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INVESTIGATION OF NATURAL OSCILLATIONS OF INHOMOGENEOUS ORTHOTROPIC CIRCULAR PLATE LYING ON INHOMOGENEOUS VISCOELASTIC FOUNDATION

Розглянуто вісісиметричну форму власних коливань ортотропної неоднорідної по радіусу кругової пластинки, що лежить на неоднорідній в'язкопружній основі. Детально вивчено випадок, коли по всьому контуру пластина жорстко затиснена. Рішення завдання будувалося із застосуванням методу поділу мінливих і методу ортогоналізації Бубнов-Гальоркіна. Проведено чисельний аналіз при конкретних значеннях характерних параметрів.

Ключові слова: *пластинка, безперервність, ортотропність, щільність, основа, частота, модулі пружності, рівняння руху.*

1. Introduction

In the building of large engineering complexes, bridges and overpasses for various purposes and in many other areas the plates of widely different configurations are used [1]. These plates are made of natural and artificial orthotropic materials.

A necessity to take into consideration environmental resistance influences during operation is appeared for stability calculation and determination of the frequency amplitude characteristics. It is obvious that consideration of orthotropicity, inhomogeneity, density and variability of the environment resistance is much complicated mathematical solution. No accounting of these same parameters can lead to significant errors (especially in dynamic problems). Given that the various branches of engineering, mechanical engineering and construction widely used inhomogeneous orthotropic plates, in this paper we solve the problem of free oscillations of a circular plate taking into consideration inhomogeneous viscoelastic resistance of environment.

2. The object of research and its technological audit

The object of research is inhomogeneous circular plate lying on inhomogeneous viscoelastic foundations.

It is assumed that the moduli of elasticity and density of the plate are continuous functions of the current radius. In this case, in contrast to the homogeneous plate, the motion equation is complex differential equation with variable coefficients.

In this regard, there is need to build an approximate analytical method solutions.

Technological audit is carried out to identify the features of inhomogeneous circular plates in terms of the calculation. This audit aims to determine the values of circular frequency in the case of rigid clamping around the plate contour.

3. The aim and objectives of research

The aim of research is to study the effect of simultaneous consideration of orthotropicity of continuously inhomogeneity at the current radius and account of inhomogeneous viscoelastic resistance of the foundation.

To achieve this aim it is necessary to perform the following tasks:

1. Identify the factors that affect the value of the circular frequency.
2. Identify the significant impact of orthotropicity, plate inhomogeneity and viscoelasticity of the medium.
3. Obtain the differential equations of motion with variable coefficients.

4. Research of existing solutions of the problem

The theoretical and experimental studies have shown that as a result of a number of reasons, elasticity moduli and density of the structural elements may be substantially continuous functions of spatial coordinates [1–3]. In this work, based on the publications [4, 5], it is assumed that the density and elastic modulus are dependent on the current radius of circular orthotropic plate.

In [1] the main problems in the theory of inhomogeneous elastic bodies in the framework of the linear theory and the specific tasks of determining the stress strain state without taking into account the environment resistance are given. In [2, 3] the basic equations of the stability of inhomogeneous orthotropic rectangular plates and cylindrical and conical shells without considering the environment resistance are introduced.

In [4] the problem of vibrations of elastic beam oscillations, lying on inhomogeneous viscoelastic foundation, is solved.

In [5] some issues of the stress strain state of homogeneous circular plates of variable thickness are considered. The conceptual theory of bending of anisotropic plates is given in a detail, but environmental impact has not been considered.

In [6–8] the issues related to the elastic foundations, which specifies the Winkler model, which is not confirmed by experiments, are given.

In [7] oscillations are considered taking into account Pasternak-type foundation resistance. In [8] the problem of inhomogeneous oscillation of transverse along the length a straight section of pipe lying on the Pasternak-type foundation. In [9–12] the problems of the dynamics of elements of homogeneous elastic and viscoelastic materials are given and many applied problems are solved. The influence of the environment resistance is not considered.

Thus, the results suggest that for the correct calculation of the amplitude and frequency characteristics it is necessary to build solution techniques taking into consideration inhomogeneity of plate and foundation.

5. Methods of research

In the course of the study we used the materials of [1–3]. The methods of separation of variables and Bubnov-Galerkin orthogonalization method, which gives effective results with homogeneous boundary conditions, were used.

6. Research results

Boundary conditions are homogeneous. Let's note that for inhomogeneous boundary conditions Bubnov-Galerkin orthogonalization method can't be used.

Foundation reaction is related to the deflection by the following relation [6, 7]:

$$q = k_1(r)W + k_2(r)\frac{\partial^2 W}{\partial t^2}, \tag{1}$$

where $k_1(r)$, $k_2(r)$ – continuous functions that characterize the properties of the foundation and are determined experimentally. The density ρ is related to the current radius as follows:

$$\rho = \rho_0 \psi(r), \tag{2}$$

where ρ_0 corresponds to the homogeneous case; $\psi(r)$ – continuous function of elastic modulus; E_1 and E_2 also depend on r. Poisson's ratios are taken as constant:

$$E_1 = E^0 f(r); \quad E_2 = E_2^0 f(r);$$

$$\nu_1 = \text{const}; \quad \nu_2 = \text{const},$$

where $f(r)$ – functions with its derivatives up to second order are continuous functions. The relationship between bending moments and deflection is expressed by the following equations [9, 10]:

$$M_1 = -D_1^0 f(r)\left(\frac{\partial^2 W}{\partial r^2} + \frac{\nu_1}{r}\frac{\partial W}{\partial r}\right),$$

$$M_2 = -D_2^0 f(r)\left(\frac{1}{r}\frac{\partial W}{\partial r} + \nu_2\frac{\partial^2 W}{\partial r^2}\right), \tag{3}$$

where

$$D_1^0 = \frac{E_1^0 h^3}{12(1-\nu_1\nu_2)}; \quad D_2^0 = \frac{E_2^0 h^3}{12(1-\nu_1\nu_2)},$$

where h – plate thickness; E_1^0, E_2^0, ν_1, ν_2 – correspond to the homogeneous case. The motion equation in this case, taking into account (1) and (2), can be written as [8, 9]:

$$\frac{\partial^2 M_1}{\partial r^2} + \frac{2}{r^2}\frac{\partial}{\partial r}\left(M_1 - \frac{1}{2}M_2\right) + k_1(r)W +$$

$$+ k_2(r)\frac{\partial^2 W}{\partial r^2} + \rho_0\psi(r)\frac{\partial^2 W}{\partial r^2} = 0. \tag{4}$$

Taking into account (3) into (4) we obtain:

$$L(W) - \bar{k}_1(r)W - \left(\bar{k}_2(r) + \rho_0\psi(r)\frac{\partial^2 W}{\partial r^2}\right) = 0, \tag{5}$$

where the following notation:

$$L(w) = f(r)\frac{\partial^4 W}{\partial r^4} + \left(2\frac{\partial f}{\partial r} + \frac{\nu_1 + 2}{r}f(r) - \frac{\beta\nu_2}{r}\right)\frac{\partial^3 W}{\partial r^3} -$$

$$-\left[\frac{\partial^2 f}{\partial r^2} + \frac{2(\nu_{i+1})}{r}\frac{\partial f}{\partial r} - \frac{\beta}{r^2}\right]\frac{\partial^2 W}{\partial r^2} + + \frac{\beta}{r^2}\left(\nu_1\frac{\partial^2 f}{\partial r^2} + \frac{1}{r^2}\right)\frac{\partial W}{\partial r},$$

where

$$\bar{k}_1(r) = k_1(r)(D_1^0)^{-1}, \ \bar{k}_2(r)(D_1^0)^{-1}, \ \beta = D_2^0(D_1^0)^{-1}.$$

As can be seen, equation (5) is complex and obtaining an accurate and complex solution even with simple inhomogeneity types $f(r)$, $\psi(r)$, $k_1(r)$, $k_2(r)$ is difficult or impossible. Therefore, solving the problem will be using approximate analytical method of solution. At the first stage we use the method of separation of variables and further Bubnov-Galerkin orthogonalization method. Solution on the first stage will be sought in the following form:

$$W(r,t) = V(r)e^{i\omega t}, \tag{6}$$

where function $V(r)$ must satisfy the boundary conditions; ω – circular frequency. Substituting (6) into equation (5) obtain:

$$\bar{L}(V) - \bar{k}_1 V - \omega^2(\bar{k}_2(r) + \bar{\rho}_0(r))V = 0. \tag{7}$$

In solving (7) we will use Bubnov-Galerkin method and a solution will be sought in the following form:

$$V(r) = \sum_{i=1}^{n} C_i \varphi_i(r), \tag{8}$$

where c_i – unknown constants, which must be determined, but every $\varphi_i(r)$ has to satisfy the relevant boundary conditions. Using (8) in (7), define the error function:

$$\eta(r) = \sum_{i=1}^{n} C_i \begin{bmatrix} \left(L(\varphi_i(r)) - \bar{k}_1\varphi_i(r) + \right. \\ \left. + \omega^2\left(\bar{k}_2(r) + \rho_0\psi(r)\right)\right) \end{bmatrix} \varphi_i(r) \neq 0.$$

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Orthogonalization conditions taking into account (9) can be written as follows:

$$\int_0^R \eta(r)\phi_q(r)rdr = 0,\ q = 1,\ 2,\ ... \qquad (10)$$

In general, ω^2 is determined from the condition that the main determinant of the system of homogeneous algebraic equations, consisting of (10) relative to constant is equal to zero (the system is linear):

$$\|\omega^2\| = 0. \qquad (11)$$

During the removal (11) relative to ω^2 let's obtain an algebraic equation of the n-th degree. Usually, however, (mainly for circular and annular plates) it is neglected the measurement of angular frequency pitch, although finding ω^2 at an arbitrary approximation does not cause particular difficulties.

In the first approximation (10) is written as follows:

$$\int_0^R \varphi_1(r)\eta(r)rdr = 0 \qquad (12)$$

or

$$\int_0^R \left[L(\varphi_1) + \bar{K}_1\varphi_1(r) - \omega^2\left(\bar{K}_2(r) + \bar{\rho}_0\psi(r)\right)\right]\varphi_1(r)rdr = 0.$$

Obtain

$$\omega^2 = \frac{\int_0^R \left[\bar{L}(\varphi_1) + \bar{K}_1\varphi_1\right]\varphi_1(r)rdr}{\int_0^R \left(\bar{K}_2(r) + \bar{\rho}_0\psi(r)\right)\varphi_1^2 rdr}. \qquad (13)$$

Let's consider the case when the plate is rigidly clamped around the contour. In this case $\varphi_1(r)$ function satisfies the following conditions:

$$\varphi_1 = 0;\quad \frac{\partial\varphi_1}{\partial r} = 0 \quad \text{at} \quad r = R.$$

Function φ_1 is selected as follows:

$$\kappa_1 = f_0\left[1 - \left(\frac{r}{R}\right)^2\right]^2, \qquad (14)$$

where f_0 – deflection value of the plate center.

From (13) at $K_2 = 0$ we obtain the solution of a similar problem for Fuss-Winkler foundation:

$$\omega^2 = \frac{\int_0^R \bar{L}_1(\varphi_1)\varphi_1(r)rdr + \int_0^R \bar{K}_1(\varphi_1)\varphi_1^2(r)rdr}{\bar{\rho}_0\int_0^R \psi(r)\varphi_1^2(r)rdr}. \qquad (15)$$

At $K_1(r) = 0$, $K_2 \neq (r)$ we obtain the solution of a similar problem, when the plate is on inhomogeneous viscous foundation:

$$\omega_2^2 = \frac{\int_0^R \bar{L}_1(\varphi_1)\varphi_1(r)rdr}{\int_0^R \left(\bar{K}_0(r) + \bar{\rho}_0\psi(r)\right)\varphi_1^2(r)rdr}. \qquad (16)$$

From (13) and (15) we obtain the following relationship between ω^2 and ω_2^2:

$$\left(\frac{\omega}{\omega_1}\right)^2 = \frac{\bar{\rho}_0\int_0^R \psi(r)\varphi_1(r)rdr}{\int_0^R \left(\bar{K}_2(r) + \bar{\rho}_0\psi(r)\right)\varphi_1^2(r)rdr}. \qquad (17)$$

The results of numerical calculation of the value of angular frequency, taking into account the variability orthotropicity of elasticity and density moduli and inhomogeneous viscoelastic resistance, are shown in Table 1 and Fig. 1. Analysis of the numerical calculation shows that the value of the angular frequency naturally depends on the function $f(r)$, $\psi(r)$, $k_1(r)$, $k_2(r)$ and approximation function.

Table 1

The results of numerical calculation of the value of angular frequency

μ = 0	$\bar{\omega}_1^2$	$\bar{\omega}_2^2$
0	1	1
0,2	0,931	0,773
0,4	0,871	0,630
0,6	0,819	0,531
0,8	0,772	0,460
1	0,730	0,405

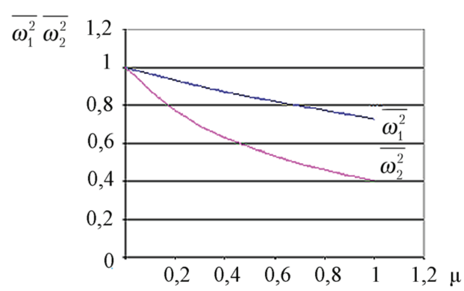

Fig. 1. Dependency graph of the dimensionless frequency values on density inhomogeneity parameter

Fig. 1 shows that the value of angular frequency depends essentially on the variable density.

Numerical analysis can be carried out at the following values of the characteristic parameters:

$$f(r) = 1 + \varepsilon\frac{r}{R};\quad \psi(r) = 1 + \mu\frac{r}{R};$$

$$K_1 = K_1^0\left(1 + \alpha\frac{r}{R}\right);\quad K_2 = K_2^0\left(1 + \alpha\frac{r}{R}\right);$$

$$\varepsilon \in [0,1];\quad \mu \in [0,1];\quad \alpha \in [0,1], \qquad (18)$$

where K_1^0 and K_2^0 correspond to the characteristics of homogeneous viscoelastic media.

For ease of analysis, calculations are made for the case of homogeneous medium without taking into consideration the viscous resistance:

$$\psi(r)=1+\varepsilon\rho;\ \psi(r)=1+\mu e^{\rho}. \tag{19}$$

$$\bar{\omega}_1^2=\frac{\int_0^1 \varphi_1^2(\rho)\rho d\rho}{\int_0^1 (1+\mu\rho)\varphi_1^2(\rho)\rho d\rho},$$

$$\rho=r\cdot R^{-1}.$$

Taking into consideration inhomogeneity significantly depends on the value of angular frequency.

7. SWOT analysis of research results

Strengths. Strengths of this research are that the decision allow to obtain specific calculation formulas. These formulas allow for known characteristics of foundation and plate to determine the value of the angular frequency.

Weaknesses. Weaknesses of this research are due to the fact that the proposed solutions are based on the fact that the decision is applied the approximate analytical methods and homogeneous boundary conditions. The reason for this is complex motion equations.

Opportunities. Opportunities for future research lies in the fact that these calculation techniques can be extended, for example, to meet the challenges of shell vibrations.

Threats. Threats for the implementation of research results are related to the fact that there are no experimental studies.

8. Conclusions

1. It is shown that the value of angular frequency is significantly affected by the following factors.

– Inhomogeneity orthotropicity at the current radius of the elastic moduli.

– Variable density at the current radius.

– Resistance of inhomogeneous viscoelastic medium.

2. The problem on natural oscillation of circular inhomogeneous orthotropic plate lying on inhomogeneous viscoelastic foundation is solved. A significant effect of orthotropicity, plate inhomogeneity and viscoelasticity of the medium are shown in Table 1 and Fig. 1.

3. A motion equation based on the inhomogeneity of plate and foundation is obtained. Equation is a complex with variable coefficients and approximate analytical methods are used to obtain an exact solution.

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ИССЛЕДОВАНИЕ СОБСТВЕННЫХ КОЛЕБАНИЙ НЕОДНОРОДНОЙ ОРТОТРОПНОЙ КРУГОВОЙ ПЛАСТИНКИ, ЛЕЖАЩЕЙ НА НЕОДНОРОДНОМ ВЯЗКОУПРУГОМ ОСНОВАНИИ

Рассмотрена осесимметричная форма собственных колебаний ортотропной неоднородного по радиусу круговой пластинки, лежащей на неоднородно вязкоупругом основании. Подробно изучен случай, когда по всему контуру пластина жестко защемлена. Решение задачи строилось с применением метода разделения переменных и метода ортогонализации Бубнов-Галеркина. Проведен численный анализ при конкретных значениях характерных параметров.

Ключевые слова: пластинка, непрерывность, ортотропность, плотность, основания, частота, модули упругости, уравнение движения.

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