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DEVELOPMENT OF A HYBRID ARTILLERY FIRE CONTROL SYSTEM BASED ON NEURAL NETWORKS AND UNCERTAINTY QUANTIFICATION METHODS

The object of research is the process of controlling the fire of artillery installations in a hybrid ballistic modeling system. The problem addressed lies in the lack of comprehensive studies of systems that combine neural network forecasting with physical iterative refinement and stochastic assessment of projectile dispersion within a single operational pipeline. This paper examines the specific features of developing a hybrid artillery fire control system based on the integration of neural networks, numerical refinement of aiming angles, and methods for quantifying the uncertainty of the ballistic model. A modular system architecture is proposed and investigated, integrating a ballistic simulator with a 4-DOF model in accordance with NATO STANAG 4355. The system is supplemented by a neural network, which generates an initial approximation of the aiming angles. For the subsequent calculation of the aiming angles, an algorithm was implemented using iterative elevation angle refinement via the Brent method and gradient azimuth correction. To assess uncertainty, polynomial chaos expansion (PCE) and Monte Carlo methods were integrated. A synthetic ballistic dataset consisting of 121107 records was generated based on 24 configurations of artillery systems. Validation of the neural network demonstrated a narrowing of the search space for aiming angles to a corridor of ±3–5°, ensuring further rapid convergence of the iterative refinement algorithm. Testing for the 2S22 "Bohdana" artillery system at a range of 20 km showed a deterministic error of 0.68 m. The PCE method achieved an error of 0.47 m, outperforming the Monte Carlo method (5.28 m) by a factor of 11.2. Analysis using the PCE method revealed anisotropic projectile dispersion: $\sigma_x = 168.85$ m, $\sigma_z = 80.84$ m, CEP50 = 147.3 m. The viability of the hybrid system has been demonstrated under ballistic simulation conditions, laying the groundwork for further validation with real-world firing data.

Keywords: *ballistic modelling, hybrid ballistic pipeline, ballistic dataset, neural networks, uncertainty quantification.*

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1. Introduction

Modern armed conflicts are characterized by highly dynamic combat operations and increased demands on the speed of response of artillery units to current situations. Artillery remains a key element in striking the enemy, accounting for a substantial share of the fire support tasks for ground forces [1, 2]. The effectiveness of artillery units directly depends on the speed and accuracy of ballistic calculations in the automated fire control systems (AFCS). The conditions of modern battlefield deployment, in particular the active use of counter-battery warfare and unmanned aerial vehicles, significantly reduce the permissible duration of artillery units' stay in firing positions [3]. Reducing the target engagement cycle from 3–5 minutes to 30–60 seconds significantly increases the survivability of artillery units and the effectiveness of firing at enemy targets [4, 5].

A key component of the AFCS is the ballistic calculation module, which must ensure high accuracy in predicting the trajectory of a projectile with minimal calculation time [6]. Traditional methods of modeling external ballistics, based on numerical integration of motion equations in accordance with NATO STANAG 4355 standard, provide high accuracy but demand considerable computing resources [7, 8]. Neural network approaches demonstrate high speed, but their approximation accuracy is insufficient for practical application without additional refinement by a physical model [9]. But a hybrid pipeline that combines neural network initial approximation with physics-based iterative refinement addresses both limitations simultaneously, enabling rapid convergence to accurate ballistic solutions within a single operational workflow [9].

At the same time, deterministic approaches to ballistic calculations do not take into account the stochastic nature of firing parameters, such as variations in the initial velocity of the projectile, fluctuations in meteorological conditions, and errors in determining the target coordinates [10, 11]. The uncertainty of the input parameters leads to the dispersion of projectiles around the aiming point, which is critically important to consider when planning fire damage [12]. Therefore, an urgent scientific task is to study hybrid methods of ballistics modeling for AFCS, which combine the advantages of neural network approximation with the accuracy of physical models and provide a quantitative assessment of the uncertainty of results within a unified operational pipeline.

A critical analysis of the literature shows the active development of three areas of artillery ballistics modeling. With regard to neural network approaches, study [13] demonstrates the effectiveness of neural models for predicting artillery rocket trajectory characteristics, achieving a relative error of ≈1% in elevation angle determination. The study addresses both the determination of firing range from a given elevation angle and the inverse problem. In study [9], a trajectory propagator based on artificial neural networks was implemented using nonlinear models with 4–5 DOF. In work [14], the CORR-CNN-BiLSTM-Attention model was proposed for predicting the trajectory of a projectile with an error of 7.9 m in firing range and 0.9 m in lateral deviation. In study [15], LSTM (Long Short-Term Memory) is used to estimate the trajectory of a projectile in the absence of GNSS (Global Navigation Satellite System) with an error of less than 10 m. At the same time, the authors of studies do not integrate the neural network with a ballistic simulator for iterative refinement of the results.

Regarding methods for quantifying uncertainty, work [16] proposed a PCE-based approach for analyzing artillery system dynamics with hybrid stochastic and interval parameters. The method provides an approximately 27-fold reduction in computation time compared to Monte Carlo, demonstrated on an example with 14 uncertain parameters. The study [17] conducted a comparative analysis of the PCE and Monte Carlo methods for stochastic wave propagation analysis, confirming the computational efficiency of the intrusive PCE approach. In study [18], a method of high-dimensional quantification of the uncertainty of projectile motion in the barrel of a self-propelled artillery unit (SAU) is implemented based on the probability density evolution method (PDEM). In paper [19], the influence of model uncertainties on impact point dispersion was investigated using the Monte Carlo method, yielding a circular error probable (CEP) of approximately 90 m. Four sources of uncertainty were considered: aerodynamic parameters, thrust curve, initial conditions, and inertial navigation system errors. It was also found that the use of graphics accelerators to parallelize the Monte Carlo method in the study [20] made it possible to significantly reduce the duration of stochastic modeling. However, it should be noted that methods for quantifying projectile motion uncertainty are mainly considered in isolation from artillery fire control algorithms.

With regard to improving numerical ballistics models and automated control systems, work [7] developed fire control algorithms for the Modular Naval Artillery Concept (MONARC), integrating the NATO STANAG 4355 ballistic core. Fire command calculations were achieved in under one second at ranges of up to 80 km. The study [21] performed a comparative analysis of 6-DOF and a modified point model of the projectile trajectory. In paper [22], the feasibility of a modified 4-DOF point model for artillery shells was analyzed. In study [8], the authors formulated a general principle for determining the accuracy of artillery fire based on a topological description of kinematic and dynamic equations, achieving an error of 2% compared to tabulated data. Paper [23] presents the PVNPG-14M automated control system for the Czech Republic's army, which complies with NATO interoperability standards.

It should be noted that Ukrainian researchers have also made a significant contribution to the development of methods for modeling artillery ballistics and AFCS. In study [24], an improved algorithm for calculating data for firing at large targets is proposed, taking into account the angle of location of the targets. In paper [25], the information and computing process of the automated control system for rocket artillery was improved by introducing a system of differential equations with weight functions for temperature and wind. The authors in research [26] improved the model and method of controlling the destruction of a target by an artillery installation with minimization of combat capability losses. In studies [27, 28], a comprehensive analysis of the current state of the AFCS was carried out and approaches to correcting artillery fire using neural networks were proposed.

An analysis of the literature revealed the following unresolved issues. The use of neural networks provides a high calculation speed, but the accuracy of the approximation is insufficient for practical application in AFCS without additional refinement by a physical model. The PCE and Monte Carlo methods are considered separately from the task of modeling the trajectory of a projectile. While each of these methods has been studied individually, no comprehensive approach exists that integrates neural network approximation, physics-based iterative refinement, and uncertainty quantification into a single operational AFCS pipeline. Thus, the problem of integrating these established methods into a unified hybrid artillery fire control system remains unresolved.

The object of research is the process of controlling the fire of artillery installations in a hybrid ballistic modeling system.

The subject of research is methods and means of controlling the fire of artillery installations that combine the prediction of aiming angles, the accurate determination of the aiming angles, and quantifying the uncertainty of ballistic model parameters.

The aim of research is to develop a hybrid artillery fire control system that integrates neural network prediction of aiming angles, physics-based numerical refinement, and methods for quantifying the uncertainty of ballistic model parameters.

To achieve the set aim, the following *objectives* must be solved:

1. To form a synthetic ballistic dataset ensuring representative coverage of input parameters, accounting for the variability of artillery systems and meteorological conditions.
2. To implement a hybrid aiming angles calculation pipeline that combines neural network initial approximation with iterative Brent-method refinement and gradient azimuth correction.
3. To integrate uncertainty quantification methods (PCE and Monte Carlo) into the hybrid pipeline and verify the complete system through comparative analysis of deterministic and stochastic approaches on a 155 mm SAU test scenario.

2. Materials and Methods

2.1. System architecture

The methodological basis of research is the integration of established ballistic modelling, neural network approximation, and uncertainty quantification methods into a hybrid artillery fire control system. The system implements a modular architecture [29] consisting of the following modules:

1. A ballistic simulator that implements a physical model of external ballistics to calculate the flight trajectory of an artillery shell.
2. A module for generating a synthetic set of ballistic data, incorporating meteorological data for training a neural network.
3. A neural network that generates an initial approximation of the elevation angle and azimuth for subsequent iterative refinement.
4. A fire control module that combines ballistic calculations, neural network predictions, and uncertainty quantification using Monte Carlo and PCE methods.

The operational workflow of the hybrid system is illustrated in Fig. 1.

The process begins with the formation of a synthetic ballistic dataset and training of the neural network predictor (Stage 1). During operation, the neural network generates an initial approximation of the aiming angles for both low and high firing modes (Stage 2). The ballistic simulator then refines the predicted angles using the Brent method for elevation, adaptive azimuth correction, and gradient-based Newton-Raphson adjustment, selecting the solution with the smallest accuracy error (Stage 3). The refined aiming angles are passed to the uncertainty quantification module, where PCE and Monte Carlo methods evaluate projectile dispersion under stochastic conditions (Stage 4). The final output includes deterministic aiming angles and probabilistic dispersion estimates, including the covariance matrix and CEP50 metric (Stage 5).

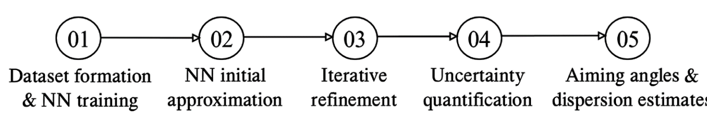

Fig. 1. Operational workflow of the hybrid artillery fire control system

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2.2. Ballistic simulator and physical model

Within the hybrid architecture, the ballistic simulator serves a dual role: as the ground-truth generator for neural network training and as the physics-based refinement engine during operation. This dual functionality requires a simulator that balances computational efficiency with sufficient physical fidelity.

The ballistic simulator implements a modified 4-DOF model in accordance with NATO STANAG 4355 standard [8, 30], describing the motion of a spin-stabilized projectile through three translational coordinates and axial rotation. The projectile trajectory is computed in a topocentric Cartesian coordinate system (x – north, y – vertical upward, z – east) by integrating a system of sixth-order ordinary differential equations [8, 24]

$$\frac{dx}{dt}=v_x,\quad \frac{dy}{dt}=v_y,\quad \frac{dz}{dt}=v_z,$$
$$\frac{dv_x}{dt}=a_x,\quad \frac{dv_y}{dt}=a_y,\quad \frac{dv_z}{dt}=a_z, \tag{1}$$

where $r=(x, y, z)$ – position vector, m; $v=(v_x, v_y, v_z)$ – velocity vector, m/s; $a=(a_x, a_y, a_z)$ – acceleration vector.

The total acceleration is formed by the superposition of gravitational force, aerodynamic drag based on tabulated Mach-dependent coefficients for Standard-HE and ERFB-BB projectile categories [30], Coriolis force, and Magnus force, computed in accordance with the standard formulation [8, 22]. Atmospheric conditions follow the ISA model [31] corrected to actual meteorological data using dimensionless density and temperature coefficients [8, 31]. The wind altitude profile is modeled by a logarithmic dependence with surface roughness parameter $z_0 = 0.03$ m [32].

2.3. Ballistic dataset generation procedure

The hybrid pipeline requires a comprehensive training dataset that captures the full complexity of the inverse ballistic problem across diverse artillery systems and environmental conditions. Unlike standalone neural network approaches that optimize for prediction accuracy alone, the dataset for the proposed hybrid system must balance two objectives: providing sufficient coverage for neural network generalization while ensuring that the generated scenarios remain within the operational envelope where iterative refinement can converge reliably.

The dataset generation procedure combines 24 unique artillery configurations (12 systems × 2 shell categories: Standard-HE and ERFB-BB) with a systematic grid of 3600 aiming angle combinations (60 elevation values from 1° to 85° × 60 azimuth values from 0° to 360°). For each combination, 60 firing scenarios are generated using the Domain Randomization method [33] through random selection of meteorological profiles and system configurations, with stochastic variations applied to the basic projectile parameters. Meteorological data archives were collected via the Open-Meteo API service for 24 regional centers in Ukraine covering the calendar year 2024 with hourly discretization. Each meteorological profile contains five parameters: air temperature, atmospheric pressure at sea level, relative humidity, wind speed, and wind direction. The ballistic trajectories are computed by numerical integration of a system of sixth-order differential equations using the 4–5th order Runge-Kutta method with an adaptive step size [34, 35] in accordance with NATO STANAG 4355 standard [8, 30]. The simulation results are then collected and filtered according to physical correctness criteria, and the final dataset is subject to statistical analysis to verify the representativeness of the sample.

2.4. Hybrid pipeline for aiming angles determination and dispersion estimation

The hybrid pipeline integrates all system components into a unified operational workflow that transforms target coordinates into aiming angles with associated dispersion estimates. While each individual component implements established techniques, their combination into a single AFCS-compatible pipeline enables simultaneous determination of deterministic aiming angles and probabilistic estimates of projectile dispersion – representing the core methodological contribution of the work.

The pipeline operates in two sequential stages. In the first stage, a neural network predictor generates an initial approximation of the aiming angles. The network maps an input vector of 17 parameters – organized into five semantic groups (target coordinates, meteorological conditions, projectile characteristics, gun parameters, and Boolean indicators) – to an output vector of 3 parameters: elevation angle in degrees (1–85°) and azimuth represented as a *sin* and *cos* pair. To improve approximation quality across the full operational envelope, the neural network predictor is structured as two independent models – one for low (1–45°) and one for high (45–85°) firing modes [9, 13].

In the second stage, iterative refinement is performed by fixing the azimuth angle, determining the elevation angle using the Brent method for a given firing range [36], and subsequently correcting the azimuth to minimize lateral deviation. Both firing modes are processed in parallel, with the solution of smallest accuracy error selected as the final result. The elevation angle is computed with an accuracy of 0.01°, and the azimuth correction is performed iteratively with adaptive tolerance [37]

$$tolerance=\max\left(\frac{0.5,\ r_{target}}{40000.0}\right), \tag{2}$$

which sets a minimum of 0.5 m for firing ranges up to 20 km and increases linearly to 1 m at 40 km. The iterative process continues until the tolerance or the maximum number of iterations (12) is reached.

The gradient correction method for the azimuth refines the aiming angles by calculating the numerical Jacobian as a local linear approximation of the nonlinear mapping of the aiming angles to the hit coordinates [37]. The 2 × 2 Jacobian matrix is calculated using finite differences with adaptive steps from 0.05° to 0.005° depending on the firing range [37]

$$J=\begin{pmatrix} \partial x/\partial\theta & \partial x/\partial\varphi \\ \partial z/\partial\theta & \partial z/\partial\varphi \end{pmatrix}, \tag{3}$$

where θ – the elevation angle; φ – the azimuth angle; x – the distance indicating the range of impact; z – the distance indicating the lateral deviation. The correction vector is determined using the Newton-Raphson method [37]

$$\Delta a=-J^{-1}\cdot\Delta r\cdot s_{scale}, \tag{4}$$

where Δr – a 2-dimensional hit error vector with components of the difference between the actual and target coordinates in terms of firing range and lateral deviation. J^{-1} is the inverse Jacobian matrix, and s_{scale} indicates the adaptive scaling coefficient that adaptively adjusts the amount of aiming angles correction depending on the current accuracy error.

Uncertainty in the system is quantified using two methods: Monte Carlo method and PCE method [17, 20]. Both methods use five independent sources of stochastic uncertainty, each following a normal distribution: ambient air temperature (σ = 5°C), atmospheric pressure (σ = 3 hPa), relative humidity (σ = 9%), surface wind speed (σ = 2 m/s), and initial projectile velocity (σ = 1% of nominal value).

The second-order PCE method uses systematic discretization through 1024 quadrature nodes formed by the tensor product of one-dimensional third-order Gauss-Hermite quadratures [38]. For each node, a ballistic simulation is performed, and the hit coordinates are approximated by decomposition based on Hermite orthogonal polynomials [38]

$$x(\xi)=\Sigma a_a \Psi_a(\xi),$$
$$z(\xi)=\Sigma b_a \Psi_a(\xi), \quad (5)$$

where $\Psi_a(\xi)$ – multidimensional Hermite polynomials, and a_a and b_a – decomposition coefficients calculated by projection onto the polynomial basis using quadrature weights [38, 39]. The statistical moments are then obtained analytically: the mathematical expectation from the zero coefficient, variances from the sum of squares of non-zero coefficients, and the covariance is defined as [38]

$$Cov\left[x,z\right]=\Sigma_a \neq_0 a_a b_a \|\Psi_a\|^2 . \quad (6)$$

The Monte Carlo method is implemented by direct probabilistic modeling of ballistic trajectories of projectiles [20]. The system generates 1024 realizations of a 5-dimensional vector of stochastic parameters through a pseudo-random generator. The transformation from standardized random variables to physical parameters is described by the system of affine transformations [19].

For each implementation, a ballistic simulation is performed with calculated aiming angles, forming a sample of 1024 pairs of hit coordinates. Based on the generated sample of hit coordinates, empirical estimates of the main statistical moments of the distribution are calculated. Empirical variances are calculated using unbiased sample estimates with Bessel correction. The empirical covariance matrix is formed from the calculated variances and covariances [37]

$$S=\begin{pmatrix} s_x^2 & s_{xz} \\ s_{xz} & s_z^2 \end{pmatrix}, \quad (7)$$

where s_x^2 – sample variance of the range coordinate; s_z^2 – sample variance of the lateral deviation coordinate; s_{xz} – sample covariance between the range and lateral deviation coordinates. Empirical standard deviations are calculated as square roots of variances and represent estimates of the characteristic value of shell dispersion around the mean point of impact.

3. Results and Discussion

3.1. Dataset characteristics and statistical validation

Before evaluating the hybrid pipeline, this section presents the characteristics of the generated dataset and validates its statistical representativeness. The quality of the training data directly affects the neural network's ability to provide bounded initial approximations across the full operational envelope, which in turn influences the performance of the subsequent iterative refinement stage.

A synthetic ballistic dataset was formed by combining 24 artillery configurations with a systematic grid of 3600 aiming angle combinations and 60 stochastic meteorological scenarios per combination, applying the Domain Randomization method [33] with random variation of key projectile and environmental parameters. After filtering according to physical realism criteria (horizontal range from –5 to 70 km, lateral deviation ≤30 km), the final dataset comprised 121107 records (56.1% of the initial volume), with a balanced distribution of 54.9% for low and 45.1% for high firing mode. Data validation included checking for missing values and outlier detection using the IQR method [40]. Fig. 2 shows histograms of the empirical distribution of four key parameters.

Statistical analysis confirms the representativeness of the dataset: air temperature follows a near-normal distribution (μ = 10.4°C, σ = 9.4°C) consistent with Ukrainian climatic conditions; atmospheric pressure is concentrated around 1018 hPa (σ = 8 hPa); impact range shows an asymmetric distribution (μ = 19.3 km, σ = 9.6 km) characteristic of artillery systems; initial velocity is multimodal (μ = 882 m/s, σ = 76 m/s) reflecting the diversity of artillery systems in the database.

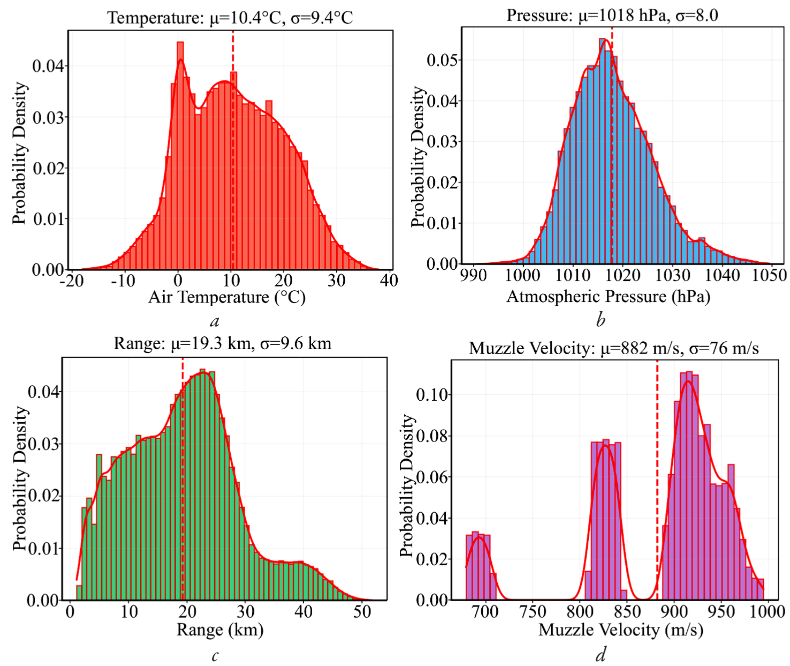

Fig. 2. Distribution histograms of dataset parameters: *a* – air temperature; *b* – atmospheric pressure; *c* – range of impact; *d* – initial velocity of the projectile

3.2. Hybrid aiming angles calculation pipeline evaluation

The evaluation of the hybrid pipeline begins with assessing the prediction quality of the neural network component, which serves as the initial approximation generator for the iterative refinement algorithm.

The neural network was trained separately for low firing mode (1–45°) and high firing mode (45–85°), as separate specialized models provide a better approximation of the inverse ballistic problem compared to a single model [41]. The dataset was divided into training, validation, and test samples in a ratio of 78/12/10%. The prediction quality was evaluated on the test sample (Table 1).

Table 1

Model accuracy metrics on the test sample

Mode	$MAE(\theta)$	$RMSE(\theta)$	$R^2(\theta)$	$MAE(\varphi)$	$RMSE(\varphi)$
Low (1–45°)	2.027	4.386	0.892	0.942	1.277
High (45–85°)	2.563	4.479	0.879	0.852	1.165

The absolute errors MAE = 2.027–2.563° for the elevation angle are insufficient for direct application in fire control: at a firing range of 25 km, such an error can result in a miss of approximately 1 km [8, 12]. However, in the context of the hybrid system, the neural network serves solely as an initial approximation generator for subsequent iterative refinement. The validation results in the form of scatter plots are shown in Fig. 3.

The predictions are concentrated along the ideal line without significant systematic bias, with the bulk of elevation angle predictions within a ±3° corridor and azimuth predictions within ±1.5°. The obtained accuracy narrows the search space from the full range of 1–85° to a corridor of ±3–5°, ensuring rapid convergence of the Brent iterative refinement in 2–3 iterations.

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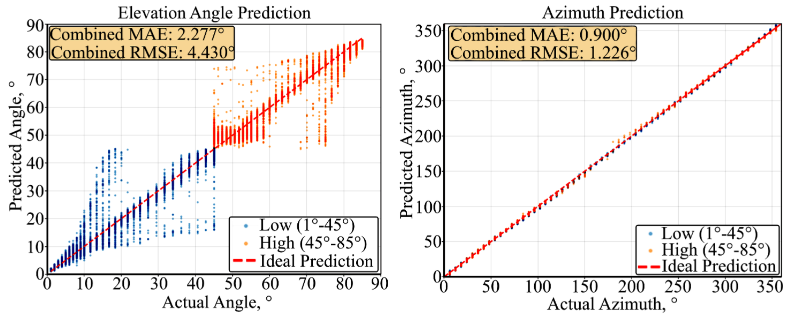

a *b*

Fig. 3. Scatter plots of predicted and actual aiming angles: *a* – elevation angle; *b* – azimuth angle

To evaluate the viability of the hybrid approach to solving the inverse ballistic problem, the fire control algorithm was tested under modelling conditions. The test scenario was configured for the 2S22 "Bohdana" 155 mm artillery system with a Standard-HE projectile under the following conditions: range to target 20 km, lateral deviation of target +2 km, relative height of target +100 m. The meteorological parameters corresponded to the standard ISA atmospheric conditions with activated Coriolis and Magnus effects, and taking into account the logarithmic wind profile.

The aiming angles calculation results for the test scenario illustrate the operational sequence of the hybrid pipeline. The neural network generated an initial approximation with an elevation angle of 24.44° and azimuth of 5.11°. After iterative refinement using the Brent method and gradient azimuth correction, the final solution was obtained: elevation angle $\theta = 25.13°$, azimuth $\varphi = 5.75°$. The total correction of the elevation angle was $\Delta_\theta = +0.69°$, and the azimuth $\Delta_\varphi = +0.64°$, which confirms the necessity of refining the neural network approximation through physics-based numerical modelling. The convergence of the iterative process was achieved in 2 iterations. Fig. 4 shows a three-dimensional visualization of the projectile flight trajectory for the calculated ballistic solution.

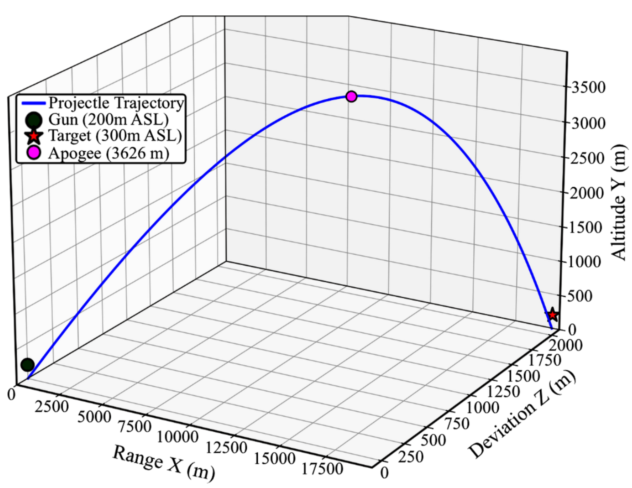

Fig. 4. Three-dimensional trajectory of the projectile

The trajectory demonstrates a characteristic parabolic shape with a maximum height (apogee) of 3626 m, reached at a horizontal distance of approximately 12.5 km from the gun position, with a flight time of approximately 51.6 s. The point of impact has coordinates $X = 19999.45$ m in firing range and $Z = 2000.39$ m in lateral deviation at a relative height of $Y = 100.00$ m, corresponding to the specified target coordinates. The resulting error is $\Delta_x = -0.55$ m in firing range, $\Delta_z = +0.39$ m in lateral deviation, and 0.68 m in the horizontal plane. The angle of fall $\theta_c = 45.40°$ and the impact velocity $V_c = 308.8$ m/s.

3.3. Uncertainty quantification results and comparative assessment

Having established the deterministic accuracy of the hybrid pipeline, the evaluation proceeds to its stochastic performance. To assess the accuracy of the ballistic solution under conditions of stochastic uncertainty, a comparative analysis of deterministic calculation and stochastic methods was performed: Polynomial Chaos Expansion (PCE) and Monte Carlo [17, 20] (Fig. 5). The integration of these methods into the fire control algorithm enables the system to provide not only point estimates of aiming angles but also quantitative assessment of projectile dispersion.

The two-dimensional distribution of hit points in the "range – lateral deviation" coordinates shows a cloud of 1024 Monte Carlo points concentrated around the target position ($X = 20000$ m, $Z = 2000$ m). The graph is overlaid with a 95% confidence ellipse constructed from the results of PCE analysis based on Gaussian-Hermite quadrature nodes [38], which covers the vast majority of Monte Carlo points. Analysis of the location of key points on the graph illustrates the sequential refinement within the hybrid pipeline. The initial prediction of the neural network (purple × mark) is at a considerable distance from the target, with coordinates approximately $X \approx 19750$ m, $Z \approx 1750$ m, which corresponds to an error of about 320 m. After iterative refinement of the aiming angles, the nominal hit (green cross) and the expected PCE hit (blue cross) practically coincide with the target position (red star). The distance between them is only 0.39 m, indicating the absence of significant systematic bias for the given test scenario.

To evaluate the hybrid artillery fire control system under stochastic conditions, a comparative analysis of three methods for calculating the coordinates of hits for the 155 mm 2S22 "Bohdana" artillery system was performed: deterministic (nominal), PCE method, and Monte Carlo method. The test scenario involved hitting a target at a firing range of 20 km with a lateral deviation of +2 km and a relative height of +100 m. The quantitative indicators of the hit results are summarized in Table 2.

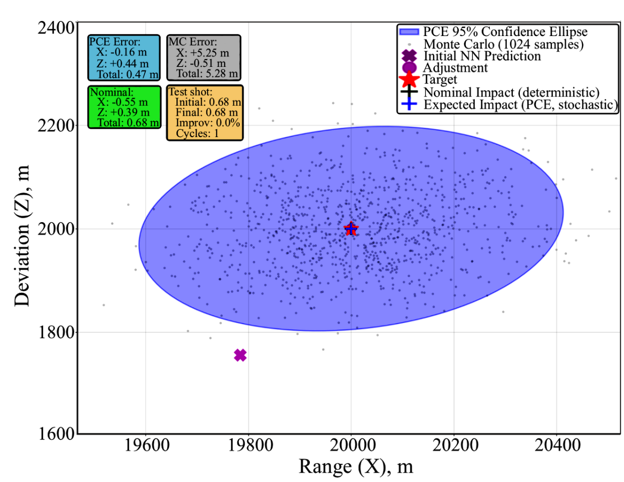

Fig. 5. Comparative analysis of deterministic calculation, PCE and Monte Carlo methods

Table 2

Comparison of the results of deterministic and stochastic methods of quantifying the uncertainty

Parameter	Designation	Nominal	PCE	Monte Carlo	Unit of measurement
Number of simulations	N	1	1024	1024	–
Hit coordinate (X)	$E[X]$	19999.45	19999.84	20005.25	m
Impact coordinate (Z)	$E[Z]$	2000.39	2000.44	1999.49	m
Error in firing range	Δ_x	−0.55	−0.16	+5.25	m
Lateral deviation error	Δ_z	+0.39	+0.44	−0.51	m
Total accuracy error	ε_{total}	0.68	0.47	5.28	m

The comparison results (Table 2) illustrate the performance of three approaches to evaluating the accuracy of the ballistic solution. The deterministic calculation provides a total accuracy error of 0.68 m when performing only one simulation in 22 s, which demonstrates the accuracy of the hybrid pipeline for calculating aiming angles under the modelling conditions employed.

The PCE method demonstrates the best accuracy with an error of 0.47 m, which is 31% less than the deterministic result and 11 times more accurate than Monte Carlo (5.28 m) with the same number of simulations – 1024. The PCE advantage is explained by the ordered arrangement of Gaussian-Hermite quadrature nodes in the uncertainty space [38, 39], while Monte Carlo random points are distributed less efficiently. The execution time of stochastic methods is comparable: 38 s for PCE and 35 s for Monte Carlo.

The consistency of the results of the deterministic approach and PCE supports the use of nominal aiming angles for the tested scenario. Beyond point accuracy, the PCE method provides analytical estimates of statistical moments that characterize projectile dispersion around the mean point of impact.

To quantify the expected projectile dispersion in standardized form, the Circular Error Probable (*CEP*) metric was calculated from the standard deviations obtained through PCE analysis (σ_x = 168.85 m for range and σ_z = 80.84 m for lateral deviation). For a two-dimensional normal distribution with the observed anisotropy ratio $\sigma_x/\sigma_z \approx 2.09$, the *CEP*50 is determined using the linear approximation method for elliptical Gaussian distributions [12]

$$CEP50 = 0.59 \cdot (\sigma_{\max} + \sigma_{\min}). \quad (8)$$

The calculated *CEP50* = 147.3 m represents the radius of a circle centered at the mean point of impact that contains 50% of all projectile impacts under the modeled atmospheric uncertainty conditions at a firing range of 20 km. The pronounced anisotropy, with range dispersion exceeding lateral dispersion by a factor of approximately two, reflects the dominant influence of initial velocity uncertainty on the longitudinal component of the trajectory, which is AFCS characteristic.

Interpretation of results. The results obtained support the viability of the proposed hybrid pipeline as an integrated approach to solving the inverse ballistic problem under modelling conditions. The key factor is the rational distribution of functions between the system components: the neural network provides a rapid initial approximation, and the ballistic simulator performs physics-based iterative refinement of the aiming angles – a concept consistent with the trajectory propagator approach proposed in work [9].

Within this pipeline, the neural network predictor serves solely as an initial approximation generator: its MAE of 2.027–2.563° for the elevation angle is insufficient for standalone fire control application, yet adequate for narrowing the search space from the full 1–85° range to a corridor of ±3–5°, ensuring convergence of the Brent refinement algorithm in 2–3 iterations [13]. The subsequent refinement stage yields a deterministic accuracy error of 0.68 m at 20 km range, consistent with the physical model implementation in accordance with NATO STANAG 4355 [8].

The integrated uncertainty quantification stage demonstrates that PCE achieves a mean impact point estimation error of 0.47 m, outperforming Monte Carlo (5.28 m) by a factor of 11.2 with the same 1024 simulations – an advantage attributed to the systematic arrangement of Gaussian-Hermite quadrature nodes [16]. The detected dispersion anisotropy (σ_x = 168.85 m, σ_z = 80.84 m) reflects the dominant influence of initial projectile velocity uncertainty on the longitudinal trajectory component, consistent with findings reported in [18].

These results are bounded by the verification conditions employed: a synthetic dataset, a simplified 4-DOF model, and a single test scenario for the 2S22 "Bohdana" at 20 km range. Within these conditions, the novelty of the work lies in the methodological integration of established techniques – neural network approximation, physics-based iterative refinement, and uncertainty quantification – into a single operational pipeline that simultaneously yields deterministic aiming angles and probabilistic dispersion estimates. The practical significance of this demonstration is accordingly limited to confirming feasibility under modelling conditions. The PCE-derived covariance matrix and *CEP*50 estimate represent outputs, which practical utility – including applicability to fire damage planning tasks – remains to be established through validation with real firing data [12, 24].

Comparison with existing results. Given that the present system was validated exclusively on synthetic data under a single modelling scenario, comparison with existing studies should be interpreted as contextual rather than conclusive, as the referenced works employ different verification bases, projectile types, and operational conditions.

Among ballistic solution calculation systems, the closest conceptual analogue is work [42], which proposes surrogate-model-based elevation angle calculation for MLRS with iterative 6-DOF refinement – an architecture structurally consistent with the proposed hybrid pipeline. Study [43] reported errors of 0.6 m in firing range using the IHHO-ELM-AdaBoost algorithm, which is of comparable order to the deterministic result obtained here, though a direct comparison is not warranted given differences in artillery type and validation methodology. The validity of using a simplified 4-DOF model for fire control tasks is supported by studies [21, 22]. For reference, trajectory navigation [15] and counter-battery systems [14] address different operational problems but indicate a general neural network accuracy level of 7–10 m in ballistic applications, confirming that standalone neural network approaches require physical refinement to achieve sub-meter accuracy.

The obtained *CEP*50 = 147.3 m at 20 km reflects dispersion from atmospheric uncertainties only, under synthetic modelling conditions, and is not directly comparable to experimentally validated figures. It is of the same order of magnitude as values reported in study [19], where unguided rocket projectiles at shorter ranges (~9 km) exhibited CEP values of 90–230 m depending on the uncertainty scenario; differences are attributable to range, projectile type, and uncertainty sources. The structural design of the developed system is consistent with NATO interoperability standards [23] and incorporates specifics relevant to Ukrainian artillery units [24, 25], while the integration of neural network methods with physics-based ballistic modelling aligns with current AFCS development trends identified in [27, 28].

3.4. Limitations and directions of research development

Limitations of the research results. The following limitations define the scope of the results obtained under modelling conditions and constrain their generalization to operational use:

1. Use of a synthetic dataset generated by numerical simulation, which provides a controlled environment for validation but does not account for the real stochastic factors of the combat environment.

2. Implementation of a simplified 4-DOF model that only takes into account the axial rotation of the projectile, assuming constant collinearity of the rotation vector with the instantaneous velocity vector, which is a simplification compared to full 6-DOF models [21].

3. The assumption of stationary meteorological conditions during the flight of the projectile is a simplification, since the wind profile can change at altitudes of 3–4 km along the trajectory of the projectiles.

4. The absence of modeling of barrel wear, which leads to a decrease in initial velocity by 0.5–1.5% after 500–1000 shots.

5. Failure to take into account the temperature regime of the barrel load and the deformation of the gun carriage during intensive firing.

6. Validation of the system only for 155 mm caliber artillery systems, which limits the generalization of results to other calibers without additional verification.

Impact of martial law conditions. Martial law conditions have exacerbated the need for modern AFCS and confirmed the importance of the reaction speed of artillery units to avoid counter-battery fire [3, 28]. At the same time, these conditions have restricted access to real firing data, which has precluded practical validation of the system beyond the modelling environment. A synthetic dataset based on meteorological observations and open-source information about artillery system characteristics was therefore used for training and initial verification. Validation against combat firing data remains a necessary step toward assessing the operational applicability of the hybrid pipeline.

Possible directions for further research. The primary and most consequential direction is the validation of the hybrid pipeline using real firing data, which would enable an assessment of its practical applicability beyond the modelling conditions employed in this research and allow for targeted adjustment of model parameters. Expanding the ballistic simulator to a full 6-DOF model, accounting for projectile nutation and precession [21], may improve prediction accuracy for high-precision ammunition and strengthen the physics-based refinement stage of the pipeline. A further promising direction is the incorporation of physically informed neural networks (PINN), which embed ballistic equations directly into the loss function [44], potentially improving the quality of the initial approximation and reducing the number of refinement iterations required. The development of an adaptive correction mechanism based on observed firing results [27] could allow the pipeline to compensate for systematic errors arising from factors not captured in the current model. Finally, alternative approximation methods – including surrogate models [42] and ensemble machine learning approaches [43] – warrant investigation as potential replacements or complements to the neural network component, with the aim of improving robustness across a wider range of artillery configurations and calibers.

4. Conclusions

1. A synthetic ballistic dataset of over 121000 records has been formed for 24 unique configurations of 12 artillery systems across two shell categories (Standard-HE and ERFB-BB). The application of the domain randomization method with random selection of meteorological profiles from real archived data for 24 regions of Ukraine ensured a statistically representative distribution of atmospheric conditions across the full range of input parameters. The formed dataset provides a controlled environment for training and initial validation of ballistic prediction models.

2. A hybrid aiming angles calculation pipeline has been implemented, combining a feedforward neural network as an initial approximation generator with physics-based iterative refinement through a two-stage numerical optimization process. The neural network, trained separately for low (1–45°) and high (45–85°) firing modes, narrows the search space to a corridor of ±3–5°, enabling rapid convergence of the iterative refinement algorithm. The iterative algorithm applies sequential elevation angle refinement via the Brent method followed by gradient-based azimuth correction, processing both firing modes in parallel and selecting the optimal solution. Testing on the 2S22 "Bohdana" scenario at 20 km range demonstrated convergence in 2 refinement cycles, achieving a final error of 0.68 m in the horizontal plane. The viability of the integrated pipeline has been demonstrated under modelling conditions, potential integration into AFCS remains subject to validation with real firing data.

3. Two uncertainty quantification methods have been integrated into the hybrid system: a 2nd-order PCE with 1024 Gauss-Hermite quadrature nodes and the Monte Carlo method with 1024 realizations, both modeling parametric uncertainty from five independent sources. Verification on the 155 mm 2S22 "Bohdana" test scenario at 20 km range demonstrated that PCE achieved a mean impact point estimation error of 0.47 m, outperforming Monte Carlo (5.28 m) by a factor of 11.2. This improvement is attributed to a systematic quadrature node arrangement providing optimal probability density coverage. The deterministic solution error was 0.68 m, and the PCE analysis yielded projectile dispersion estimates of $\sigma_x = 168.85$ m, $\sigma_z = 80.84$ m, and $CEP50 = 147.3$ m. The results demonstrate the viability of the proposed hybrid approach under modelling conditions using synthetic data. The derived covariance matrix and *CEP*50 estimate demonstrate that the pipeline produces quantitative dispersion outputs compatible with standardized AFCS metrics, though their operational validity remains subject to verification with real firing data.

Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship, or other, that could influence the research and its results presented in this article.

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The research was performed without financial support.

Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

During the preparation of this manuscript, the authors utilized DeepL and Grammarly for grammar verification and spelling correction. Claude (Sonnet 4.5) was employed to assist with the analysis, summarization, and formatting of selected references. Following the use of these tools, the authors thoroughly reviewed and edited all content as necessary and assume full responsibility for the final publication.

Authors' contributions

Yurii Rubel: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Writing – review and editing; ***Yurii Hrytsiuk***: Methodology, Resources, Supervision, Project administration, Writing – review and editing.

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