. The corresponding relation has the form

$$\phi_{ij}(t) = \sum_{\substack{k \in E \\ \alpha \neq i}} P_{ik} \int_0^t f_{ik}(\tau) \phi_{kj}(t - \tau) \mathrm{d}\tau. \quad (5)$$

Relations (4) and (5) together define a system of integral equations for $\Phi_{ij}(t)$. An indicator has been introduced

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases}$$

that allows to combine (4) and (5). This yields

$$\phi_{ij}(t) = \delta_{ij} \psi_i(t) + \sum_{\substack{k \in E \\ k \neq i}} P_{ik} \int_0^t f_{ik}(\tau) \phi_{kj}(t - \tau) \mathrm{d}\tau. \quad (6)$$

The Laplace transform can theoretically be used to obtain a solution to system of equations (6). In this case

$$\phi_i^*(s) = \delta_{ij} \psi_i^*(s) + \sum_{\substack{k \in E \\ k \neq i}} P_{ik} f_{ik}^*(s) \phi_{kj}^*(s)_{i \in E, j \in E}. \quad (7)$$

System of equations (7) is written in matrix form. For this purpose, matrices

$$\phi(t) = \left(\phi_{ij}(t_i)\right), \quad Q = \left(P_{ij}\right), \quad f(t) = \left(f_{ij}(t)\right),$$

$$\omega(t) = \left(\delta_{ij} f_i(t)\right), \; F(t) = \left(\delta_{ij} F_j(t)\right), \; \psi(t) = \left(\delta_{ij} \psi_i(t)\right),$$

and the corresponding Laplace transform matrices are introduced. Furthermore, to simplify further relations, a special matrix multiplication operation, denoted by the symbol *O*, is introduced. According to this operation, for square matrices *A*, *B*, and *C* of the same dimension, the notation $C = AOB$ means that $C_{ij} = \alpha_{ij} \beta_{ij}$, $i \in E$, $j \in E$. Then, relation (7) takes the form

$$\phi^*(s) = \psi^*(s) + \left[POf^*(s)\right] \phi^*(s),$$

from which

$$\phi^*(s) = \left[1 + POf^*(s)\right]^{-1} \psi^*(s). \quad (8)$$

The matrix obtained as a result of operation (8) is then subjected to the inverse Laplace transform, the result of which determines the solution to the problem of finding $\Phi_{ij}(t), i \in E, j \in E$.

Thus, the theoretical possibility of analyzing semi-Markov systems exists. However, the practical implementation of this possibility is associated with the need to perform certain specific, non-standard computational procedures, as shown in the example. In particular, obtaining a solution to the system of integral equations (6) is non-trivial. Moreover, in most cases, only an approximate solution is possible, the error level of which is difficult to predict. As a result, a tabular description of the sought-after function is obtained. This result cannot satisfy the researcher, since it does not contain the most important thing – a description of the analytical dependencies of the parameters of the sought-after probability distribution densities of the system state at any point in time on the parameters of the functions describing the probabilities of the system transitions (initial data).

The real, obvious need to apply semi-Markov models to solve a variety of practical problems is fundamental, and, therefore, the corresponding results are widely discussed. In [1], general approaches to the problem of analyzing stochastic systems using Markov and semi-Markov models are considered. Using a problem in reliability theory as an example, the problem of determining the distribution law for the duration of a system's stay in a selected set of admissible states is formulated. The solution has been completed only for the specific case under consideration. The same problem is considered in [2], and in [3], an inventory control problem is analyzed in which the intervals between requests and service durations are described by semi-Markov processes. As a result, for this specific problem, only a probabilistic description of the queue length was obtained. In [4], a probabilistic description of the inventory level was obtained for a similar problem. In [5], the average duration of stay of a semi-Markov system in possible states is calculated and analyzed. In [6], the problem of processing data on failure rates and uptime is solved. Based on this, an empirical probability density function of the uptime distribution is obtained.

The relevance of the problem of developing technologies for analyzing and optimizing semi-Markov systems is discussed and emphasized. In [7], the problem of optimizing a technical maintenance system is posed and considered. A technology for calculating the start time of maintenance is developed. The same problem of optimizing a maintenance system is considered in [8]. In [9], the problem of optimizing a maintenance system is considered in more detail. The difficulties of solving the problem are analyzed. The desired result was obtained only after radically simplifying the problem by reducing it to a Markov model. The paper [10] considers the problem of modeling the operation of systems. Optimization possibilities are explored only at the level of formulating the corresponding problems.

An analysis of these works on current issues in the development, construction, research, and operation of real stochastic systems provides grounds for several conclusions:

– the vast majority of stochastic systems used in practice are semi-Markov systems;

– there is no general, theoretically sound method for accurately studying such systems.

Hence, the object, aim, and objectives of the research follow.

The object of research is a method for solving analysis and optimization problems for any semi-Markov system.

The aim of research is to develop a reliable and universal method for solving analysis and optimization problems for semi-Markov systems.

To achieve this aim, it is necessary to solve the following objectives:

– for each state of the system, calculate histograms of the durations of the system's stay before transitioning to another state;
– each of the resulting histograms is approximated by an Erlang distribution of the appropriate order;
– use the obtained distribution densities, which determine the dynamics of the probabilities of the system's existence in each of the possible states, to form a system of differential equations for the probabilities of states at any point in time;
– develop a computational technology for solving the resulting system of differential equations.

2. Materials and Methods

The description of the proposed method is presented in a sequence corresponding to the numbering of the problems formulated above. This method must be capable of implementing a combination of the following technologies. The computational scheme must be oriented toward processing the initial durations of the studied system's stay in each of its possible states before its transition to any other state. The result of this processing will be a set of corresponding histograms. Each of these histograms is smoothed by a suitable distribution density or their additive convolution. The resulting set of densities is used to form a mathematical model of the system's behavior in the dynamics of its operation. The resulting mathematical model is a graph of states and transitions controlled by the corresponding transition probability matrix. Using the resulting model, a system of differential equations is formulated for functions determining the probabilities of the system's stay in each of its possible states at any given time. The solution to this system of equations determines the behavioral dynamics of the analyzed system.

3. Results and Discussion

The proposed method for studying semi-Markov objects differs from known methods in that it takes a special approach to selecting the distribution densities of random variables used in the mathematical model. The following requirements are imposed on these densities. First, they must possess high approximation capabilities for maximum accuracy in reproducing histograms. This means that the corresponding functions must be at least two-parameter. Second, the basic functions of the resulting mathematical model of the behavior of the analyzed system must be suitable for describing the behavior of Markov systems. Two-parameter functions describing Erlang distributions of arbitrary order simultaneously satisfy both of these requirements. The fact is that an Erlang flow of any order has a unique property: this flow is a sifted simplest Poisson flow. Therefore, with its use, the corresponding mathematical models of the system from semi-Markov ones are easily transformed into Markov ones, the analysis of which is carried out without difficulty by solving the Kolmogorov-Chapman system of linear differential equations [1, 2]. The strong analytical capabilities of this approach can be further enhanced by using compositions of Erlang models for analysis, which naturally improves the accuracy of the approximation. In this case, the original semi-Markov mathematical model is transformed into a multi-stream Markov model.

Thus, the general plan for solving the problem is as follows. First, histograms of random values of intervals between transitions are formed based on the results of processing a series of observations. Next, a function is selected that represents the distribution density of the random value of the interval between these events. After this, the numerical values of the parameters of the desired distribution densities from the class of Erlang distributions are sequentially found using the least-squares method. This method and the corresponding computational technology provide an analytical description of the distribution densities of random values of the duration of a semi-Markov system's stay in any possible state before transitioning to any other state. The desired analytical description is realized by approximating the corresponding histograms with distribution densities from the class of Erlang distributions of the appropriately chosen order. An important remark should be made regarding the usefulness of the result obtained above. The proposed technology for approximating real histograms of random variables with Erlang distributions would not be so useful and would hardly be widely used were it not for one key property of these Erlang distributions. It was mentioned that a stream of events, which intervals have an Erlang distribution of any order, is a sifted simplest (exponential) stream. This means that to obtain, for example, a first-order Erlang stream, one must select every odd (or every even) event (i. e., events that follow one another, not consecutively, but every other event) from a simplest stream of the same intensity. It is precisely this property of Erlang distributions that makes it possible to analyze non-Markov systems using Markov methods. Thus, it is possible to move on to describing a technology for constructing models of systems, which behavior is described using Erlang streams of a given order.

The simplest variants of constructing queueing systems, which event streams are described by Erlang distributions are considered. Let's assume that a first-order Erlang stream arrives at the input of a single-channel system, and the servicing is exponential. The state and transition graph in such a system is shown in Fig. 1.

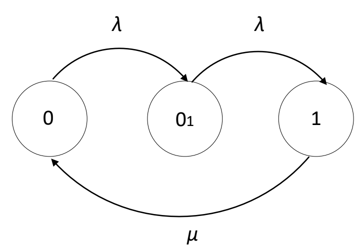

Fig. 1. System state and transition graph

In Fig. 1: 0 – the only channel in the system is free; O_1 – the request to be screened, which does not occupy the channel, has arrived at the system input; 1 – the channel is busy with servicing; λ – the incoming flow rate; μ – the service rate.

A system of linear algebraic equations is introduced that describe the system's behavior using state probabilities P_O, P_{O_1}, P_1:

$$
\begin{aligned}
&-\lambda P_0 + \mu P_1 = 0,\\
&-\lambda P_{0_1} + \mu P_0 = 0,\\
&\lambda P_{0_1} + \mu P_1 = 0,\\
&P_0 + P_{0_1} + P_1 = 1.
\end{aligned} \qquad (9)
$$

This system is solved by expressing P_{0_1} and P_1 in terms of P_0. As a result:

$$
P_1 = \frac{\lambda}{\mu} P_0,
$$
$$
P_{01} = P_0.
$$

By substituting the resulting expressions for P_{01} and P_1 into the normalization condition, let's obtain

$$
P_0 + \frac{\lambda}{\mu} P_0 + P_1 = 2P_0 + \frac{\lambda}{\mu} P_0 = P_0\left(2 + \frac{\lambda}{\mu}\right) = 1.
$$

Hence

$$P_0=\frac{1}{2+\frac{\lambda}{\mu}}=\frac{\mu}{2\mu+\lambda},$$

$$P_{01}=P_0=\frac{1}{2+\frac{\lambda}{\mu}}=\frac{\mu}{2\mu+\lambda},$$

$$P_1=\frac{\lambda}{\mu}.\frac{1}{2+\frac{\lambda}{\mu}}=\frac{\lambda}{2\mu+\lambda}.$$

The desired distribution of the final probabilities of the system is obtained.

Below, let's consider a slightly more complex problem in which the service flow is also a first-order Erlang flow. The system state and transition graph is shown in Fig. 2.

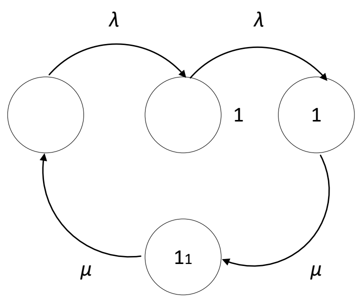

Fig. 2. System state and transition graph

In Fig. 2: 0 – the only channel is free; O_1 – the request has arrived at the system input that does not cause the channel to be occupied; 1 – the channel is busy servicing; 1_1 – the state to be screened out, the channel is busy servicing.

A system of linear algebraic equations describing the system's behavior is introduced:

$$\begin{aligned}&-\lambda P_0+\mu P_{1_1}=0,\\&\lambda P_0-\lambda P_{0_1}=0,\\&\lambda P_{0_1}-\mu P_1=0,\\&\mu P_1-\mu P_{1_1}=0,\\&P_0+P_{0_1}+P_1+P_{1_1}=1.\end{aligned}\qquad(10)$$

Expressing all system states through P_0, let's obtain:

$$P_{1_1}=\frac{\lambda}{\mu}P_0,$$

$$P_{0_1}=P_0,$$

$$P_1=\frac{\lambda}{\mu}P_{0_1}=\frac{\lambda}{\mu}P_0.$$

The resulting expressions are substituted into the normalization condition, and is determined by P_0. In this case

$$P_0+\frac{\lambda}{\mu}P_0+P_0+\frac{\lambda}{\mu}P_0=2P_0+2\frac{\lambda}{\mu}P_0=2P_0\left(1+\frac{\lambda}{\mu}\right)=1.$$

Hence

$$P_0=\frac{1}{2\left(1+\frac{\lambda}{\mu}\right)}=\frac{\mu}{2(\lambda+\mu)}.$$

Then

$$P_{1_1}=\frac{\lambda}{\mu}P_0=\frac{\lambda}{\mu}\cdot\frac{1}{\left(1+\frac{\lambda}{\mu}\right)2}=\frac{\lambda}{2(\lambda+\mu)},$$

$$P_{0_1}=P_0\cdot P_1=\frac{\lambda}{\mu}P_{0_1}=\frac{\lambda}{\mu}P_0=\frac{\lambda}{\mu}\cdot\frac{\mu}{2(\lambda+\mu)}=\frac{\lambda}{2(\lambda+\mu)}.$$

The desired state probability distribution is obtained.

Finally, an even more complex model is considered. Let a first-order Erlang flow arrive at the input of a three-channel semi-Markov system, and let the servicing of requests be exponential. The system state and transition graph is shown in Fig. 3.

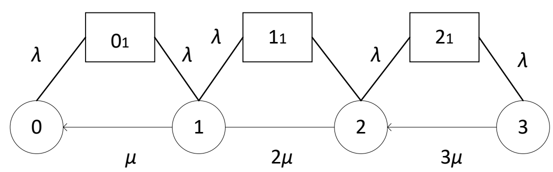

Fig. 3. State and transition graph of a three-channel semi-Markov system with a first-order Erlang flow as input. Service is exponential

Thus, in acc ordance with the unique property of Erlang flows, the order of this flow for $k=1$ is reflected in the graph by the appearance of intermediate states $(0_1,1_1,2_1)$, that are eliminated. The corresponding system of linear algebraic equations then takes the following form:

$$\begin{aligned}&-\lambda P_0+\mu P_1=0,\\&\lambda P_0-\lambda P_{0_1}=0,\\&\lambda P_{0_1}+2\mu P_2-\lambda P_1-\mu P_1=0,\\&\lambda P_1-\lambda P_{1_1}=0,\\&\lambda P_{1_1}+3\mu P_3-\lambda P_2-2\mu P_2=0,\\&\lambda P_2-\lambda P_{2_1}=0,\\&\lambda P_{2_1}-3\mu P_3.\end{aligned}\qquad(11)$$

By summing the second equation with the third, the fourth equation with the fifth, and the sixth equation with the seventh, let's obtain the following system of four equations:

$$\begin{aligned}&-\lambda P_0+\mu P_1=0,\\&\lambda P_0+2\mu P_2-\lambda P_1-\mu P_1=0,\\&\lambda P_1+3\mu P_3-\lambda P_2-2\mu P_2=0,\\&\lambda P_2-3\mu P=0.\end{aligned}\qquad(12)$$

The following set of parameters is introduced

$$Z_k=\lambda P_{k-1}-k\mu P_k, k=1,2,3,4.$$

Using these parameters, system (12) is written as follows:

$$\begin{aligned}&Z_1=0,\\&Z_1-Z_2=0,\\&Z_2-Z_3=0,\\&Z_3=0.\end{aligned}$$

Hence

$$Z_1=Z_2=Z_3=0,$$

or

$$\lambda P_{k-1}-k\mu P_k=0, k=1,2,3.$$

These relations yield the following recurrence formulas

$$P_k = \frac{\lambda}{k\mu} P_{k-1}, k = 1,2,3.$$

Hence

$$P_1 = \frac{\lambda}{\mu} P_0,$$
$$P_2 = \frac{\lambda}{2\mu} P_1 = \frac{\lambda^2}{2!\mu^2} P_0,$$
$$P_3 = \frac{\lambda}{3\mu} P_2 = \frac{\lambda^3}{3!\mu^3} P_0. \quad (13)$$

It is noted that the second, fourth, and sixth equations of system (13) yield:

$$P_0 = P_{01},$$
$$P_1 = P_{21},$$
$$P_2 = P_{21}. \quad (14)$$

From relations (13) and (14), taking into account the normalization condition, let's obtain the value P_0

$$\sum_{k=0}^{3} P_k + \sum_{k=0}^{2} P_{k1} = 2P_0 \sum_{k=0}^{2} \frac{\lambda^k}{k!\mu^k} + P_3 = 1,$$

hence

$$P_0 = \frac{1}{2\left(\sum_{k=0}^{3} \frac{\lambda^k}{k!\mu^k}\right) + P_3}.$$

Then

$$P_k = \frac{\frac{\lambda^k}{k!\mu^k}}{2\sum_{k=0}^{3} \frac{\lambda^k}{k!\mu^k} + P_3}, k = 0,1,2,3. \quad (15)$$

The solution to the problem is complete. It should be noted that the computational scheme for solving specific problems discussed above and implemented is in no way tied to the specifics of their formulation, confirming the universality of the method. This allows for a radical expansion of the capabilities of existing methods for solving a variety of similar problems, which are strictly limited to the study of Markov systems. The proposed method can be used for any semi-Markov system with arbitrary distribution laws for the duration of stay in each state before transitioning to any other state. The possibility of developing a general method for studying semi-Markov systems is the result of the constructive use of the properties of Erlang event flows. The high efficiency of the proposed method for studying systems is determined by the simplicity of its practical implementation and the controllable accuracy of the obtained results.

A natural limitation: the stated problem must be well-posed, that is, the number of experiments must exceed the number of factors. Further research is aimed at extending the method to cases where the initial data are not clearly defined.

4. Conclusions

1. The universal method for analyzing and optimizing semi-Markov systems is proposed and substantiated. The method can be implemented for any initial data with respect to the distribution laws for the durations of the system's stay in states before departure.

2. The two-stage computational procedure was developed and used to solve the problem. In the first stage of implementing the method, statistical processing of the experimental data is performed to obtain histogram approximations using the Erlang distribution or, if necessary, their additive convolution.

3. In the second stage of the procedure, a technology for obtaining a system of linear differential equations for the probabilities of system states at any point in time is substantiated. The complexity of these equations is independent of the nature and characteristics of the initial data.

4. The solution to these equations determines the desired distribution densities of the system's stay in each of the possible states, as well as the probability distribution of the system's states at any point in time.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship or other, which could affect the study and its results presented in this article.

Financing

The study was performed without financial support.

Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

Authors' contributions

Lev Raskin: Conceptualization, Methodology, Project administration; ***Larysa Sukhomlyn***: Validation, Investigation; ***Viacheslav Karpenko***: Supervision, Validation; ***Dmytro Sokolov***: Writing – review and editing, Supervision; ***Vitalii Vlasenko***: Formal analysis.

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