UDC 519.23
DOI: 10.15587/2312-8372.2016.75572

Тупко О. С.,
Тупко Н. П.,
Васильєва Н. С.

ПОРІВНЯЛЬНИЙ АНАЛІЗ МЕТОДІВ ФОРМУВАННЯ ПОРТФЕЛЯ ЦІННИХ ПАПЕРІВ НА БАЗІ КОПУЛ ТА МАРКОВІЦА

У статті виконано порівняльний аналіз методів формування портфелю цінних паперів: класичного методу Марковіца та методу на базі копул. Продемонстровано, що підхід на базі копул дозволяє уникнути некоректних припущень класичного методу і більш гнучко описувати залежність між випадковими величинами. Результати апробовані на даних Нью-Йоркської фондової біржі (NYSE).

Ключові слова: *портфель цінних паперів, фінансовий ризик, копула, ієрархічна копула, архімедова копула.*

1. Introduction

Nowadays, globalization is the leading trend in the world. Among other things, it takes place in a financial market. Today, people have the opportunity to invest in the financial markets of any country that are closely related. This raises the problem of investments management so as to minimize their risks and maximize returns. According to Henry Markowitz theory, Modern Portfolio Theory (MPT), in this case, it makes sense to diversify assets. Diversification is the process of reducing risk through capital allocation among various investment objects. In terms of portfolio management, it means construction of the diversified portfolio. Therefore, since the mid-1990s diversified portfolio optimization problems play a significant role in an investing. The financial crisis of 2007–2008 has demonstrated that classic portfolio optimization techniques are not sufficient anymore. Therefore there is a need to study alternative and correct investment methods, in particular, those ones, that are on copula approach based.

2. The object of research and its technical audit

The object of research — securities (stocks), investment portfolio.

In finance, an investment is a monetary asset purchased with the idea that the asset will provide income in the future or appreciate and be sold at a higher price [1].

An investor could be either an organization (insurance company, pension fund, corporation, charitable organization, etc.) or individual (both directly and through an investment company).

There are a variety of securities' types. One of them is a stock, which will be used to construct investment portfolios in this work. A stock is a type of security that signifies ownership in a corporation and represents a claim on a part of the corporation's assets and earnings [2].

One of the most basic and important concepts of investing is a diversification. The concept of diversification does not guarantee high returns, but it reduces risk or volatility by investing in a variety of assets. Therefore it is more reasonable to invest in optimized diversified portfolio.

Despite the first approaches to portfolio optimization were formed in the 50's of last century, but they were widely used just since the mid-1990s. However, the financial crisis of 2007–2008 has shown that previous methods of optimization portfolio (including the classic Markowitz methods) are too optimistic and not sufficient anymore. Consequently there is a need to study alternative and correct methods. Specifically, this paper studied the method based on copula functions.

From a financial point of view, current investment is the accumulation of assets in the form of securities (stocks, bonds, options, etc.) in order to obtain funds in the future.

As an investor can serve organizations (insurance companies, pension funds, corporations, charitable organizations, etc.) and individuals (both directly and through an Investment Company).

3. The purpose and the problems of research

The purpose of research — copula application for portfolio construction analysis, namely, an effectiveness of the copula-based portfolio optimization examination in compared with the classical method.

The problems of research:

— mean-variance optimization of an investment portfolio;

— restoration of stock returns distribution using copula functions;

— assessment of portfolio risk, Value-at-Risk (VaR), by the restored distribution;

— portfolio optimization based on copula functions using VaR.

4. Literature review

The first concepts of portfolio management, the main purpose of which is an optimal portfolio construction, was presented by Henry Markowitz in his paper «Portfolio

Selection», 1952 [3]. Henry Markowitz Modern Portfolio Theory (MPT) explains how risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk (or minimize market risk based on a given level of expected return). Specifically, it gives formulation of an optimization problem, where an expected return is used as a target function, a portfolio return variance is used as a constraint. In case of the minimization problem, portfolio return variance is used as a target function, an expected return is used as a constraint. According to the theory, it's possible to construct an «efficient frontier» of optimal portfolios (solution of an optimization problem) offering the maximum possible expected return for a given level of risk.

Apart from some MPT concepts, such as diversification and optimization problem formulation, which are classical in modern investing, it assumes a normal distribution of portfolio stock returns and a variance as a portfolio risk measures.

These assumptions are simple to understand and easy to implement, but they do not represent the real processes in the modern volatile economy. Their inadequacy became apparent after the global financial crisis of 2007–2008. The fact is that in terms of volatile financial markets probability of extreme events is rather high, that means stock returns distributions have heavy tails, which is not typical for normal ones. Therefore, the assumption of normality of the distribution cannot be used anymore.

Correlation between random variables (stock returns) is more complex than linear as well. The occurrence of extreme events for one stock causes extreme events for another one with high probability, which means tail dependence. This dependence cannot be described in full with the Pearson correlation coefficient, which is closely related to variance. That is to say, variance as a measure of risk does not take into account the probability of extreme events, such as a simultaneous increase or decrease of stock returns.

Such argumentation was partly published in [4], but there are a number of other works, that also criticize MPT assumptions. Hence, for correct portfolio optimization, the assumption about the normality of portfolio stock returns distribution must be rejected and a correct measure of risk must be chosen.

The risk can be calculated in several ways. In early 1990s VaR was presented [5] as a measure of risk and since then it is widely used. VaR is also a recommended approach to market risk assessment by the Basle Committee on Banking Supervision [6]. VaR satisfies properties of monotonicity, uniformity, and translational invariance. Unlike variance, VaR takes into account the probability of extreme events. However, it was demonstrated [7] that VaR does not satisfy the condition subadditivity, and therefore is not coherent risk measure. However, VaR is widely spread as a risk measure and will be used in this paper.

VaR assessment is based on a random variable distribution (in our case a random vector is a vector of portfolio stock returns). Since the assumption of the normal distribution of stock returns is incorrect, to calculate VaR of the portfolio the true distribution must be defined. In the general case, the problem of the distribution determination is not a trivial problem. One of such methods is that based on copulas. According to [8] copula is a multivariate distribution function supported in $[0,1]^n$. The main advantage of copula is that one can separate the marginal of a multivariate distribution from their dependence structure. Thus, it is possible to model the marginal separately and choose copula to represent the dependence structure between them. The concept of copula is based on Shklar theorem [9].

In [10–14] they can conclude that since 2007–2008 elliptical copulas become popular for the purpose of market risk assessment. However, Gaussian copula ignores tail dependence. T-copula is the best among the elliptical ones and Archimedes copulas assess risk more accurately than the elliptical.

Let us consider one-parameter Archimedean copulas, whose advantages are simplicity of an assessment and simulation of multivariate distributions with tail dependency (copula parameter is explicitly defined by Kendall rank correlation coefficient). The only one parameter provides on the one hand easy interpretation and on the other the same amount of dependence between random variables.

The one-parameter problem could be successfully solved through hierarchical copulas. With hierarchical Archimedean copulas, it is possible to describe dependence structures more flexible while keeping simplicity of understanding.

To use hierarchical Archimedes copula its structure must be first determined. In the most of corresponding papers copula structure is defined by the author [15]. In contrast to these ones, in [16, 17] a formal approach to determining the structure was firstly suggested. The approach is based on complete monotonicy. To be more concrete, on the results of [18, 19], according to which copula parameter at each hierarchical level must be less than at the previous one in case of Clayton, Gumbel and Frank hierarchical copulas. This means that dependence (in terms of rank correlation) must decrease with hierarchical level increasing.

To sum up described above, firstly, the classical methods of portfolio optimization are too optimistic and therefore inadequate in today's volatile economy. Secondly, the copula-based method (hierarchical Archimedean copulas) is an alternative to traditional methods.

There are modern papers, where non-hierarchical copulas for portfolio optimization are studied. For example, author researches the effectiveness of t-copula in comparison with Markowitz method for portfolio optimization [8]. There are also papers in which copula-functions are studied with the purpose of portfolio risk estimation [4].

The novelty of this work is the application of hierarchical Archimedean copulas to portfolio optimization and comparison of Markowitz and copula-based portfolios returns.

5. Materials and methods of research

In this paper, the comparison analysis of classical portfolio optimization (Markowitz method) and copula-based optimization methods was made.

The last one is based on MPT concepts: portfolio diversification, optimization problem in general and expected return as a constraint. But unlike Markowitz method the assumptions of normal distribution of returns and a variance as a measure of portfolio risk were declined. In contrast to them, restoration of portfolio return distribution with copulas is made, VaR is chosen as a measure of risk, portfolio optimization problem is transformed into stochastic one and is solved by the relevant methods.

Let us consider Markowitz method first. The mathematical problem is the following.

Let:

d — stock number in a portfolio;

μ_i — expected value of i^{th} stock in a portfolio;

σ_{ij} — covariance between i^{th} and j^{th} stock in a portfolio;

R — minimum expected portfolio return (defines by an investor);

ω_i — weight of i^{th} stock in a portfolio.

Than according to mean-variance model (Markowitz model) the portfolio optimization problem is following:

$$\begin{cases} \sum_{i=1}^{d}\sum_{j=1}^{d}\omega_i\omega_j\sigma_{ij} \to \min\limits_{\omega}, \\ \sum_{i=1}^{d}\omega_i\mu_i \ge R, \\ \sum_{i=1}^{d}\omega_i, \\ \omega_i \ge 0, \forall i = 1,\ldots,d. \end{cases} \quad (1)$$

The mathematical expectation, which is bounded below:

$$E\left[\omega^B R\right] = \omega^B\mu = \omega_1\mu_1 + \omega_2\mu_2 + \ldots + \omega_d\mu_d = \sum_{i=1}^{d}\omega_i\mu_i, \quad (2)$$

where $\left(R^1R^2\ldots R^d\right)^B$ — random vector of stock returns; ω^B — vector of stocks' weights in the portfolio.

This problem is convex quadratic optimization problem and can be solved with appropriate deterministic optimization algorithms.

Consider copula-based portfolio method. As it was written above, VaR is accepted as a measure of risk, and consequently as a target function in the method. VaR is a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over some investment horizon [8]:

$$\mathrm{VaR}_\alpha = \inf\left\{r \in \mathbb{R} \,|\, F_{Returns}(r) \le 1-\alpha\right\}, \quad (3)$$

where $F_{Returns}$ — portfolio return distribution function; α — confidence level; r — portfolio return value.

With VaR_α as a target function, Markowitz portfolio optimization problem transforms into the following stochastic optimization problem:

$$\begin{cases} \mathrm{VaR}_\alpha(\omega) \to \min\limits_{\omega}, \\ \sum_{i=1}^{d}\omega_i\mu_i \ge R, \\ \sum_{i=1}^{d}\omega_i, \\ \omega_i \ge 0, \forall i = 1,\ldots,d. \end{cases} \quad (4)$$

To calculate VaR portfolio return, distribution function needs to be defined. If random variables are independent, joint distribution function equals a product of marginal distribution functions. But generally, joint distribution function assessment problem can be solved with copula-functions. In this paper, a hierarchical Archimedean copula was considered.

In order to define a structure of the copula, R package HAC was used [20]. In [20] the algorithm, which was suggested in [16, 17], was implemented. As it was mentioned in the previous section, at each hierarchical level random variables with the highest dependence in terms of Kendall's rank correlation are grouped.

The algorithm consists of the following steps. In the first stage, parameters of pair-copulas given family (in our case Gumbel) for all possible combinations of two variables were calculated. Then those variables with the highest level of the parameters are grouped. Grouping more than two variables is possible only if the difference between the estimated parameters is less than a given value (in the paper, this value was 0,15). Grouped random variables are considered as a whole, and the algorithm is repeated until all variables will not be aggregated.

Copula parameters evaluation at every hierarchical level is implemented through the semiparametric method.

When copula-function is finally defined, it is possible to calculate the VaR and to solve the stochastic optimization problem. To this purpose modified Nelder-Mead method or NMSS (Nelder-Mead Simplex Search) [21] was used, namely its implementation in R package neldermead [22]. NMSS is a direct search method for unconstrainted deterministic optimization problems. However, it is widely used for stochastic optimization problems.

Since the method the method finds only local extrema, it was decided to run it 10 times with different initial points and choose the best solution. NMSS is used for unconstrainted optimization problem, constrained optimization problem (4) was transformed into unconstrainted one (5).

$$\begin{aligned} &\mathrm{VaR}_\alpha(\omega) - \sum_{j=1}^{d+1}\lambda_j \min\left(0, g_j(\omega) - c_j(\omega)\right) \to \min_{\omega}, \\ &\lambda_j \ge 0, j = 1,\ldots,d+1. \end{aligned} \quad (5)$$

In order to diversify portfolio, an input sample was represented by stocks of ten different companies from four different sectors of economic activity, which are trading on New York Stock Exchange (NYSE).

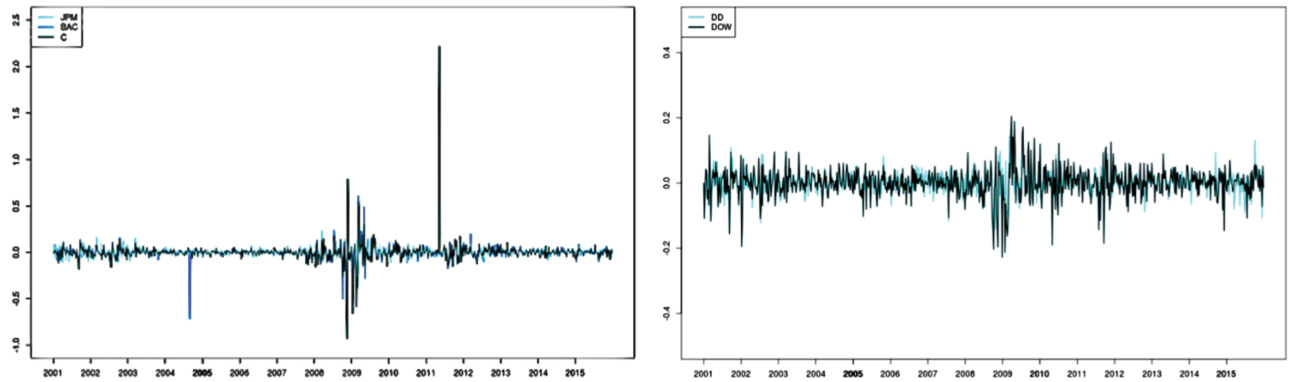
6. Research results

In the research an input sample was represented by weekly stocks prices of ten different companies from four different sectors of economics in period from 2001–2015, that are trading on New York Stock Exchange (NYSE). Data source is Yahoo Finance.

The following stocks were considered:

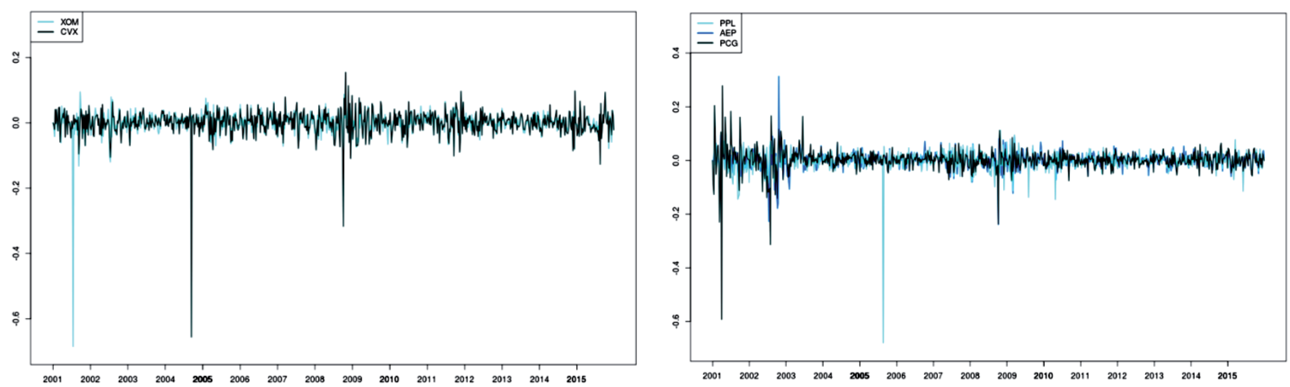
- finance — JPMorgan Chase & Co. (JPM), Citigroup Inc (C), Bank of America Corp (BAC);
- chemical industry — E. I. Du Pont De Nemours & Co. (DD), Dow Chemical Company (DOW);
- oil industry — Exxon Mobil Corporation (XOM), Chevron Corporation (CVX);
- electroenergetics — American Electric Power Company, Inc (AEP), Ppl Corporation (PPL), PG & E Company (PCG).

In this research it is more reasonable to work with arithmetic stock returns (Fig. 1, 2). As it was expected, the sudden changes in the dynamics of arithmetic stock returns were observed in the period from late 2007 to late 2009, which is justified because of the subprime mortgage crisis in 2007 and the great recession in 2008.



a *b*

Fig. 1. Finance (*a*) and chemical industry (*b*)



a *b*

Fig. 2. Oil industry (*a*) and electroenergetics (*b*)

Taking into consideration the dynamics of arithmetic stock returns, a hypothesis about a structural change of returns distributions was set up. Therefore Kolmogorov-Smirnov test of the equality of probability distributions for two subsamples: from 2001 to 2009 and from 2010 to 2015 with confidence level was $\alpha = 0{,}05$ was made and a Kolmogorov-Smirnov statistics (D) were obtained (Table 1).

As a result, null hypothesis about equity of distributions for JPM, DD, PPL stocks was declined at confidence level $\alpha = 0{,}05$.

Table 1

Kolmogorov-Smirnov test results

Stock	D
JPM	0,0995
C	0,0764
BAC	0,0827
DD	0,1047
DOW	0,0633
XOM	0,0833
CVX	0,0491
AEP	0,0971
PPL	0,1133
PCG	0,0953

Then next the sample from 2010 to 2015.

In the next step, let us consider the dynamics of dependency between shares inside each of the four sectors and between the sectors. To do this, let us build graphics of rank correlation coefficient τ-Kendall at time horizon of 2001–2015 years with sliding window method Fig. 3, 4. Considering that the hypothesis of equality of distributions at the time of the 2001–2009 and 2010–2015 years (from 2010/01/04 to 2015/12/28, a total 313 weeks) was rejected, a 313 week window (6 years) was used to build τ-Kendall.

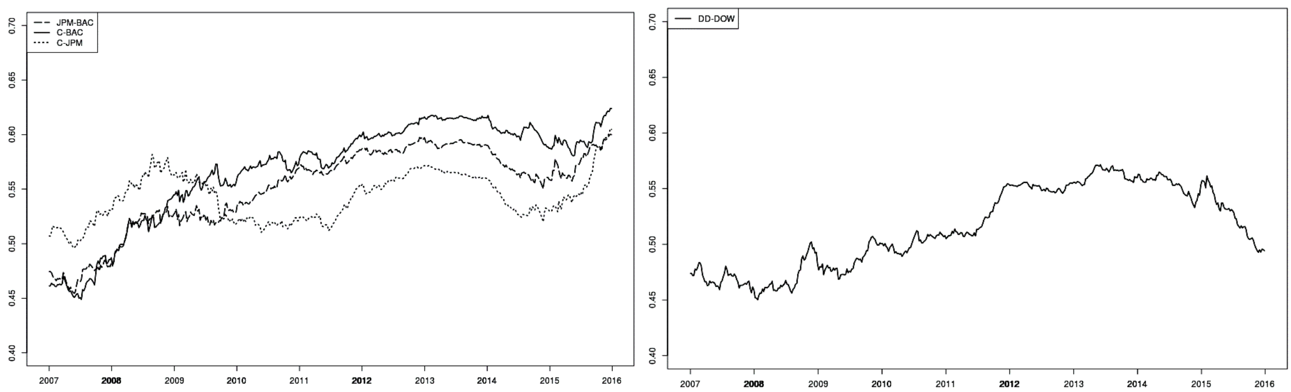
From the graphics (Fig. 3, 4) can be drawn the conclusion, that dependence degree inside each of the sectors changed with time (mostly increased) and stayed relatively high (from 0,40 to 0,64) for all the period. Next let us consider dependence between different sectors Fig. 5–7.

Dependence between sectors was also changing with time, τ-Kendall was mostly increasing, except dependence between chemical industry and electroenergetics, oil industry and electroenergetics. Generally speaking, despite increase of dependence between sectors, dependence inside them is lot higher.

So there are the following results:

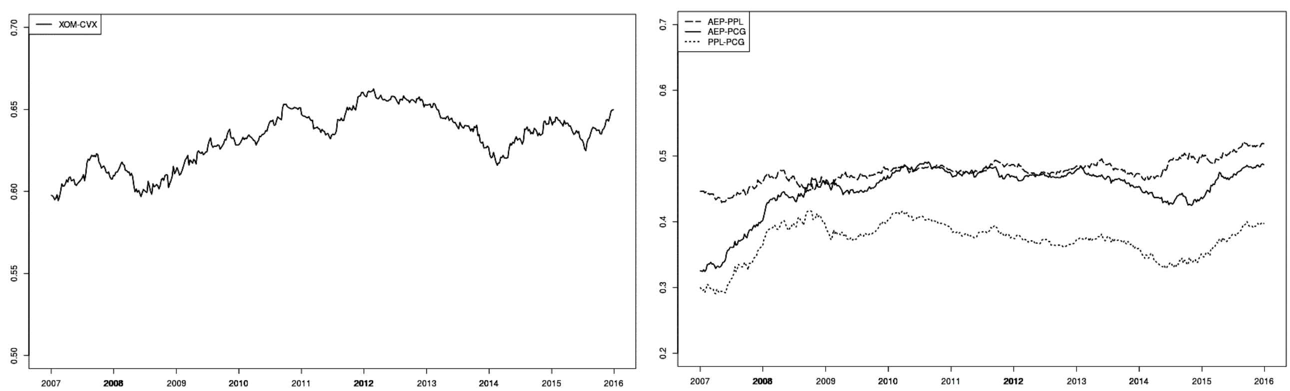
— There is a structural shift of some stocks distribution on the sample 2001–2015. Thus the distribution definition will be made, based on the 2010–2014 years sample.

— Stock returns are dependent random variables according to dynamics of rank correlation coefficient, value of which is not less than 0,15. Therefore, the use of independent copula is not applicable. Apart from that the height of dependence inside and between sectors differs, so the usage of hierarchical copulas is quite reasonable.



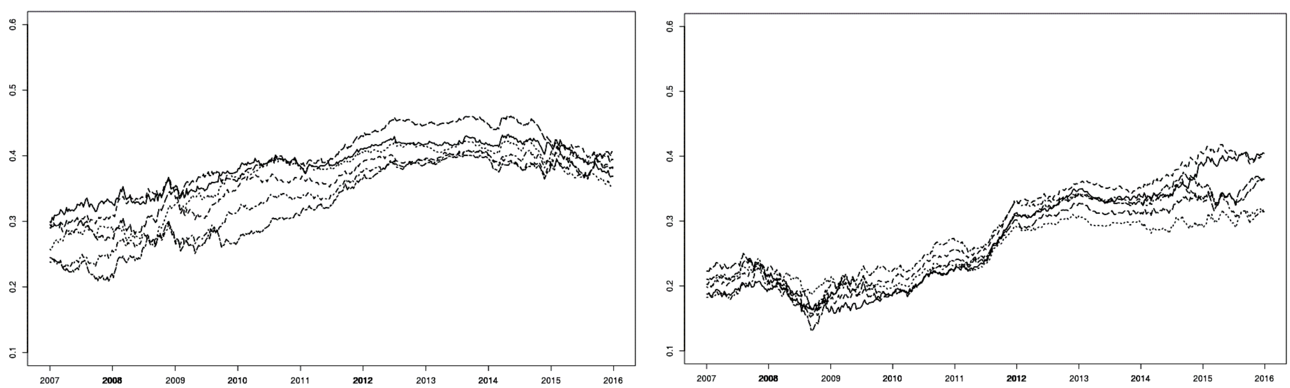
a *b*

Fig. 3. Finance (*a*) and chemical industry (*b*)



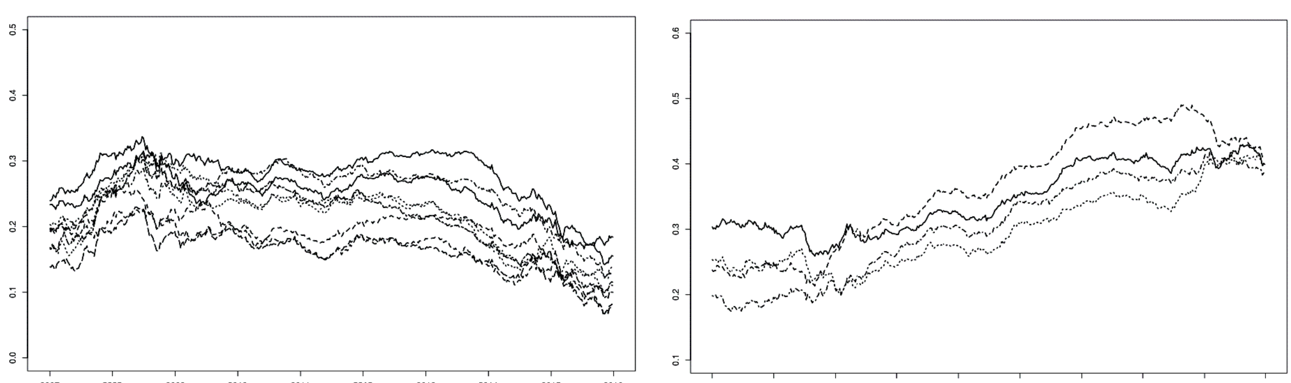
a *b*

Fig. 4. Oil industry (*a*) and electroenergetics (*b*)



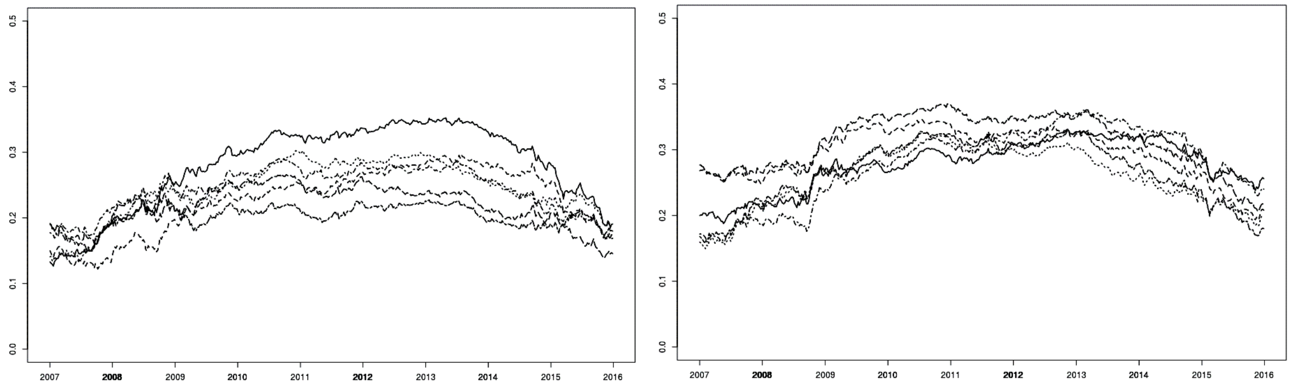
a *b*

Fig. 5. Finance and chemical sector (*a*); finance and oil industry (*b*)



a *b*

Fig. 6. Electroenergetics and finance (*a*); chemical and oil industries (*b*)



a *b*

Fig. 7. Electroenergetics and chemical industry (*a*); electroenergetics and oil industry (*b*)

In the next stage, copula will be constructed to restore portfolio stock returns distribution.

The the sample of 2009–2015 should first be divided into two sub-samples: training (2009–2014 years) and testing (2015 year).

The structure and copula parameters determination was based on [17, 18] and the relevant implementation: R package HAC [21].

As a result, hierarchical copula, the dependence on each level of which was decreasing, was built (Fig. 8).

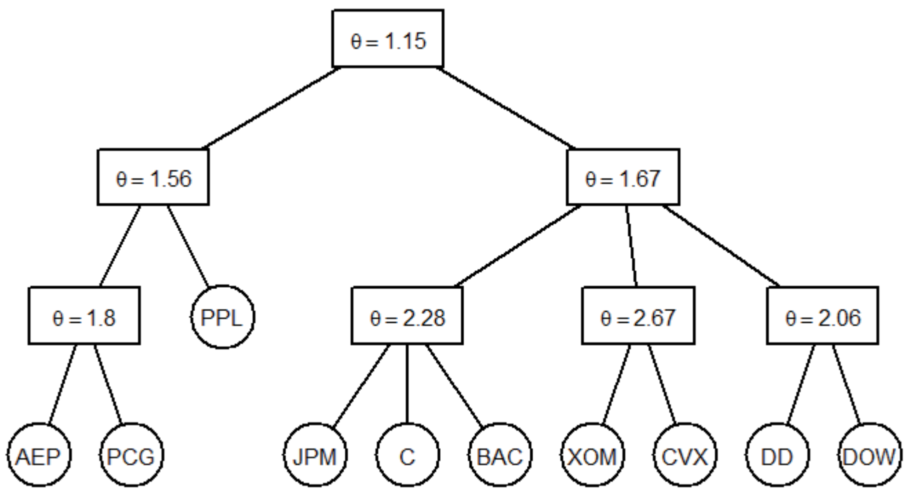


Fig. 8. Structure of the hierarchical copula

The greatest homogeneity (in terms of degree of dependence) showed stocks of companies that belong to the same sector, as it is evidenced by Fig. 3, 4. Therefore, the lower level of hierarchy is represented by four copulas, each of which corresponds to its sector (except electroenergetics which aggregates two of the three companies: AEP, PCG, on the lower level).

At the next level of the hierarchy was expected to see either an aggregation of all four sectors by a copula, or combining chemical or oil industries, according to their vertical dependence. However, in addition to the expected aggregation of chemical and oil industries, the highest degree of dependence has demonstrated the finance, chemical and oil industries. At the second hierarchical level electroenergetics was fully merged. At the third and final level copulas of the other deeper levels was aggregated with the single one.

With the built copula, which actually determines the joint distribution function of stock returns $\left(R^1 R^2 \ldots R^d\right)^B$, it is possible to estimate VaR of portfolio.

VaR estimation is performed in two stages. First, simulation of random sample based on estimated distribution function (copula), sample range was chosen $n = 1000$, and evaluation of portfolio return for each of n vectors $\omega_1 R^1 + \omega_2 R^2 + \ldots + \omega_d R^d$. Secondly, portfolio VaR_α evaluation with a given confidence level α (in this paper $\alpha = 0{,}99$) based on n simulated portfolio return values (R function *quantile*).

So, copula-approach provides risk estimation just with simulation method.

To optimize portfolio based on copulas, Nelder-Mead method (NMSS) was used (R package Nelder-Mead [22]). NMSS is used to solve stochastic optimization problems, at each step of which objective function VaR is evaluated with simulation method [23].

Since VaR_α estimates risk counting on one week, to test on one-year sample (52 weeks) portfolio was constructed dynamically:

– Stochastic optimization problem is solved on test sample 2009–2014. As a result, optimal weights of stocks $\omega^1 = \left(\omega_1^1\ \omega_2^1 \ldots \omega_d^1\right)$ counting on one week ahead are found.

– At the next step one more value of stock returns of the first week 2015 is added to the test sample. Based on the new test sample, optimization problem is saved in the same way and optimal weights of portfolio $\omega^2 = \left(\omega_1^2\ \omega_2^2 \ldots \omega_d^2\right)$ counting on one week ahead are found the algorithm goes on until the end of 2015.

As a result of dynamic portfolio optimization, matrix (10×52) with optimal weights for each of 52 weeks 2015 as elements was got. Let us visualize portfolio return-time (year 2015) diagram (Fig. 9).

Total arithmetic portfolio return equals 12 % of entirely invested sum during 2015.

Much as the previous one, Markowitz portfolio was being constructed dynamically with R package for convex quadratic optimization *quadprog*. Markowitz portfolio returns are visualized below (Fig. 10).

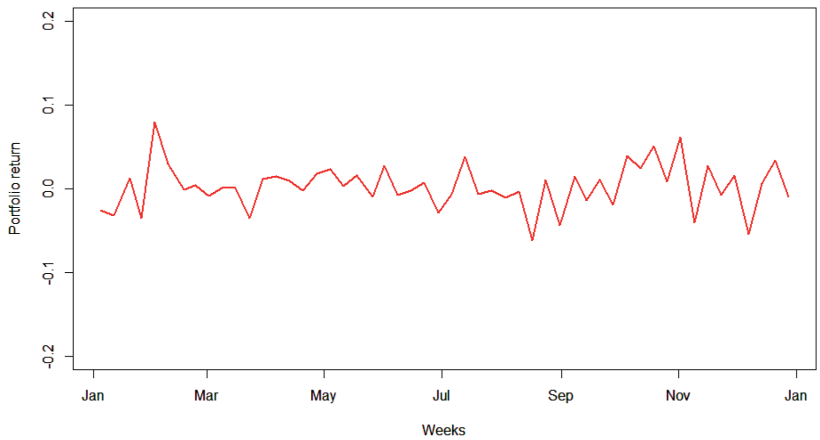


Fig. 9. Copula-based portfolio return-time (year 2015) diagram

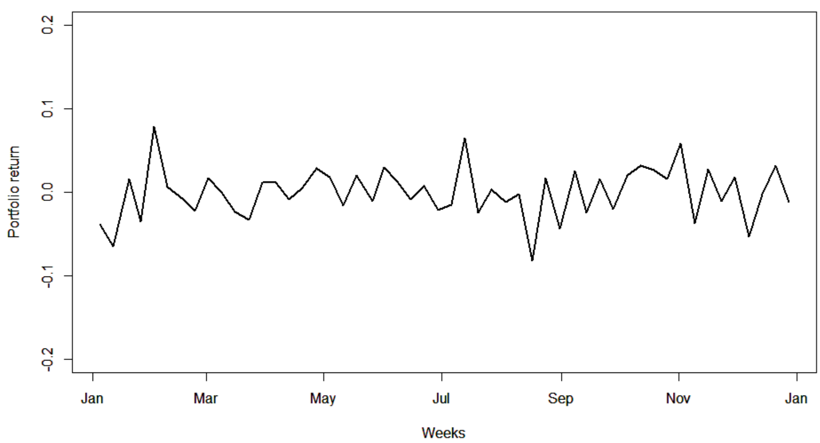


Fig. 10. Markowitz portfolio return-time (year 2015) diagram

Total arithmetic Markowitz portfolio return equals −4,1 %, which means 4,1 % loss of entirely invested sum during 2015.

In the result we can conclude that copula-based method is much more effective (12,1 %) than Markowitz one (−4,1 %).

7. SWOT-analysis of research results

The study proved the effactiveness of Archimedean copula for the purpose of portfolio construction in comparison with the classical method. It was clearly demonstrated that the copula-based portfolio provides significantly higher return than Markowitz portfolio on a one-year sample. According to this, copula-based method can be successfully applied to solve the current problems of investment management in today's volatile economy.

In this paper two optimization methods [22, 24] were used. The working time of each of them was aprx. 15–20 minutes, which is relatively high, taking into account a dynamic optimization. Thus, to implement described model it is necessary to develop new more effective optimization methods or modernization of old ones.

Since VaR as a risk measure is quite widespread, in this paper optimization with VaR as a target function was made. Later, however, it may be appropriate in addition to VaR risk measure to form optimal portfolio in terms of risk measure such as Expected Shortfall (ES).

8. Conclusions

As a result of research:

1. A dynamic optimization of Markowitz portfolio during the 2015 was made. As a result Markowitz portfolio has demonstrated losses for 4,4 % from invested sum. Markowitz approach is a classic one in modern investment and was considered as a basis for comparison with a new method.

2. Before restoration of stock returns distribution restoring, preliminary analysis of input data was made. As a result, structure shift on 2001–2015 samples was detected. Therefore copula-based distribution restoration was made on 2010–2014 samples. The dependence between stocks inside sectors and between them was also previously studied. Since stock returns appeared to be interdependent, the usage of independent copula was excluded. In addition to that degree of dependence appeared to be different between stocks that denoted used hierarchical copulas. At the next step, structure and parameters of hierarchical copula was determined with appropriate R packages.

3. Hence stock returns distribution was restored with hierarchical Archimedean copulas, it was possible to estimate portfolio risk VaR (with simulation method) and to minimize portfolio using VaR, that mean optimization problem solving.

4. Dynamic copula-based portfolio optimization during 2015 was made, at each step of which optimization problem using VaR with modified Nelder-Mead method was solved. As a result, return of copula-based portfolio was 12,1 % from invested sum.

In conclusion, a copula-based approach, which much more effective than Markowitz one, was designed.

References

1. Investment [Electronic resource] // Investopedia, LLC. — 2003. — Available at: \www/URL: http://www.investopedia.com/terms/i/investment.asp#ixzz4EhUH0JRl. — Title screen.
2. Stock [Electronic resource] // Investopedia, LLC. — 2003. — Available at: \www/URL: http://www.investopedia.com/terms/s/stock.asp#ixzz4EhZKi2Hd. — Title screen.
3. Markowitz, H. Portfolio selection [Text] / H. Markowitz // The Journal of Finance. — 1952. — Vol. 7, № 1. — P. 77–91.
4. Penikas, G. I. Ierarhicheskie kopuly v formirovanii riskov investitsionnogo portfelia [Text] / G. I. Penikas // Journal of Applied Econometrics. — 2014. — Vol. 35, № 3. — P. 18–38.
5. RiskMetrics — Technical Document [Electronic resource] / J. P. Morgan/Reuters. — Ed. 4. — New York: Morgan Guaranty Trust Company, 1996. — Available at: \www/URL: http://pascal.iseg.utl.pt/~aafonso/eif/rm/TD4ePt_2.pdf
6. International Convergence of Capital Measurement and Capital Standards. A Revised Framework [Electronic resource] / Basel Committee on Banking Supervision. — Bank for International Settlements, 2004. — Available at: \www/URL: http://www.bis.org/publ/bcbs107.pdf
7. Artzner, P. Coherent Measures of Risk [Text] / P. Artzner, F. Delbaen, J.-M. Eber, D. Heath // Mathematical Finance. — 1999. — Vol. 9, № 3. — P. 203–228. doi:10.1111/1467-9965.00068

8. Karadag, D. T. Portfolio risk calculation and stochastic portfolio optimization by a copula based approach [Electronic resource] / D. T. Karadag. — Graduate Program in Industrial Engineering, Bogazici University, 2008. — Available at: \www/URL: http://www.ie.boun.edu.tr/~hormannw/BounQuantitiveFinance/Thesis/karadag.pdf
9. Sklar, A. Fonctions de répartition à n dimensions et leurs marges [Text] / A. Sklar // Publ. Inst. Statist. Univ. Paris. — 1959. — № 8. — P. 229–231.
10. Daul, S. The Grouped t-Copula with an Application to Credit Risk [Electronic resource] / S. Daul, E. G. De Giorgi, F. Lindskog, A. McNeil // SSRN Electronic Journal. — 2003. — Available at: \www/URL: http://doi.org/10.2139/ssrn.1358956
11. Luo, X. The t copula with multiple parameters of degrees of freedom: bivariate characteristics and application to risk management [Text] / X. Luo, P. V. Shevchenko // Quantitative Finance. — 2010. — Vol. 10, № 9. — P. 1039–1054. doi:10.1080/14697680903085544
12. Hofert, M. Efficiently sampling nested Archimedean copulas [Text] / M. Hofert // Computational Statistics & Data Analysis. — 2011. — Vol. 55, № 1. — P. 57–70. doi:10.1016/j.csda.2010.04.025
13. Garcia, R. Dependence structure and extreme comovements in international equity and bond markets [Text] / R. Garcia, G. Tsafack // Journal of Banking & Finance. — 2011. — Vol. 35, № 8. — P. 1954–1970. doi:10.1016/j.jbankfin.2011.01.003
14. Patton, A. J. A review of copula models for economic time series [Text] / A. J. Patton // Journal of Multivariate Analysis. — 2012. — Vol. 110. — P. 4–18. doi:10.1016/j.jmva.2012.02.021
15. Savu, C. Hierarchical Archimedean Copulas [Electronic resource]: Discussion paper / C. Savu, M. Trede. — University of Muenster, 2006. — Available at: \www/URL: httm://www.uni-konstanz.de/micfinma/nference/Files/papers/Savu_Trede.pdf
16. Okhrin, O. Hierarchical Archimedean Copulae: The HAC Package [Text] / O. Okhrin, A. Ristig // Journal of Statistical Software. — 2014. — Vol. 58, № 4. — P. 1–20. doi:10.18637/jss.v058.i04
17. Okhrin, O. On the structure and estimation of hierarchical Archimedean copulas [Text] / O. Okhrin, Y. Okhrin, W. Schmid // Journal of Econometrics. — 2013. — Vol. 173, № 2. — P. 189–204. doi:10.1016/j.jeconom.2012.12.001
18. McNeil, A. J. Sampling nested Archimedean copulas [Text] / A. J. McNeil // Journal of Statistical Computation and Simulation. — 2008. — Vol. 78, № 6. — P. 567–581. doi:10.1080/00949650701255834
19. Nelsen, R. B. An introduction to copulas [Text] / R. B. Nelsen. — Ed. 2. — Portland: Springer, 2006. — 262 p. doi:10.1007/0-387-28678-0
20. HAC: Estimation, Simulation and Visualization of Hierarchical Archimedean Copulae (HAC) [Electronic resource] // CRAN. — 12.10.2015. — Available at: \www/URL: https://cran.r-project.org/web/packages/HAC/index.html. — Title screen.
21. Nelder — Mead method [Electronic resource] // Wikipedia. — 16 July 2016. — Available at: \www/URL: https://en.wikipedia.org/wiki/Nelder%E2%80%93Mead_method. — Title screen.
22. Neldermead: R port of the Scilab neldermead module [Electronic resource] // CRAN. — 11.01.2015. — Available at: \www/URL: https://cran.r-project.org/web/packages/neldermead/index.html. — Title screen.
23. Copula: Multivariate Dependence with Copulas [Electronic resource] // CRAN. — 25.07.2016. — Available at: \www/URL: https://cran.r-project.org/web/packages/copula/index.html. — Title screen.
24. Quadprog: Functions to solve Quadratic Programming Problems [Electronic resource] // CRAN. — 17.04.2013. — Available at: \www/URL: https://cran.r-project.org/web/packages/quadprog/index.html. — Title screen.

СРАВНИТЕЛЬНЫЙ АНАЛИЗ МЕТОДОВ ФОРМИРОВАНИЯ ПОРТФЕЛЯ ЦЕННЫХ БУМАГ НА БАЗЕ КОПУЛ И МАРКОВИЦА

В статье проведён сравнительный анализ методов формирования портфеля ценных бумаг: классического метода Марковица и метода на базе копул. Продемонстрировано, что подход на базе копул позволяет избежать некорректных предположений классического метода и более гибко описывать зависимость между случайными величинами. Результаты апробированы на данных Нью-Йоркской фондовой биржи (NYSE).

Ключевые слова: портфель ценных бумаг, финансовый риск, копула, иерархическая копула, архимедова копула.

Тупко Ольга Сергіївна, *Навчально-науковий комплекс «Інститут прикладного системного аналізу», Національний технічний університет України «Київський політехнічний інститут», Україна,* ***e-mail: olha.tupko@gmail.com****.*

Тупко Наталя Петрівна, *кандидат фізико-математичних наук, доцент, кафедра вищої і обчислювальної математики, Національний авіаційний університет, Київ, Україна,*
e-mail: natupko@rambler.ru*.*

Васильєва Наталія Семенівна, *кандидат фізико-математичних наук, доцент, кафедра вищої математики, Одеська державна академія будівництва та архітектури, Україна.*

Тупко Ольга Сергеевна, *Учебно-научный комплекс «Институт прикладного системного анализа», Национальный технический университет Украины «Киевский политехнический институт», Украина.*

Тупко Наталья Петровна, *кандидат физико-математических наук, доцент, кафедра высшей и вычислительной математики, Национальный авиационный университет, Киев, Украина.*

Васильева Наталия Семёновна, *кандидат физико-математических наук, доцент, кафедра высшей математики, Одесская академия строительства и архитектуры, Украина.*

Tupko Olha, *Educational-Scientific Complex «Institute for Applied System Analysis», National Technical University of Ukraine «Kyiv Polytechnic Institute», Kyiv, Ukraine,*
e-mail: olha.tupko@gmail.com*.*

Tupko Natalia, *National Aviation University, Kyiv, Ukraine,*
e-mail: natupko@rambler.ru*.*

Vasil'eva Nataliia, *Odessa State Academy of Civil Engineering and Architecture, Ukraine*