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APPLICATION OF EQUATIONS OF THE PLASTICITY THEORY IN THE PROCESSES OF PROSESSING POROUS BODY PRESSURE

In the article the results of the application of the equations of the plasticity theory in the processes of processing of the powder body pressure such as a porous sleeve were presented. The mathematical model describing the plastic behavior of a porous body is preferred. For this purpose, the hypothesis on the use of the Beltram plastic flow in the construction of a model of the plastic behavior of the porous body was used, diagrams of tangential and normal stresses and the contact surface of the body were constructed. On the basis of the Beltram hypothesis, the plasticity condition in which, the three-dimensional space of principal stresses, the equation is an ellipsoid. An equation of plastic friction of a porous powder body during re-compaction is obtained. In the case when the components of the stress deviator are known, when using certain boundary conditions, it becomes possible to find the components of the stress tensors. Testing of the processes of deformation of a porous body was carried out on cylindrical powder samples with dimensions D×H = 10×10 mm. after primary pressing, their relative density was ρ *= 0.68. As a result of the experimental tests of repeated pressing of the sleeve, a graph of the dependence of the yield strength of the iron-based material on the relative density of the sleeve was built, which is in good agreement with the calculated data. Further tests carried out on the porous bushings are further compacted by pulling. The zones of plastic deformation in the powder sleeve are determined. Diagrams of normal* σr *and tangential* τ_{rx} *stresses of the tangent to the contact surface along the r = const line are constructed. The proposed method for determining the stresses and plastic behavior of iron-based porous powder bushings can be used for other stationary methods of deformation of powder materials (rolling, extrusion, drawing, etc.).*
Keywords: *body, sleeve, theory of plasticity, stress state, normal stress, shear stress, plastic flow.*

Рустамова С.М., Мамедов А.Т. Застосування рівнянь теорії пластичності пористому тілу в процесах обробки тиском. *У статті представлені результати застосування рівнянь теорії пластичності до процесів обробки тиском порошкового тіла типу пористої втулки. Віддано перевагу математичній моделі, що описує пластичну поведінку пористого тіла. З цією метою використана гіпотеза про застосування пластичної течії Бельтрама при побудові моделі пластичної поведінки пористого тіла, побудовані епюри дотичної і нормальної напруг, контактної поверхні тіла. На основі гіпотези Бельтрама виражено умову пластичності, в якій в тривимірному просторі головних напруг рівняння є еліпсоїдом. Отримано рівняння пластичної течії пористого порошкового тіла при повторному ущільненні. Для випадку, коли відомі компоненти девіатора напруг, при використанні деяких граничних умов стає можливим знаходження компонентів тензорів напруг. Випробування процесів деформування пористого тіла проведено на циліндричних порошкових зразках розмірами D×H = 10×10 мм. Після первинного пресування їх відносна щільність становила* ρ *= 0,68. В результаті експериментальних випробувань повторного пресування втулки побудований графік залежності межі текучості матеріалу на основі заліза від відносної щільності втулки, який добре узгоджуються з розрахунковими даними. Проведені випробування пористих втулок в умовах додаткового ущільнення шляхом протягування. Визначено зони пластичних деформа-*

[1] *Phd Student, senior lecturer, Azerbaijan Technical University, Baku, sevilrustamova70@mail.ru*
[2] *Dsc (Engineering), professor, Azerbaijan Technical University, Baku, ariff-1947@mail.ru*

цій в порошковій втулці. Побудовано епюри нормальних σ_r і дотичних τ_{rx} напруг дотичної на контактну поверхню уздовж τ = const лінії. Запропонований метод визначення напруг і пластичної поведінки пористих порошкових втулок на основі заліза може бути використаний при інших стаціонарних методах деформування порошкових матеріалів (прокатці, видавлюванні, волочіння і ін.). Результати повторного деформування спечених пористих втулок на основі заліза показали істотне зміцнення матеріалу після додаткової обробки.
Ключові слова: *тіло, втулка, теорія пластичності, напружений стан, нормальна напруга, дотична напруга, пластична течія.*

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Description of the problem. The operations of shaping and hardening - calibration, which are promising directions in the powder metallurgy, are based on plastic deformation of powder blanks. This, in many cases, by reducing the residual porosity, strengthening the base of the material and increasing the surface finish of the part, can significantly improve the performance of materials. However, for the correct execution of the processes of pressure treatment of powder materials in the process of plastic shaping, it is necessary to monitor their behavior. This, in turn, makes it necessary to build a model describing the results of experiments carried out in a plastic porous body. The main distinctive feature of the plastic deformation of the strength of the body is the presence of permanent deformation. This is mainly reflected in Poisson's ratio. The complexity of the problem posed also lies in the fact that in the process of plastic deformation, the mechanical strengthening of a porous body is caused by both a change in porosity and a change in the basic strength of the material [1].

Analysis of recent research and publications. To describe the plastic behavior of a porous body, first of all it is necessary to accept the limiting state condition and the material plasticity condition.

In the general case of the stress state, the plasticity condition is described as follows

$$f(\sigma_{ij}, S) = 0, \qquad (1)$$

where $f(\sigma_{ij})$ is equation of hypersurface plasticity; σ_{ij} – component of the stress tensor; *S* is the relative density.

In case of plastic deformation, the condition for detecting irreversible plastic deformation can be written as follows:

$$\frac{d_f}{d\sigma_o} \neq 0, \qquad (2)$$

where $\sigma_o - \sigma_o \, {}^{1}\!/_{3} J_1$ the ratio is the average stress related to the first invariant of the stress tensor.

Therefore, the plasticity condition for compressible bodies must include the first invariant of the stress tensor *J* or the average stress, then the plasticity condition of the compressible body has the following form:

$$f(J_1 J_2^1 S) = 0 \quad \text{or} \quad f(\sigma_0 J_2^1) = 0, \qquad (3)$$

where J_2^1 is the second invariant of the stress tensor deviator.

There are some models that, giving approximate description of the plastic behavior of a porous body in one or another step [2-6]. The more complete model is the model of a hardened porous body proposed in [6]. Here, the state of the medium is characterized by two hardening parameters - the current porosity θ and the intensity of deformations of the material base - γ. In some cases, it is difficult to determine the value of - γ Therefore, the porous body models proposed in [2-4] are of great interests, in which only one parameter is included in the plasticity condition - the current porosity or the relative density of the body. As shown in [2-4], this simplification does not lead to a significant discrepancy between the information obtained theoretically and experimentally.

In our study, we used the hypothesis of the application of the Beltram plastic state when constructing a model of the plastic behavior of a porous body [7]. On the basis of this hypothesis, the material transforms into a plastic state when the total specific deformation energy reaches a certain limiting value. In this case, it is proposed that Hooke's law is valid before the onset of plasticity.

Based on the Beltram hypothesis, the plasticity condition is expressed as follows:

$$\frac{2}{3}(1+\mu)\sigma_i^2 + 3(1-2\mu)\sigma_0^2 = \mathrm{Y}^2, \tag{4}$$

where $\sigma_i = (3J_2^1)^{1/2}$ is the intensity of stress; Y is the current yield stress of the porous body in the state of linear stress.

In equation (4) Y and μ are functions of the relative density of the body.

In the three-dimensional space of the principal stresses, equation (4) is an Ellersoid. The law between the rates of deformations and stresses induced by the yield surface is expressed as follows:

$$\dot{E}_{ij} = 2\lambda\left[(1+\mu)\sigma_{ij} - \delta_{ij}3\mu\sigma_o\right], \tag{5}$$

where $\dot{E}_{ij}$ is strain rate tensor component; δ_{ij} – Kronecker symbol.

$$2\lambda = \frac{H_i}{\sigma_i}, \tag{6}$$

here

$$H_i = \left[\sqrt{6/2}(1+\mu)\right]\sqrt{(\dot{E}_{ij} - \delta_{ij}\dot{E}_o)(\dot{E}_{ij} - \delta_{ij}\dot{E}_o)} \tag{7}$$

is the intensity of the deformation rate, and

$$\dot{E}_o = 1/3\delta_{ij}\dot{E}_{ij} \tag{8}$$

is the rate of volumetric deformation.

From relation (5), taking into account (6), the equation of the relationship between the components of the stress deviator and the strain rate deviator is derived:

$$S_{ij} = \frac{\sigma_i}{(1+\mu)H_i}\dot{e}_{ij}, \tag{9}$$

where $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_o$ are the components of stress deviator; $\dot{\ell}_{ij} = \dot{E}_{ij} - \delta_{ij}$ – components of the strain rate deviator from relation (5), the following equations can be derived:

$$\delta_o = \frac{\phi}{1-2\mu}\sigma_i, \tag{10}$$

here $\phi = \frac{\dot{E}_o}{H_i}$.

Let's write the expression (4) in the following form

$$\mathrm{Y}^2 = \left[\frac{2}{3}(1+\mu) + \frac{3\phi^2}{1-2\mu}\right]\sigma_i^2.$$

From here

$$\sigma_i = \mathrm{Y}\sqrt{\frac{1}{\frac{2}{3}(1+\mu) + \frac{3\phi^2}{1-2\mu}}} = \omega\mathrm{Y}, \tag{11}$$

where

$$\omega = \sqrt{\frac{1}{\frac{2}{3}(1+\mu) + \frac{3\phi^2}{1-2\mu}}} = \omega\mathrm{Y}. \tag{12}$$

We express the equation of plastic flow of a porous body (9) in the following form

$$S_{ic} = \frac{\omega}{1+\mu}\frac{\mathrm{Y}}{H_i}\dot{E}_{ij}. \tag{13}$$

If the components of the stress deviator are known, then expressing the differential equations by balancing σ_{ij}, c = 0 and using some boundary conditions, one can find the components of the stress tensors. We used the considered model of a plastic porous body to determine the stresses during deforming broaching of bushings made of iron powder of the PZh2M3 grade. The deforming broach is a stationary process of pressure treatment of powder materials, since at any point the stress-strain state of the body is determined by its coordinates and does not depend on time.

Purpose of the article. A preliminary test of powder cylindrical samples with dimensions $D \times H$ = 10×10 mm was carried out. Before testing, the relative density of the samples was ρ = 0.68.

Compression tests were carried out on a GP-250 press. To lubricate the contact surfaces, fluoroplastic gaskets were used as a lubricant. In the process of upsetting, the diameter and height of the sample were measured, as a result, a graph was plotted Y = Y(S) (Fig. 1).

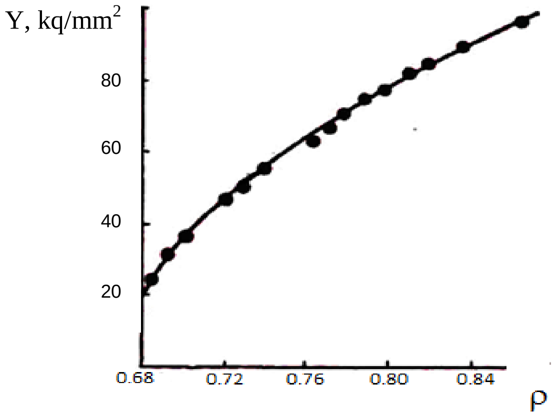

Fig. 1 – Dependence of the yield point on the relative density of the sleeve. The dependence of the coefficient on the relative density is well expressed by the following experimental relationship: $\mu = 0.5S^2$

At the next stage of the study, powder bushings with an outer diameter D_0 = 23 mm, an inner diameter d_0 = 15 mm, and a height H_0 = 18 mm were used. The relative density of the samples was S_0 = 0.70. The sleeves were prepared by cutting in the meridian plane. A coordinate sect with cell sizes was applied to the cut plane. 0.5-0.5 mm with a corundum needle on a UIM-21 instrumental microscope.

Presentation of the main material. The deformation broach was carried out in a UME-10TM machine at a speed V_0 = 5 mm/min. (interference $i = (d - d_0)/d_0 = 0.027$). The taper angle of the mandrel intake was α = 4 degrees, and the width of the calibrating clip was 1.5 mm. The broaching was carried out until the sleeve grew (Fig. 2), then the deformation was stopped, the sleeves were removed from the holder, and the coordinates of the deformed nodal points of the mesh were measured using a UIM-21 microscope. Figure 2 shows the boundaries of deformation zones during a deforming broach that changes its direction.

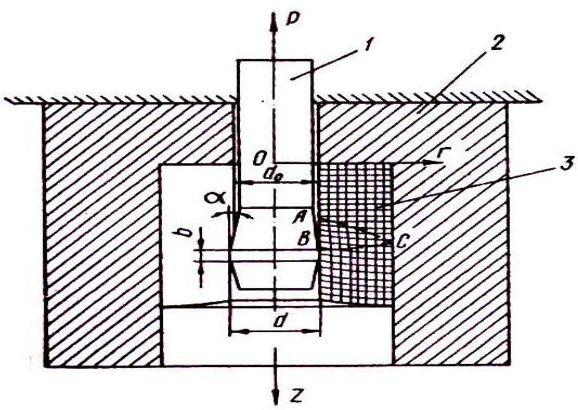

Fig. 2 – Chipping deforming holes in the powder sleeve: 1 – mandrel; 2 – clip; 3 – sleeve; ABC – plastic deformation zone

Using the following ratio, we found the relative density

$$S = S_o \frac{W_o}{W}, \tag{14}$$

where W_0 is the volume before the deformation of the annular element formed by the rotation of the grid cell around the axis of the sleeve; W is the volume of the deformed annular element.

The rate of change of the location of the leaks in the direction is determined using the flow function.

$$V_r = \frac{1}{r}\frac{d\psi}{dz}\frac{S_o}{S}; \quad V_z = -\frac{1}{r}\frac{d\psi}{dr}\frac{S_o}{S}, \tag{15}$$

where r and z are coordinates of points, in them the velocities V_r and V_z are determined; S is the relative density at this point; $\psi = 1/2\pi$; Q is the flow function, and $Q = S_0V_0$ is the volume of the substance passing through the annular surface in unit time.

$S_0 = \pi(r_{oj}^2 - r_0)$, where j is the ordinal number of the flow line determined by the coordinates of the point; r_{oj} is the distance before the flow line arrives from the bushing axis to the deformation center; $r_o = d_o/2$ – inner radius of the undeformed part of the sleeve. Then the calculations were carried out in the nodes of the computational grid with a step of 0.5 mm, superimposed on the figure of the deformed coordinate grid. S and ψ were determined by line interpolation at the nodal points of the regular grid. The found values of the relative densities S were used to find Y and μ.

The tensor components of the strain rates were determined by the following relation

$$\dot{E}_0 = \frac{dV_r}{d_r}; \quad \dot{E}_z = \frac{dV_r}{d_z}; \quad \dot{E}_o = \frac{V_r}{r}; \quad \dot{E}_{rz} = \frac{dV_z}{d_r} + \frac{dV_r}{d_2}. \tag{16}$$

$$\dot{l}_r = \dot{E}_r - \dot{E}_o; \quad \dot{l}_z = \dot{E}_z - \dot{E}_o; \quad \dot{l}_o = \dot{E}_o - \dot{E}_o, \tag{17}$$

here, $\dot{E}_o = \frac{1}{3}(E_r + E_z + E_\theta)$ is the rate of volumetric deformation.

Then the components of the stress deviator were determined S_r, S_z, S_θ. For axisymmetric deformation, relation (13) takes the following form.

$$S_r = \frac{\omega}{(1+\mu)}\frac{\text{Y}}{H_i}\dot{\ell}_r; \quad S_z = \frac{\omega}{(1+\mu)}\frac{\text{Y}}{H_i}\dot{\ell}_z;$$

$$S_\theta = \frac{\omega}{(1+\mu)}\frac{\text{Y}}{H_i}\dot{\ell}_\theta; \quad \tau_{rz} = \frac{\omega}{2(1+\mu)}\frac{\text{Y}}{H_i}\dot{\eta}_{rz}, \tag{18}$$

here

$$H_i = \frac{\sqrt{2}}{2(1+\mu)}\sqrt{(\dot{E}_r - \dot{E}_z)^2 + (\dot{E}_z - \dot{E}_\theta)^2 + (E_\theta - E_r)^2 + \frac{3}{2}\eta_{rz}^2}. \tag{19}$$

Let us write down the relations

$$\sigma_r = S_r + \sigma_\theta; \quad \sigma_z = S_z + \sigma_\theta; \quad \sigma_\theta = S_\theta + \sigma_\theta. \tag{20}$$

To determine the average stresses σ_θ in the deformation zone, we use the differential stability equations

$$\frac{d\sigma_r}{d_r} + \frac{d\tau_{rz}}{d_z} + \frac{\sigma_z - \sigma_\theta}{r} = 0; \quad \frac{d\tau_{rz}}{d_r} + \frac{d\sigma_z}{d_z} + \frac{\tau_{rz}}{r} = 0; \tag{21}$$

after substitution of relation (20), they obtain the following form

$$\frac{d\sigma_\theta}{d_r} + \frac{dS_r}{d_z} + \frac{d\tau_{rz}}{d_z} + \frac{S_r - S_o}{r} = 0; \quad \frac{d\sigma_\theta}{d_z} + \frac{dS_z}{d_z} + \frac{d\tau_{rz}}{d_r} + \frac{\tau_{rz}}{r} = 0. \tag{22}$$

Accordingly, integrating Eqs. (22) in the direction of the r and z axes and using the boundary condition at point A to be the radial stress σ_r σr equal to zero, we can find the average stresses σ_θ, and then using relations (20), we find the stresses σ_r, σ_z, σ_θ. During deforming pulling, the stresses were calculated on a computer. When calculating random derivatives at the nodes of a regular grid, the finite difference method was used. Numerical integration was performed based on the Simpson formula [8, 9].

Figure 3 shows the diagrams of normal Q_r and tangential τ_{rz} stresses along the r = constant contact line of the tangential contact surface. Normal stresses in the direction of the *z* axis grow rapidly and get the highest values at the exit of the deformation zone. Shear stresses are also exposed to the highest values at the exit of the deformation zone.

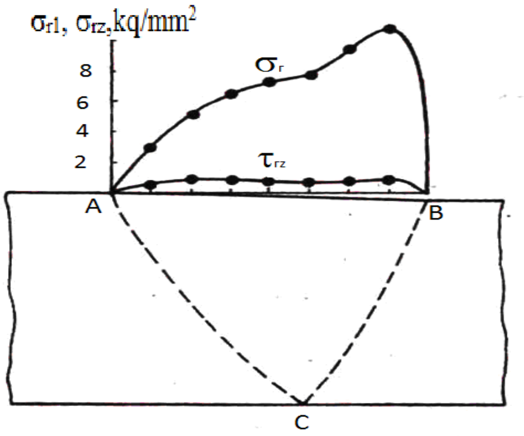

Fig. 3 – Diagrams of normal σ_r and tangential τ_{rz} stresses tangent to the contact surface along the r = const line

Due to the smallness of the angle α, these stresses at the first approximation can be considered contact deformations. Consequently, there is no proportional relationship between the contact normal and shear stresses. Therefore, when calculating the energy-power parameters of the process, it is necessary to give an advantage to the law of friction not according to Coulomb, but according to Sieber.

The proposed method for determining stresses can be used for other stationary methods of processing powder materials by pressure (rolling, extrusion, drawing, and others).

Conclusions

1. The model based on Beltram's hypothesis on the use of the plastic state of a plastic porous body was presented and a control that connects stresses and strain rates was derived.

2. The method was developed for determining stresses in stationary processes of pressure treatment of powder materials, based on the use of a coordinate grid and the obtained equations of plastic flow of a porous body to find the region of deformation rates. The method is applied to calculate stresses during deforming broaching of holes in porous bushings obtained by cold pressing by intermediate sintering from iron powder PZh2M3.

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Reviewer: N.Sh. Ismailov
Dsc (Engineering), professor, Azerbaijan Technical University

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