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# SUBSTITUTION OF THE METHOD FOR ASSESSING GENERALIZED DYNAMIC INSTABILITY OF PARAMETERS OF THE GAS ENVIRONMENT OF PREMISES FOR EARLY FIRE WARNING

*The object of research is the dynamics of the gas environment at the early stages of material combustion in premises. The problem to be solved is to develop a method of generalized dynamic instability of the gas environment, based on the local Kantz function and its time derivative, to detect early unstable regimes of the gas environment preceding the development of a fire. A method for assessing the generalized dynamic instability of the gas environment of premises is proposed, focused on the parameters of early fire warning. Generalized dynamic instability is understood as a cumulative characteristic that expresses the level of local sensitivity of the dynamics of time series of gas environment parameters to excitation, as well as the rate of change of this sensitivity in time. The local in time variant of the Kantz method and its time derivative are used as the basis. The method is tested on experimental data of the current concentration of carbon monoxide, which are obtained under the conditions of modeling the ignition of materials. It is shown that the local Kantz function and its derivative demonstrate pronounced changes in the transient regimes of the gas environment in the absence of significant excesses of the permissible thresholds of the dangerous measured parameter. The results obtained allow to consider the proposed measure of generalized dynamic instability as an additional diagnostic feature in early fire warning systems. The dynamics of the proposed measure for the initial stages of ignition of alcohol, paper and textiles are studied. The relationship between the dynamic content of carbon monoxide and the change in the generalized measure is analyzed. The results obtained indicate the efficiency of the method and show that, despite the differences in the kinetics of gas release and the nature of combustion, the dynamic response of the generalized measure of dynamic instability is universal.*

**Keywords:** *generalized dynamic instability, local Kantz function, ignition of materials, gas environment, fire warning.*

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## 1. Introduction

Fires are one of the greatest sources of danger to humans and most objects. Fires result in loss of life and damage or complete destruction of property and objects [1]. An effective method of fire control is their prevention through the prompt identification of ignition sources of materials (IM) and their timely extinguishment. Otherwise, an IM source leads to a fire, which causes significant losses [2] and colossal environmental damage [3]. Therefore, the prompt identification of IM sources is considered one of the key areas of fire safety for various facilities. In this regard, early detection of fires in premises remains a key objective of fire protection systems, since it is in the initial stages of an IM that the greatest potential exists for preventing the development of emergency situations and minimizing the consequences. Traditional fire detection systems based on threshold monitoring of combustion product concentrations, temperature, or smoke demonstrate high reliability in the advanced stages of a fire; however, their sensitivity to early transient processes in the gas environment (GE) is often limited. In the initial stages of a fire, changes in GE parameters, such as carbon monoxide (CO) concentrations, can occur at background levels and

not exceed established thresholds [4]. Under these conditions, the GE maintains an externally stationary nature, while its internal dynamics can transition to a non-stationary regime due to local sources of gas emission, changes in heat and mass exchange conditions, and premise ventilation characteristics. Detecting such transient regimes is a complex task that goes beyond the scope of classical threshold methods. One promising area of time series analysis is the use of dynamic stability and instability characteristics, reflecting the system's sensitivity to small disturbances. In this case, the use of the Kantz method for estimating the exponential divergence of trajectories in phase space is of particular interest. However, in its original formulation, the Kantz method is focused primarily on global dynamics characteristics and does not take into account local changes in the regime over time, typical of GE transient processes the early stages of a fire. The IM in the premises of objects leads to the appearance of various hazardous parameters (HP) in the GE. The main HPs are usually considered to be the CO concentration, the specific optical density of smoke and the GE temperature [5]. It should be noted that the traditional features of the GE HP dynamics during IM, used in existing fire protection systems, are not capable of detecting early IM sources in premises. Therefore,

non-traditional features of the GE HP dynamics that respond to the IM occurrence become relevant. A measure of the generalized instability of the dynamics of the GE parameters can be considered as such a feature. The aforementioned instability of the parameter dynamics during fires is due to the complexity and impossibility of adequately mathematically describing the physical processes occurring in the GE under the influence of numerous irreversible physicochemical mechanisms, both in the materials themselves and in the building's enclosing structures. Therefore, the generalized instability of the GE HP dynamics caused by fires can be considered as an IM indicator in various types of premises, allowing for the prevention of costly fires.

To study the instability of dynamics in various systems based on experimental observations of parameters, a proven toolkit based on Lyapunov exponents has been developed [6, 7]. The degree of instability of the system dynamics is characterized by one of the largest Lyapunov exponents. In this regard, studies of the features of the largest Lyapunov exponent for the GE HP dynamics are relevant. In [8], an approach to the IM operational detection based on observations of one arbitrary GE HP using several sensors placed in a premise and subsequent network processing of the measurement information is developed. However, this approach is limited to the use of traditional features of the GE HP dynamics without taking into account its nonlinear features. A similar approach, but in the case of simultaneous observation of various GS parameters is considered in [9]. The same limitations are characteristic of this approach. In this case, possible nonlinearity, chaos and instability of the GE HP dynamics are not considered. In [10], it is proposed to use the spectral features of the GE HP dynamics to identify IM. However, the use of this indicator of the GE HP dynamics significantly narrows the class of acceptable GE HP dynamics, as well as the possibilities for identifying IM. The features of the GE HP dynamics, characterized by the dependence of the rate of heat release on the intensity of real combustion of plantation wood, are studied in [11]. Similar studies for various types of wood were carried out in [12]. However, in [11, 12] the study of dependencies is limited to the dependencies between the average values of the rate of heat release and the intensity of real combustion of materials. The indicated dependencies for the real combustion of organic glass and cypress are carried out in [13].

However, in [11–13], the instability of the GE HP dynamics is not considered or investigated. The use of non-traditional third-order spectra of the GE HP dynamics for the operational IM detection is considered in [14]. It is shown that such spectra generally allow one to identify the correlation between the corresponding frequency components caused by the nonlinearity of the GE HP dynamics. However, it is noted that the correlation of the frequency components significantly depends on the energy of the observed HP. The use of third-order spectra turns out to be quite complex in practice and is limited by the stationarity intervals of GE HP dynamics. The use of features of the average bicoherence of the GE HP dynamics for the IM detection is considered in [15]. It is noted that the use of average bicoherence values can generally be used as an IM sign. However, due to the alternating nature, it is difficult to interpret the instability of the GE HP dynamics during IM. In [16], the features of determining bicoherence based on a single realization and an ensemble of realizations of observation data are compared. It is noted that the determination of bicoherence by an ensemble of realizations is more effective in terms of identifying the IM. However, the Lyapunov indices and the instability indices of the GE HP dynamics are not considered or studied in [16]. The use of the PI feature defined by the empirical cumulative distribution function of the current recurrence of the GE HP state is considered in [17]. However, its use is associated with the implementation of a complex computational procedure, which significantly reduces efficiency. In [18], the IM feature is studied in the form of the probability of the absence of recurrence of the GE HP state vector increments based on the empirical cumulative distribution function. However, the implementation of this feature is

associated with complex calculations, which significantly limits its use in practice. The results of the study of the GE HP dynamics during IM fire tests are considered in [19].

It is noted that in order to increase the reliability of detecting IM, it is necessary to take into account the combined dynamics of the CO concentration and the specific optical density of smoke. In this case, the maximum Lyapunov exponent, which characterizes the chaos and dynamic GE HP instability, is not considered. In [20], the results of an experimental study of the mutual GE HP relationship during IM are presented. However, the mutual relationship is estimated by correlation, which, as is known, only estimates the degree of linear relationship. The Lyapunov exponents [21], which characterize the chaotic features and instability of the GE HP dynamics, are also not considered. The methodology for predicting the GE HP increments for the prompt IM detection is considered in [22]. However, the studies are limited to considering the simplest parametric Brown model, the parameters of which significantly depend on the analyzed GE HP dynamics. Moreover, the simplest Brown model belongs to the class of linear models. This significantly limits its application in real conditions. The use of the sample variance of the GE HP parameters in intervals of reliable absence and presence of IM as a fire warning indicator is considered in [23]. However, this IM indicator has limited operational effectiveness, determined by the specified duration of the GE HP observation interval.

Thus, it follows that the application of the largest Lyapunov exponent and measures of GE HP dynamic instability in the early IM stages has not been sufficiently studied for indoor fire prevention tasks. Therefore, a method for estimating the generalized dynamic instability of indoor GE parameters is relevant for early fire warning tasks.

*The object of this research* is the GE dynamics in a premise in the early IM stages.

*The aim of the research* is to substantiate a method for generalized dynamic instability of a gas environment based on the local Kantz function and its time derivative for identifying early non-stationary processes preceding the development of a fire in a premise.

Research objectives:

1. To develop a time-local version of the Kantz method and determine the contribution of the local Kantz function and its derivative to the formation of a generalized instability measure.
2. To investigate the behavior of a generalized measure of dynamic instability on experimental time series of CO concentrations obtained by modeling the early stages of material combustion.

## 2. Materials and Methods

Fires involving combustion products are accompanied not only by rapid ignition reactions, but also by smoldering combustion reactions, which usually develop quite slowly. Therefore, fires in buildings are usually divided into two types: flaming and smoldering. Compared to flaming fires, smoldering fires emit a relatively small amount of thermal energy [24]. However, after the IM onset involving combustion products, the process of their decomposition in the form of smoldering leads to a significant release of various toxic gases and volatile compounds into the GE [25, 26]. Moreover, the released toxic gases and volatile compounds pose a significant danger to humans even before the appearance of dense smoke and open flame [5]. In fact, inhalation of toxic combustion products is the leading cause of death in fires [27, 28]. Synthetic materials are widely used in various building materials, furniture, and electrical wire insulation. These materials emit more toxic substances, especially in the presence of flame retardants [29]. Identifying the moments of increase of toxic compounds above the background level at the early IM stages allows to quickly improve the level of fire safety of premises with low costs for their elimination. It is known that any combustion of a material causes its destruction and changes in its physical properties, as well as the dynamic GE HP instability in

a premise. Following [5, 24–29], different amounts of toxic combustion products and heat are released in the GE of a premise at different moments in time. This means that the early IM detection can be considered based on the analysis of the dynamics instability of various GE parameters. At the initial IM stage, the level of reactions and mechanisms of the material combustion process is usually insufficient for their prompt detection using threshold methods. In this formulation, the GE of premises is proposed to be considered as a certain complex dynamic system with moving unstable modes. Moreover, the IM occurrence will cause a corresponding instability of the regime of such a dynamic system. Therefore, early fire warning can be carried out based on the assessment of a certain measure of the generalized dynamic instability of the GE. This problem can be solved by analyzing the Lyapunov exponents for the GE HP dynamics. In this paper, the measure of generalized dynamic instability is understood as a cumulative characteristic that takes into account both the level of local sensitivity of the time series dynamics to initial disturbances (the local Kantz function) and the rate of its change over time.

It is known that Lyapunov exponents are widely used to identify unstable regimes in complex dynamic systems of various natures [6]. However, these exponents cannot be applied to non-stationary systems, which include the considered indoor GE when a ground fault occurs. The fundamental property of most real dynamic systems is the chaos regime [30, 31]. Non-periodic chaotic oscillations are the most typical for most different real dynamic systems [32, 33]. In general, the largest Lyapunov exponent, which characterizes the degree of chaos in dynamic systems, is determined over an infinite stationarity interval. However, the time interval for observing the parameters of dynamic systems is usually limited in time, and the observations themselves are subject to the influence of noise. This generally reduces the accuracy of the canonical definition of the Lyapunov exponent, complicating the assessment of its reliability and interpretation [34]. It should be noted that, to estimate the largest Lyapunov exponent in the classical formulation, the Kantz method is applied to stationary dynamic systems and is focused on the analysis of the asymptotic behavior of trajectory divergence over significant observation time intervals. In this paper, the Kantz method is used in a different context: not to restore the global Lyapunov exponent, but as a tool for local time analysis of changes in the dynamic sensitivity of the GE HP time series. The main focus is on the shape and temporal variability of the local Kantz function, as well as the rate of its change, which allows for the identification of transient regimes not described within the classical approach.

The study materials for the purpose of testing the proposed method for assessing the generalized dynamic instability of the GE parameter dynamics were the results of observing the output signal of a sensor measuring the GE CO concentration in a laboratory chamber simulating an unsealed premise, taking into account the main similarity criterion [35, 36]. To measure the temporal changes in the GE CO concentration in the chamber, a CO-B4 sensor (Alphasense, UK) was used [37]. The method was tested by igniting test materials (TM) such as alcohol, paper, and textiles. The ignition time was considered known a priori. Measurements of the CO concentration in the chamber GE during TM ignition were carried out at discrete moments in time with an interval of 0.1 seconds. The TM choice was determined by various features of their ignition reactions and CO generation in the chamber GE [38]. The measurement results were considered as a causal time series determined by a set  $\{x(k)\}$  of CO concentration values at discrete moments in time for each TM.

### 3. Results and Discussion

#### 3.1. Development of a method for assessing the generalized dynamic instability of the gas environment parameters in premises

As a measure of the dynamic the GE HP instability based on a discrete time series of their observations, it is proposed to use an indicator sensitive to the divergence of close trajectories in the phase space.

A positive value of the indicator will indicate local disturbances in the GE dynamics, while a zero or negative value will indicate marginal and local stability of the dynamics. The results of an analysis of known algorithms for identifying the divergence of initially close trajectories and the specifics of their application are contained in [39, 40]. The key here is the reconstruction of the phase space based on a single observed sample of states of the dynamic system under study, using the delay method, the selection of a suitable algorithm, and an estimate of the largest Lyapunov exponent [41–45]. A positive value of the obtained indicator is usually associated with the chaotic regime of the dynamic system under study that generates the aforementioned observations. Important aspects of the application of known algorithms include: the size of the observation sample, as well as the embedding parameters. However, among the algorithms [41–45], only two allow one to estimate the divergence of trajectories from a data sample without knowing the operator that generates the observed sample. These are the algorithms of Kantz [39, 43, 46] and Rosenstein [42]. The canonical Kantz algorithm boils down to calculating the average distance between adjacent points of the trajectory over time and estimating the largest Lyapunov exponent by fitting the function of the change in the average distance between adjacent points to an exponential dependence. However, the Kantz algorithm does not directly estimate the largest Lyapunov exponent. The Rosenstein algorithm is also based on calculating the average distance between adjacent points of the trajectory over time, but the largest Lyapunov exponent is estimated by the slope of the average divergence between adjacent points of the trajectory on a double logarithmic scale. The advantage of the Rosenstein algorithm is that the average distance is calculated between several adjacent points. This ensures increased stability of the algorithm to noisy data. Also known is the Kantz-Rosenstein algorithm [44], which is a modification of the indicated algorithms and combines the advantages of the algorithms noted above. However, the use of the algorithms [42–44] assumes a priori knowledge of the embedding parameters or their determination from a data sample. The solution to the latter problem presents significant difficulties, which increase significantly in the case of noisy observations. In addition, it is necessary to know in advance the state of the dynamic system under study and select the starting point at the beginning of the region of the studied state. In the case under consideration, the specified conditions are not met, since the one-dimensional sample of observed data  $x_1, x_2, x_3, \dots, x_N$  is noisy and is taken from the region of the dynamics of a system with an unknown state. In closed spaces, IM leads not only to an increase in the concentrations of combustion products, but also to a change in the dynamics of the GE parameters. Therefore, to prevent a fire, it is important to promptly detect the indicated changes in the dynamics, and not just the excess of the concentrations of combustion products above threshold levels. In this regard, a method for estimating the generalized dynamic instability of the GE parameters is proposed. Let an arbitrary GS parameter be represented by measurements in the form of a corresponding discrete series

$$x_k = x(t_k), t_k = k\Delta t, \quad (1)$$

where  $\Delta t$  – the sampling step. At time  $k$ , only observations  $\{x_1, \dots, x_k\}$  are available; future values are not available. The goal of the method is to construct real-time estimates reflecting the local dynamic instability of an arbitrary GE HP based on its measurements, represented as a corresponding time series (1). To analyze the nonlinear dynamics of time series (1), the phase space embedding method is used according to Takens' theorem [47]

$$X_i = (x_i, x_{i-\tau_e}, \dots, x_{i-(m-1)\tau_e}),$$

where  $m$  – the embedding dimension;  $\tau_e$  – the embedding delay. Correct reconstruction requires that the  $i \geq (m-1)\tau_e + 1$  be satisfied.

To ensure causality and adapt to changing conditions, a sliding window of length  $W$  is introduced at time  $k$

$$J_k(\tau) = \{i: \max(k-W+1, (m-1)\tau_c + 1) \leq i \leq k\}.$$

This means that all calculations are performed only on data from the  $J_k$  set, eliminating the use of future values of the series. Local (window) methods of time series analysis have recently been considered an effective approach to analyzing non-stationary time series [43]. The classical method for estimating trajectory divergence was proposed by Kantz and is used to estimate the largest Lyapunov exponents. For the causal formulation, a localized Kantz window function is introduced. For this purpose, a subset of indices is determined for a given lag  $\tau$

$$J_k(\tau) = \{i \in J_k : i + \tau \leq k\}.$$

For each point  $X_i, i \in J_k(\tau)$ , a fixed number of nearest neighbors  $K$  within  $J_k(\tau)$  is selected according to the Euclidean metric

$$N_k(i; k) \subset J_k(\tau).$$

Then, the canonical window function will be defined as

$$S_k(\tau) = \frac{1}{|J_k(\tau)|} \sum_{i \in J_k(\tau)} \ln \left( \frac{1}{k} \sum_{i \in N_k(i; k)} \|X_{i+\tau} - X_{j+\tau}\| \right), \quad (2)$$

where  $\|\cdot\|$  – the Euclidean norm. This local function describes the average logarithm of the divergence of close trajectories in the window and preserves the causal properties of the analysis. Given representation (2), the local rate of divergence of points will be determined by the lag derivative

$$\lambda_k(\tau) = \frac{d}{d\tau} S_k(\tau). \quad (3)$$

For a robust numerical estimate of this derivative, filtering and differentiation are applied using the Savitzky-Golay method [48], which is widely used to identify trends in noisy signals of sensor systems [49]. In this case, the generalized measure of instability at time  $k$  will be determined by the maximum of local velocities over the range of lags

$$A_k = \max_{\tau \in [1, \tau^*]} \lambda_k(\tau), \quad (4)$$

where  $\tau^*$  is chosen in the quasi-linear region of function  $S_k(\tau)$ . This allows one to obtain a scalar characteristic sensitive to the structure of the local dynamics. This approach is related to the ideas of local analysis of dynamic systems described by non-stationary series [48]. In the engineering formulation of early fire warning, the value of  $A_k$  is interpreted as follows:

- $A_k \approx 0$ : nearly stationary, background mode of the parameter dynamics;
- $A_k > 0$ : the beginning of a dynamic transition, indicating a disruption of the stable dynamics of the parameter;
- a steady increase in  $A_k$  indicates the development of uncontrolled processes characteristic of the initial stage of a fire and the development of a fire.

Unlike classical threshold detectors based on the excess of absolute values of the GE HP above the corresponding levels, the proposed method captures structural changes in the HP dynamics that may occur before the appearance of significant absolute effects. The proposed method does not require stationarity and uses only the data available at time  $k$ . Unlike threshold detectors focused on absolute parameter levels, the method captures dynamic transitions. The use of a local Kantz window function allows adaptation to the changing background and

noise of the GE HP measuring sensors. It should be noted that, in the classical formulation, the well-known Kantz method is used to estimate the largest Lyapunov exponent of stationary modes of dynamic systems and is focused on the analysis of the asymptotic behavior of trajectory divergence over large time intervals. In this paper, the Kantz method is used not to estimate the asymptotic Lyapunov exponent, but as a tool for identifying local changes in time in dynamic sensitivity based on non-stationary time series of current causal observations of an arbitrary GE HP. The primary focus is on the shape and temporal variability of the local Kantz function, as well as the rate of its change, which enables the identification of hazardous transient non-stationary GE HP modes that are not described within the classical approach. To test the proposed method, the dynamics of CO concentrations were considered as the GE HP. The dynamics of CO concentrations in the GE of premises is an integral indicator of the incomplete oxidation of the ignited material. Therefore, it reflects not only the IM fact and its combustion, but also the thermal degradation of the ignited material and the oxygen balance conditions. CO dynamics reflect the mechanisms of smoldering combustion, pyrolysis followed by ignition, flame combustion under oxygen deficiency, localized overheating and thermal and oxidative degradation, self-heating and spontaneous combustion of the material, as well as flashover and ventilation-controlled fires. The transition from a steady state to a deterministic instability mode, characteristic of the onset of the exothermic phase, is particularly informative.

### 3.2. Study of the behavior of a generalized measure of dynamic instability on experimental time series of CO concentrations

The results of testing the proposed method for the GE CO dynamics in a laboratory chamber during the combustion of alcohol, paper, and textiles are shown in Fig. 1–3, respectively. The value of  $k$  in each figure characterizes the current moment in time, determined by  $k\Delta t$ . The green curves represent the experimental dynamics of  $CO_k$  concentration (ppm), the red curves represent the calculated window function  $S_k$  (2), and the blue curves represent the generalized measure of instability  $A_k$  (4), corresponding to alcohol, paper, and textiles.

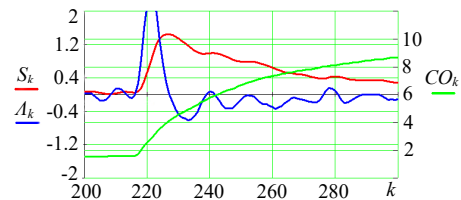


Fig. 1. Experimental dynamics of CO concentration, window function (2), and generalized instability measure (4) during alcohol ignition

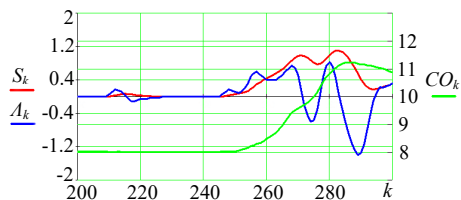


Fig. 2. Experimental dynamics of CO concentration, window function (2), and generalized instability measure (4) during paper ignition

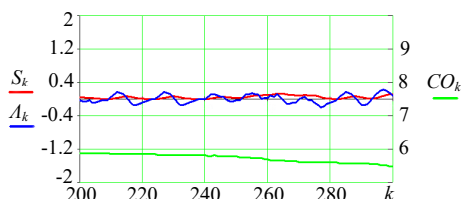


Fig. 3. Experimental dynamics of CO concentration, window function (2), and generalized instability measure (4) during textile ignition

To test the method, forced ignition of the corresponding TM was performed in the interval  $k \in (210, \dots, 230)$ . The corresponding window functions (2) and generalized instability measures (4) were calculated for  $m = 3$ ,  $\tau_c = 0.1$  s,  $K = 2$ , and  $W = 9$  for alcohol, paper, and textiles. A comparison of the presented results revealed that, despite differences in gas evolution kinetics and combustion behavior, the dynamic response of the generalized instability measure is universal for alcohol, paper, and textiles. For alcohol, with its rapid gas evolution,  $S_k$  spikes are characterized by greater amplitude and shorter duration, while for paper and textiles, they are more extended. This difference reflects the complex physical properties of the actual decomposition and combustion processes of these materials and confirms the sensitivity of the proposed method to the type of GE CO instability source during ignition. At the same time, the presence of qualitatively similar instability features for the considered TMs demonstrates the potential universality of the proposed approach for analyzing the GE CO instability under various early ignition scenarios.

### 3.3. Discussion

The obtained results confirm that the transition of the GE HP state of a premise from a background stable mode to an unstable state can be accompanied by significant changes in their dynamic properties without a pronounced increase in absolute values [50, 51]. A similar qualitative picture was observed for alcohol, paper, and textiles. The local Kantz function  $S_k$  remained at a level close to the background mode, while its time derivative  $\Delta k$  demonstrated sharp local increases (coinciding with the moment of violation of the GE stationarity). This indicates that in the early stages of a fire, the rate of change of the dynamic mode of the GE HP plays a key role. From a physical point of view, this is explained by the appearance of local sources of gas emission and a change in the conditions of their transfer to the premise GE. This leads to a rapid redistribution of the phase GE HP structure while maintaining moderate concentrations of combustion products. From a practical point of view, the proposed method should be considered as an additional analytical module in fire monitoring systems, functioning in parallel with traditional threshold algorithms. The use of a generalized instability measure increases the system's sensitivity to early transient conditions without modifying the hardware or reducing the reliability of fire alarm systems at advanced stages of a fire. Promising areas for further research include expanding the set of analyzed GE HP, studying the proposed method's robustness to various types of interference, and developing criteria for the automated interpretation of the joint behavior of the  $S_k(\tau)$  and  $\Delta k$  functions. Of additional interest is the analysis of spatially distributed changes, which allows for the consideration of the heterogeneity of GE parameters in premises. However, this approach has several limitations. First, the analysis results depend on the choice of phase reconstruction parameters and the sliding window length. Although the parameters were fixed in this study, their optimal choice may depend on the characteristics of a specific premise, the sensor type, and ventilation conditions. Second, the method is focused on identifying GE HP instability and is not intended for the unambiguous identification of fires or the classification of ignition sources. In particular, changes in dynamic instability can also be caused by other factors, such as abrupt changes in the ventilation regime or external disturbances unrelated to the IM. Third, the analysis was performed based on a single GE parameter – CO concentration. Although the method is formally applicable to other parameters, including the concentrations of other gases or the temperature and optical characteristics of the GE, its effectiveness in such cases requires separate experimental verification.

## 4. Conclusions

1. A localized version of the Kantz method is developed, and the contribution of the local Kantz function and its derivative to the formation of a generalized measure of gaseous instability is determined.

The concept of generalized dynamic instability of a gaseous environment is introduced. Its physical meaning is defined in the context of the transition from a background state to a non-stationary state of a gaseous environment upon the onset of a material fire. An approach for the rapid detection of localized instability of gaseous environment parameters is proposed, based on the combined consideration of the dynamics of the generalized instability measure and its time derivative. This approach enables the identification of short-term instability modes in a gaseous environment, which may precede a noticeable increase in the concentration of various combustion products. This approach is not aimed at replacing existing fire alarm systems, but rather at expanding their functionality through additional analysis of the dynamic properties of the measured hazardous gaseous parameters.

2. The dynamics of the generalized measure of dynamic instability and its derivative were studied using experimental time series of CO concentrations obtained by simulating the early stages of alcohol, paper, and textile combustion. The relationship between the dynamics of CO concentration and the change in the generalized measure of instability and its derivative over time in transient gas environments was analyzed. It was found that the average logarithm of the divergence of close trajectories at the onset of alcohol, paper, and textile combustion was 0.4, 0.015, and 0.09, while the generalized measure of instability was 2.3, 0.8, and 0.2, respectively. The obtained results confirm the efficiency of the method and demonstrate that, despite the differences in the kinetics of gas evolution and the nature of alcohol, paper, and textile combustion, the dynamic response of the generalized measure of instability is universal, allowing for differentiation between the ignited materials. An assessment was made of the use of a generalized measure of dynamic instability as an additional diagnostic feature of a dangerous parameter of a gas environment during ignition of materials for early warning of fires in premises.

### Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship or other, that could influence the research and its results presented in this article.

### Financing

The research was conducted without financial support.

### Data availability

Data will be made available on reasonable request.

### Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies in creating the submitted work.

### Authors' contributions

**Boris Pospelov:** Methodology, Project administration; **Igor Tolok:** Project administration; **Evgeniy Rybka:** Software, Writing – review and editing; **Ihor Morozov:** Formal analysis; **Yurii Kozar:** Investigation; **Olekciï Krainiukov:** Supervision; **Volodymyr Volovyk:** Data curation; **Olga Levada:** Writing – original draft; **Maksym Harifullin:** Conceptualization; **Natalia Bed:** Resources.

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