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THE STUDY OF ONE MODEL OF RATIONAL CONSUMER BEHAVIOR ON THE MARKET

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V. Lisovska, V. Stasiuk. Study one of the model of rational consumer behavior in the market.

The article investigates the function of consumer utility with a standard budget constraint. In this paper, we study the example of the consumer choice of essential goods and luxury goods, the question of whether demand for essential goods will be limited if the income tends to infinity is investigated. The basis of consumer choice of the buyer is its advantages. The choice of the consumer is determined both by its advantages, and by the price of the chosen product, and also by its income. It is assumed that such a choice brings the buyer the greatest benefit in terms of budget constraints.

In this paper, we consider one of the models of consumer choice, which characterizes the features of the consumer's optimal choice in different situations. At the same time, we believe that one product is cheaper than another. In particular, the question is whether these products are interchangeable (mutually polled), or normal (valuable or inferior) or Giffen goods. It also turns out the question of whether the demand for each product depends on changes in prices for another product.

The function of consumer utility of a certain type is investigated. An example of consumer choice of essential goods and luxury goods is considered. In particular, we consider a consumer set of two products. It is proved that if the income tends to infinity, then the demand for essential goods is limited and tends to a constant value, and the demand for luxury goods increases unlimitedly.

The relationship is proved, which means that at the point of contact of the budget line to its indifference curve, the slopes (angular coefficients) of the lines (the indifference curve and the budget line) are the same. It is shown that the demand for the first product does not depend on the price of the second product, and the demand for the second product depends on the price of the first product.

Лісовська В. П., Стасюк В. Д. Дослідження однієї моделі раціональної поведінки споживача на ринку.

У статті досліджується функція споживчої корисності при стандартному бюджетному обмеженні. В даній роботі вивчається приклад споживчого вибору товарів першої потреби і товарів розкоші, досліджується питання про те, чи буде попит на товари першої потреби обмеженим, якщо дохід прямує до нескінченності. Основою споживчого вибору покупця є його переваги. Вибір споживача визначається як його перевагами, так і ціною обраної продукції, а також його доходом. При цьому припускається, що такий вибір приносить покупцеві найбільшу корисність в умовах бюджетного обмеження.

В даній роботі розглядається одна з моделей споживчого вибору, що характеризує особливості оптимального вибору споживача в різних ситуаціях. При цьому вважаємо, що один товар дешевше іншого. Зокрема, вивчається питання про те, чи є вказані товари взаємозамінними (взаємодоповнювальними), чи нормальними (цінними або малоцінними) або товарами Гіффена. Також з'ясовується питання про те, чи залежить попит на кожний товар від зміни цін на інший товар.

Досліджується функція споживчої корисності певного вигляду. Розглядається приклад вибору споживачем товарів першої потреби і товарів розкоші. Зокрема, розглядається споживчий набір із двох товарів. Доведено, що якщо дохід прямує до нескінченності, то попит на товари першої потреби є обмеженим і прямує до сталої величини, а попит на товари розкоші необмежено зростає. Доведено співвідношення, яке означає, що в точці дотику бюджетної прямої до її кривої байдужості, нахили (кутові коефіцієнти) ліній (кривої байдужості та бюджетної прямої) однакові. Показано, що попит на перший товар не залежить від ціни на другий товар, а попит на другий товар залежить від ціни на перший товар.

Лисовская В. П., Стасюк В. Д. Исследование одной модели рационального поведения потребителя на рынке.

В статье исследуется функция потребительской полезности при стандартном бюджетном ограничении. В данной работе изучается пример потребительского выбора товаров первой необходимости и товаров роскоши, исследуется вопрос о том, будет ли спрос на товары первой необходимости ограниченным, если доход стремится к бесконечности. Основой потребительского выбора покупателя являются его преимущества. Выбор потребителя определяется как его преимуществами, так и ценой выбраной продукции, а также его доходом. При этом предполагается, что такой выбор приносит покупателю наибольшую пользу в условиях бюджетного ограничения.

В данной работе рассматривается одна из моделей потребительского выбора, что характеризует особенности оптимального выбора потребителя в разных ситуациях. При этом считаем, что один товар дешевле другого. В частности, изучается вопрос о том, являются ли указанные товары взаимозаменяемыми (взаимоопыляемыми), или нормальными (ценными или малоценными) или товарами Гиффена. Также выясняется вопрос о том, зависит ли спрос на каждый товар от изменения цен на другой товар.

Исследуется функция потребительской полезности определенного вида. Рассматривается пример выбора потребителем товаров первой необходимости и товаров роскоши. В частности, рассматривается потребительский набор из двух товаров. Доказано, что если доход стремится к бесконечности, то спрос на товары первой необходимости является ограниченным и стремится к постоянной величине, а спрос на товары роскоши неограниченно возрастает.

Доказано соотношение, которое означает, что в точке касания бюджетной прямой к ее кривой безразличия, наклоны (угловые коэффициенты) линий (кривой безразличия и бюджетной прямой) одинаковы. Показано, что спрос на первый товар не зависит от цены на второй товар, а спрос на второй товар зависит от цены на первый товар.

Articulation of issue. One of the major elements of any economic system are the consumers. There are different mathematical models of consumer behavior of a separate person. The major problem of the rational housekeeping by the consumer lies in the question how much of available goods s/he should buy during the certain period of time, having the stated prices and consumer income (Ponomarenko, Perestiuk and Burym, 2004).

Assume that the I - is a consumer income (capital), which she is going to spend on purchasing of goods. The prices of the goods are considered to be stated. Taking into consideration the structure of prices, income, and personal advantages, the consumer buys the certain amount of the goods, and mathematical model of such his/her behavior is considered to be the consumer choice model. Let's review the consumption bundle that consists of two goods. It is widely thought that the consumer can say about these two bundles, that one of them is more desirable, than the other one, or the consumer just does not see the difference between them. The sum of these consumer bundles (x_1, x_2) consists of a function $U(x_1, x_2)$ (the so-called consumer utility function). Each consumer has its own utility function. The task of the rational consumer behavior on the market lies in choice of such consumer bundle (x_1^*, x_2^*), which maximize its utility function with the stated limitation of budget (Zamkov, Tolstopiatenko and Cheremnykh, 2001).

Analysis of the last publications related to the issue. General theory of the consumer choice has been analyzed in particular, in works of Zamkov O., Tolstopiatenko A., Cheremnykh Yu. (2001), Kolemaev V. (2005), Lange O. (1967), Ponomarenko O., Perestiuk M., Burym V. (1995, 2004), Tsurikov A., Tsurikov V. (2002, 2004).

Kolemaev (2005), in which the number of the most widespread mathematical economic models, related to the theories of consumption, production, saving, general economic equilibrium, etc have been analyzed. The necessity of exact quantitative models while housekeeping and managing of business operations have been realized long ago. Consumers are the one of the major elements of each economic system. Mathematical models of consumer behavior (that can be a person, family, household, etc, members of which have one for all of them total budget) have been analyzed in publications of well-known English economist Stone J. R., Slutskyi E. E., Kolemaieva V. A. and others. A number of important examples, which bring into the open the peculiarities of optimal consumer choice in different situation (Ponomarenko, Perestiuk and Burym, 2004) have been analyzed in this scientific work.

The example of consumer choice of the goods of prime necessity and luxury goods have been studied in this research paper, as well as the question about whether the demand for the goods of prime necessity can be limited, if the income $I \to +\infty$, and also the issue whether the specified goods are interchangeable (complementary goods), or normal ones (costly or of low value) or, maybe, Giffen goods. In this paper also has been done an effort to elucidate a question, whether the demand for each product depends on the price changes on the other product.

Problem definition. In this scientific paper has been analyzed one of the models of consumer choice, which characterizes the peculiarities of the optimal choice in different situations.

Let's analyze the example of consumer choice in the space of two goods R_+^2 in the case of utility, which has been given by the function

$$U(x_1, x_2) = x_1^{\alpha} \cdot x_2^{\beta-\alpha} \cdot (x_1 + \beta - \alpha)^{-\beta}, \; x_1, x_2 \geq 0, \tag{1}$$

where α, β meet the requirements: $\alpha > 0, \beta > \alpha$. Therewith we consider the one good to be cheaper than the other. We need to find out the demand function x_i, which maximize the utility function (1) with the stated budget limitation:

$$\sum_{i=1}^{2} p_i \cdot x_i \leq I, \tag{2}$$

where p_i - is the price for unit i of the certain good, I - is the amount of profit, as well as to find out, whether the demand for each good depends on the price changes on the other products, and also to investigate the demand functions for $I \to +\infty$. Also we are going to clear up the question whether the goods are interchangeable (so-called complementary goods), or normal ones (costly or of low value) or, for example, Giffen goods.

General material of the research.

The consumer with the general multiplicative function is under the review in the Stone consumer model

$$U(x_1, x_2, ..., x_n) = c \cdot \prod_{i=1}^{n} (x_i - \bar{x}_i)^{\alpha_i}, \tag{3}$$

in particular, the next Stone demand function has been studying

$$U(y_1, y_2) = (y_1 - a_1)^{c_1} \cdot (y_2 - a_2)^{c_2}, \tag{4}$$

where (a_1, a_2, c_1, c_2) - are the defined constants, moreover $c_1 + c_2 = 1$.

Let's discuss, in particular, the example of the choice of goods of primary necessity and luxury goods done by the consumer. Let's consolidate the goods of primary necessity into complex goods No. 1, and luxury goods - into complex goods No. 2. Let the x_1, x_2 - to be the amount of units of the goods No. 1 and No. 2, and p_1, p_2 - are the prices of the one separate unit of goods No. 1 and No. 2 correspondingly. The choice of the goods of the primary necessity and luxury goods can be represented by the next utility function (1).

Let's consider that the income (capital) I of the consumer, which he is ready to spend to purchase the goods, exceeds the cost of minimum subsistence basket $\vec{x} = (x_1, x_2) \geq 0$, which means, that the standard budget limitation takes place

$$p_1 x_1 + p_2 x_2 \leq I. \tag{5}$$

The problem of the consumer choice can be substituted by the problem of the constrained extremum of the function (1) upon the condition (5). Let's determine the demand functions $x_1^* = \varphi_1(p_1, p_2, I),\ x_2^* = \varphi_2(p_1, p_2, I)$ in consequence for the goods No. 1 and No. 2, which characterizes the optimal consumer choice. This problem resolve into the search for the maximum of utility function (1) with the budget limitation (5).

Let's use the Kuhn-Tucker theorem, which has given all necessary and sufficient conditions for the optimality of the solution x_1^*, x_2^* of the problem (1), (5). Lagrange function upon the conditions (1), (5) looks like (Lisovska, Perestiuk, 2009)

$$L(x_1, x_2, \lambda) = U(x_1, x_2) + \lambda(I - \sum_{i=1}^{2} p_i x_i), \tag{6}$$

which means

$$L(x_1, x_2, \lambda) = x_1^{\alpha} \cdot x_2^{\beta-\alpha} \cdot (x_1 + \beta - \alpha) + \lambda(I - p_1 x_1 - p_2 x_2). \tag{7}$$

Let's find the partial differential coefficient $\frac{\partial L}{\partial x_1}, \frac{\partial L}{\partial x_2}, \frac{\partial L}{\partial \lambda}$:

$$\frac{\partial L}{\partial x_1} = x_2^{\beta-\alpha}(\alpha \cdot x_1^{\alpha-1}(x_1 + \beta - \alpha)^{-\beta} + x_1^{\alpha}(-\beta)\cdot(x_1 + \beta - \alpha))^{-\beta-1} - \lambda p_1,$$

$$\frac{\partial L}{\partial x_2} = x_1^{\alpha} \cdot (x_1 + \beta - \alpha)^{-\beta} \cdot (\beta - \alpha) \cdot x_2^{\beta-\alpha-1} - \lambda p_2,$$

$$\frac{\partial L}{\partial \lambda} = I - p_1 x_1 - p_2 x_2.$$

The necessary conditions for the optimality of the solution x_1^*, x_2^* and multiplier λ^* are the equality to zero of all partial differential coefficients for all variables (*Lisovska, Perestiuk, 2012*): $\frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0, \frac{\partial L}{\partial \lambda} = 0.$

The optimal consumption (x_1^*, x_2^*) satisfy the system of equations:

$$\begin{cases} (\alpha \dfrac{x_1^{\alpha}}{x_1} \cdot (x_1 + \beta - \alpha)^{-\beta} + x_1^{\alpha} \cdot \dfrac{(x_1 + \beta - \alpha)^{-\beta}}{x_1 + \beta - \alpha} \cdot (-\beta)) x_2^{\beta-\alpha} - \lambda p_1 = 0, \\ x_1^{\alpha} \dfrac{x_2^{\beta-\alpha}}{x_2} \cdot (x_1 + \beta - \alpha)^{-\beta} \cdot (\beta - \alpha) - \lambda p_2 = 0, \\ I - p_1 x_1 - p_2 x_2 = 0. \end{cases} \tag{8}$$

The solution of the system (8) lies on the line $I - p_1 x_1 - p_2 x_2 = 0$, or $\left(\dfrac{x_1}{I/p_1} + \dfrac{x_2}{I/p_2} = 1 \right)$, the so-called budget-constraint line. A pair of data values (x_1^*, x_2^*), that are the optimal solution of the system (8) with the meaning λ^* is the adherent point of the budget-constraint line to the indifference curve $U(x_1, x_2) = U(x_1^*, x_2^*) = const$.

Taking into account (1), the system (8) will appear as

$$\begin{cases} \dfrac{\alpha}{x_1}\cdot U + \dfrac{U}{x_1+\beta-\alpha}\cdot(-\beta) - \lambda p_1 = 0, \\ \dfrac{U}{x_2}\cdot(\beta-\alpha) - \lambda p_2 = 0, \\ p_2 x_2 = I - p_1 x_1, \end{cases} \quad \text{or} \quad \begin{cases} U\left(\dfrac{\alpha}{x_1} - \dfrac{\beta}{x_1+\beta-\alpha}\right) = \lambda p_1, \\ U\cdot\dfrac{\beta-\alpha}{x_2} = \lambda p_2, \\ p_2 x_2 = I - p_1 x_1, \end{cases} \quad \text{from where}$$

$$\begin{cases} \dfrac{U}{\lambda} = p_1\cdot\dfrac{x_1(x_1+\beta-\alpha)}{\alpha(x_1+\beta-\alpha)-x_1\beta}, \\ \dfrac{U}{\lambda} = \dfrac{p_2 x_2}{\beta-\alpha}, \\ p_2 x_2 = I - p_1 x_1. \end{cases} \tag{9}$$

Let's insert $p_2 x_2$ from the third equation of the last system into the second one and then $\dfrac{U}{\lambda}$ from the second one into the first one, and will find

$$\begin{cases} p_2 x_2 = I - p_1 x_1, \\ \dfrac{U}{\lambda} = \dfrac{I - p_1 x_1}{\beta-\alpha}, \\ \dfrac{I-p_1 x_1}{\beta-\alpha} = \dfrac{p_1 x_1(x_1+\beta-\alpha)}{\alpha x_1 + \alpha(\beta-\alpha) - \beta x_1}. \end{cases} \tag{10}$$

Now let's solve the last equation of the system (10) in relation to x_1 :

$$\frac{I-p_1 x_1}{\beta-\alpha} = \frac{p_1 x_1(x_1+\beta-\alpha)}{\alpha x_1 + \alpha(\beta-\alpha) - \beta x_1},$$

$$(I - p_1 x_1)\cdot(\alpha-\beta)\cdot(x_1-\alpha) = -(\alpha-\beta)p_1 x_1(x_1+\beta-\alpha).$$

As far as $\beta > \alpha$ (according to the condition), we are going ot divide the last equation onto $\alpha-\beta$, will get $(I-p_1x_1)\cdot(x_1-\alpha) = -p_1 x_1(x_1+\beta-\alpha)$, or

$I\cdot x_1 - p_1 x_1^2 - I\alpha + \alpha p_1 x_1 + p_1 x_1^2 + p_1\beta x_1 - p_1\alpha x_1 = 0$, $\quad x_1(I + p_1\beta) = I\alpha$, from where

$$x_1 = \frac{I\alpha}{I + \beta\, p_1}. \tag{11}$$

Will insert (11) into the first equation of the system (10), will carry out the sequential transformations and will find out x_2 :

$$x_2 = \frac{I - p_1\cdot\dfrac{I\alpha}{I+\beta p_1}}{p_2} = \frac{I(I+\beta p_1 - \alpha p_1)}{p_2(I+\beta p_1)},$$

$$x_2 = \frac{I(I + p_1(\beta - \alpha))}{p_2(I + \beta p_1)}. \tag{12}$$

As far as

$$\lim_{I \to +\infty} x_2^* = \lim_{I \to +\infty} \frac{I^2 + Ip_1(\beta - \alpha)}{p_2 I + p_2 p_1 \beta} = [\frac{\infty}{\infty}] = \lim_{I \to +\infty} \frac{1 + \frac{p_1}{I}(\beta - \alpha)}{p_2(\frac{1}{I} + \frac{\beta p_1}{I^2})} = \infty,$$

$\lim_{I \to +\infty} x_1^* = \lim_{I \to +\infty} \frac{I\alpha}{I + \beta p_1} = [\frac{\infty}{\infty}] = \lim_{I \to +\infty} \frac{\alpha}{1 + \frac{\beta p_1}{I}} = \alpha$, that is why the second good is "the luxury good", and the first one is not "the luxury good". It is "the good of the primary necessity".

So, the demand function for the goods of primary necessity x_1^* can be identified through the formula (11), herewith we have the next situation, in which the demand for the first good does not depend on the price for the second one, and the demand function for luxury goods x_2^* - according to the formula (12), from which we can see, that the demand for the second good depends on the price for the first one.

Then the optimal Lagrange multiplier

$$\lambda^* = \frac{U \cdot (\beta - \alpha)}{p_2 x_2} = \frac{U(\beta - \alpha) \cdot p_2(I + \beta p_1)}{p_2 I(I + p_1(\beta - \alpha))} = \frac{(\beta - \alpha) \cdot (I + \beta p_1)}{I(I + p_1(\beta - \alpha))} \cdot U.$$

As far as

$$\frac{\partial x_1^*}{\partial p_1} = -\frac{I\alpha\beta}{(I + p_1\beta)^2} < 0 \quad \text{and} \quad \frac{\partial x_2^*}{\partial p_2} = -\frac{I(I + p_1(\beta - \alpha))}{p_2^2(I + p_1\beta)} < 0,$$

then each good (the first one and the second one) are normal, which means that the increasing of the price leads to the decreasing of the demand on this good.

Let's analyze the demand function (10) of the first good. Its partial differential coefficient for the income looks like

$\frac{\partial x_1^*}{\partial I} = \frac{\alpha(I + \beta p_1) - I\alpha}{(I + p_1\beta)^2} = \frac{\alpha\beta p_1}{(I + \beta p_1)^2}$ and $\frac{\partial x_1^*}{\partial I} > 0$, because α, β, p_1 (and because of that their product also) more than zero. In such a case the demand for the first good with the increasing of the income increases, that is the first good is normal, moreover, costly good. Otherwise, the decrease of income leads to the decrease of demand for the first and for the second goods. The normal good cannot be the Giffen good.

Will find out $\frac{\partial x_2}{\partial I}$:

$$\frac{\partial x_2}{\partial I} = \frac{(2I + p_1(\beta - \alpha))(I + p_1\beta) - I(I + p_1(\beta - \alpha))}{(I + p_1\beta)^2} \cdot \frac{1}{p_2} =$$

$$= \frac{2I^2 + I(\beta - \alpha)p_1 + 2Ip_1\beta + p_1^2\beta(\beta - \alpha) - I^2 - I(\beta - \alpha)p_1}{p_2(I + p_1\beta)^2} = \frac{I^2 + 2Ip_1\beta + p_1^2\beta(\beta - \alpha)}{p_2(I + p_1\beta)^2} > 0,$$

because $\beta > \alpha$.

So, the second good is the normal good too, and also costly good.

Will find out $\frac{\partial x_2}{\partial p_1}$ and $\frac{\partial x_2}{\partial p_2}$:

$$\frac{\partial x_2}{\partial p_2}=0,$$

$$\frac{\partial x_2}{\partial p_1}=\frac{1}{p_2}\cdot\frac{I(\beta-\alpha)(I+\beta p_1)-(I^2+Ip_1(\beta-\alpha))\beta}{(I+p_1\beta)^2}=\frac{I}{p_2(I+p_1\beta)^2}(I(\beta-\alpha)+p_1\beta(\beta-\alpha)-$$

$$-(I+p_1(\beta-\alpha))\beta)=\frac{I}{p_2(I+p_1\beta)^2}(I\beta-I\alpha+p_1\beta^2-p_1\alpha\beta-I\beta-p_1\beta^2+p_1\alpha\beta)=$$

$$=-\frac{I^2\alpha}{p_2(I+p_1\beta)^2}<0.$$

Indirect consumer utility function looks like:

$$V(p,I)=U(x_1^*,x_2^*)=(x_1^*)^{\alpha}(x_2^*)^{\beta-\alpha}(x_1^*+\beta-\alpha)^{-\beta}=$$

$$=\left(\frac{I\alpha}{I+p_1\beta}\right)^{\alpha}\left(\frac{I(I+p_1(\beta-\alpha))}{p_2(I+p_1\beta)}\right)^{\beta-\alpha}\left(\frac{I\alpha}{I+p_1\beta}+\beta-\alpha\right)^{-\beta}=$$

$$=\frac{I^{\alpha}\alpha^{\alpha}I^{\beta-\alpha}(I+p_1(\beta-\alpha))^{\beta-\alpha}(I+p_1\beta)^{\beta}}{(I+p_1\beta)^{\alpha+\beta-\alpha}p_2^{\beta-\alpha}(I\alpha+I\beta-I\alpha+p_1\beta(\beta-\alpha))^{\beta}}=\frac{I^{\beta}\alpha^{\alpha}p_2^{\alpha}(I+p_1(\beta-\alpha))^{\beta-\alpha}}{p_2^{\beta}(\beta(I+p_1(\beta-\alpha)))^{\beta}},$$

which means

$$U(x_1^*,x_2^*)=\left(\frac{I}{p_2\beta}\right)^{\beta}\frac{(\alpha p_2)^{\alpha}}{(I+p_1(\beta-\alpha))^{\alpha}}\ . \qquad (13)$$

After the differentiating of the last equation for the variable I, will get

$$\frac{\partial V}{\partial I}=\frac{(\alpha p_2)^{\alpha}}{(\beta p_2)^{\beta}}\frac{\partial}{\partial I}\left(\frac{I^{\beta}}{(I+p_1(\beta-\alpha))^{\alpha}}\right)=$$

$$=\frac{\alpha^{\alpha}}{\beta^{\beta}}p_2^{\alpha-\beta}\frac{\beta I^{\beta-1}(I+p_1(\beta-\alpha))^{\alpha}-\alpha(I+p_1(\beta-\alpha))^{\alpha-1}I\beta}{(I+p_1(\beta-\alpha))^{2\alpha}}=$$

$$=\frac{\alpha^{\alpha}}{\beta^{\beta}}p_2^{\alpha-\beta}\frac{I^{\beta-1}(\beta-\alpha)(I+p_1\beta)}{(I+p_1(\beta-\alpha))^{\alpha+1}},$$

along with that $\frac{\partial V}{\partial I}>0.$

From the second equation of the system (9), taking into consideration (11)- (13) will find out

$$\lambda=\frac{U(\beta-\alpha)}{I-p_1x_1}=\frac{\beta-\alpha}{I-p_1\dfrac{I\alpha}{I+p_1\beta}}\cdot\left(\frac{I}{p_2\beta}\right)^{\beta}\frac{(\alpha p_2)^{\alpha}}{(I+p_1(\beta-\alpha))^{\alpha}}=$$

$$=\frac{(\beta-\alpha)(I+p_1\beta)}{I^2+Ip_1\beta-Ip_1\alpha}\cdot\frac{I^{\beta}(\alpha p_2)^{\alpha}}{(p_2\beta)^{\beta}(I+p_1(\beta-\alpha))^{\alpha}}=\frac{I^{\beta-1}(\beta-\alpha)(I+p_1\beta)}{(I+p_1(\beta-\alpha))^{\alpha+1}}\cdot\frac{\alpha^{\alpha}}{\beta^{\beta}}\cdot p_2^{\alpha-\beta}=\Lambda(p_1,p_2,I)-$$

co-boundary consumer income utility function.

So, now we have the next, $\Lambda=\frac{\partial V}{\partial I}.$

Let's find the co-boundary consumer utilities from $i-$those good, and, taking into consideration (11), (12), will get the correlation

$$\frac{1}{p_1}\cdot\frac{\partial U}{\partial x_1^*}=\frac{1}{p_2}\cdot\frac{\partial U}{\partial x_2^*}. \qquad (14)$$

Really,

$$\frac{\partial U}{\partial x_1^*}=\alpha(x_1^*)^{\alpha-1}(x_2^*)^{\beta-\alpha}(x_1^*+\beta-\alpha)^{-\beta}+(x_1^*)^{\alpha}(x_2^*)^{\beta-\alpha}(-\beta)(x_1^*+\beta-\alpha)^{-\beta-1}=$$

$$=\frac{(x_1^*)^{\alpha-1}(x_2^*)^{\beta-\alpha}}{(x_1^*+\beta-\alpha)^{\beta+1}}(x_1^*-\alpha)(\alpha-\beta)=\frac{(I\alpha)^{\alpha-1}}{(I+p_1\beta)^{\alpha-1}}\cdot\frac{I^{\beta-\alpha}(I+p_1(\beta-\alpha))^{\beta-\alpha}}{p_2^{\beta-\alpha}(I+p_1\beta)^{\beta-\alpha}}\times$$

$$\times\frac{(\alpha-\beta)}{\left(\dfrac{I\beta-I\alpha+p_1\beta^2-\alpha\beta p_1+I\alpha}{I+p_1\beta}\right)^{\beta+1}}\cdot\frac{I\alpha-I\alpha-\alpha\beta p_1}{I+p_1\beta}=\frac{I^{\alpha-1}\alpha^{\alpha-1}}{(I+p_1\beta)^{\alpha-1}}\cdot\frac{I^{\beta-\alpha}(I+p_1(\beta-\alpha))^{\beta-\alpha}}{p_2^{\beta-\alpha}(I+p_1\beta)^{\beta-\alpha}}\times$$

$$\times\frac{(\alpha-\beta)(-\alpha p_1\beta)}{\left(\dfrac{(I-\alpha p_1+p_1\beta)\beta}{I+p_1\beta}\right)^{\beta+1}}=\frac{I^{\beta-1}(I+p_1(\beta-\alpha))^{\beta-\alpha}\alpha^{\alpha-1}\alpha(\beta-\alpha)p_1\beta}{(I+p_1\beta)^{\beta-1-\beta-1}p_2^{\beta-\alpha}(I+p_1\beta)(\beta^{\beta+1}(I+p_1(\beta-\alpha))^{\beta+1}}=$$

$$=\frac{\alpha^{\alpha}(\beta-\alpha)p_1I^{\beta-1}(I+p_1\beta)}{\beta^{\beta}p_2^{\beta-\alpha}(I+p_1(\beta-\alpha))^{\alpha+1}}>0.$$

$$\frac{\partial U}{\partial x_2^*}=(x_1^*)^{\alpha}(x_1^*+\beta-\alpha)^{-\beta}(\beta-\alpha)(x_2^*)^{\beta-\alpha-1}=\left(\frac{I\alpha}{I+p_1\beta}\right)^{\alpha}\frac{(\beta-\alpha)\left(\dfrac{I(I+p_1(\beta-\alpha))}{p_2(I+p_1\beta)}\right)^{\beta-\alpha-1}}{\left(\beta-\alpha+\dfrac{I\alpha}{I+p_1\beta}\right)^{\beta}}=$$

$$=\frac{I^{\alpha}\alpha^{\alpha}}{(I+p_1\beta)^{\alpha}}\cdot\frac{(\beta-\alpha)I^{\beta-\alpha-1}(I+p_1(\beta-\alpha))^{\beta-\alpha-1}}{\left(\dfrac{I\beta-I\alpha+p_1\beta^2-p_1\alpha\beta+I\alpha}{I+p_1\beta}\right)^{\beta}p_2^{\beta-\alpha-1}(I+p_1\beta)^{\beta-\alpha-1}}=\frac{\alpha^{\alpha}(\beta-\alpha)I^{\beta-1}(I+p_1\beta)}{\beta^{\beta}p_2^{\beta-\alpha-1}(I+p_1(\beta-\alpha))^{\alpha+1}}.$$

So,

$$p_2\cdot\frac{\partial U}{\partial x_1^*}=p_1\cdot\frac{\partial U}{\partial x_2^*},\quad \text{or}\quad \frac{1}{p_1}\cdot\frac{\partial U}{\partial x_1^*}=\frac{1}{p_2}\cdot\frac{\partial U}{\partial x_2^*}.$$

Summary. The set of two goods have been analyzed. The consumer utility function have been investigated (1). Have been proved, that for $I\to+\infty$ the demand for the goods of the primary necessity is restricted and tend to α, and the demand for the luxury goods is unrestrictedly increasing. Has been proved the condition (14), which means, that in adherent point of budget-constraint line, which can be described by the equation $I-p_1x_1^*-p_2x_2^*=0$, to its the indifference curve $U(x_1^*,x_2^*)=const$ slope terms of the lines (of the indifference curve and budget-constraint line) are similar.

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