MATHEMATICAL MODEL OF THE CURRENCY MARKET, BASED ON FRACTAL ANALYSIS AND ANN CLUSTERING ALGORITHMS

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Abstract. In the modern economic situation, stock returns and the deviations like market collapses depend on many external factors. When studying financial time series, in order to build a reliable model, it is necessary to use various types of hypotheses regarding the structure of market processes. The main task of a financial investor is to search for information able to predict the immediate future of the asset being studied. Market behavior is modeled both on the basis of theoretical arguments about the nature of market processes and the experimentally gained market knowledge. Basically, the study of financial time series uses methods of statistical analysis. In this paper, a method for determining trends of a time series of financial indicators based on fractal and cluster analyzes is proposed. This procedure was considered on the example of the study of time series of quotations of foreign currency exchange rates, in particular, the euro from 2010 to 2017. The authors found monthly fractal dimensions of the studied time series and identified the relationship between trends and fractal dimension. Next, a cluster analysis was conducted on fractal dimensions using algorithms based on the use of artificial neural networks (ANN), on the basis of which the types of trends, such as stability, pre-crisis period and crisis situations were determined. To verify the proposed algorithm for determining the trend of a time series of financial quotations, the euro rate was considered in the period from January 2018 to August 2018. This technique, proposed by the authors for determining the types of periods of time series of financial assets and considered on the example of the euro exchange rate quotations, is universal and can be applied both for securities and financial indicators of commercial enterprises.
Keywords: fractal analysis, cluster analysis, system stability, crisis situations, artificial neural networks (ANN).

Introduction. The current unstable economic situation calls for developing and improving methods for predicting crisis situations for the stock and securities market. Of considerable interest in the field of modeling economic processes is fractal analysis - a mathematical algorithm for identifying a single numerical parameter for describing multilevel structures, which are, in particular, dynamic economic systems. It is known that the financial market before and after the crisis is described by the same laws as phase transitions in physics and technical systems. At the same time, the dynamics of stock indices around the "critical point" are similar to a cardiogram or a seismograph curve. E. Peters, R. M. Cronauer, E. Feder write in their works that next to the "critical point" the chart of stock indices is a fractal [1], [2], [3]. For these objects, the fractal dimension is the index of curve complexity [4], [5].

This paper describes the method of joint application of fractal and cluster analyzes used in the study to predict and assess the degree of stability of the economic system. The developed method was considered on the example of the analysis of foreign currency quotations. The study of the euro in the period 2010-2017 allowed establishing a connection of its state with an indicator of the fractal dimension. In addition, using the mechanisms of artificial neural networks [6], it was possible to form a set of clusters containing time gaps with different levels of stability of the studied exchange rate, and also to use this information to automatically classify new data.

Methods. The basis of the fractal analysis of a time series of economic indicators is processing by scaling a fractal object imposed with the geometric carriers with different cell sizes [7]. It is necessary to begin to measure the length of any curved line. The measurement process itself is to calculate the number that the selected scale fits on the curve. Then the scale changes, and the measurement process is repeated. Denote the scale of measurement as δ. Calculate the number of steps $N(\delta)$ required to measure the curve from one end to the other. The product of the measured number of steps $N(\delta)$ by the pre-selected scale δ determines the desired length $L(\delta)$:

$$L(\delta)= N(\delta)\cdot\delta.$$

Between the number of steps $N(\delta)$ and the scale δ there is the following power-law dependence:

$$(\ ) = \ \cdot \ ^{-}\ .$$

Taking logarithm of the last equation, we get:

$$\ln \ (\ ) = \ln \ + \ \cdot \ln\frac{1}{}. \qquad (1)$$

For any curve that is a fractal, it is true that there is a linear relationship between ln N and $\rightarrow$ 0, we get

$$= \lim_{\rightarrow 0} \frac{\ln \ (\ )}{\ ^{-1}}.$$

D is the fractal dimension of the object under consideration. In other words, the coefficient for the variable ln $1/\delta$ in the linear dependence (1) is the fractal dimension.

As a curve, which length is to be measured, we choose the line of the time series of quotations of financial indicators. For individual periods of a time series of financial indicators, such as a month, quarter, etc., the fractal dimension is calculated in the manner described above. We obtain a set of data containing information about the selection of objects.

The obtained absolute values of the fractal dimensions themselves are not of particular interest for the study. Despite the presence of a clear connection between periods with different economic situations and fractal dimension values, it is impossible to establish specific intervals of the latter values, which could indicate crisis, pre-crisis and post-crisis or stable periods, since their values may significantly depend on the units of measurement of quotations. Thus, the values of fractal dimensions obtained during calculations with the euro and Chinese yuan, in similar situations, could differ by an order of magnitude (at the same time, the graphs of courses during periods of crisis in Russia are very similar). The problem lies in the difference of scales along the Y axis, that is, the direct value of a particular currency in the two cases considered. Thus it is impossible to fix the specific value of the obtained indicators as corresponding to a critical, pre-crisis or stable situation. The most significant factor is the relative changes in the values found and their correlation with certain periods of the economic situation in the corresponding period of time. In this connection, it became necessary to automate the assignment of the belonging of one or another value of fractal dimensions to a specific group of values (cluster). For this purpose, the authors applied the technology of artificial neural networks, which allowed them to carry out cluster analysis and relatively homogeneously group the objects [8]. We shall note that the task of clustering relates to statistical processing, as well as to a wide class of unsupervised learning tasks. In the course of clustering, the input vectors (images) are placed in categories (clusters) so that the vectors that are similar in a certain sense are in the same category. At the same time, the difference from the similar classification problem is that the set of categories is not initially defined and is determined during the training of the neural network. It also requires an economic assessment of the distribution of objects in clusters. There are various clustering methods. However, those of them that are based on the use of artificial neural networks have certain advantages. Being a development of classical clustering methods, they are a more flexible tool when applied to large arrays of data, and are also easier to automate.

To verify the hypothesis proposed in this study, the Wolfram "Mathematica" mathematical package [9], [10] was used. It calculated the values of the fractal dimensions for each time interval and produced a cluster analysis, which includes the step of building the clustering function for the analyzed data. Subsequently, this function was used to automatically classify newly received data without the need for economic research.

Results. For experimental verification of the methods of fractal analysis, the open data of the archives of foreign exchange rates from the Finam website (www.finam.ru) were used. In particular, the data on the EURRUB_TOM instrument, which shows the contract value for a lot in euros for Russian rubles with a maturity of obligations on the first working day following the day of trading on the Moscow Exchange, was investigated. Data was collected with a frequency of 1 day for the period from 12/30/2009 to 12/21/2017. In order to ensure a uniform distribution of measurements of the course on the timeline, on the days of the absence of trading (weekends and holidays), data from the nearest previous working day were substituted. The obtained data are presented in the form of a graph of quotations in Fig. 1.

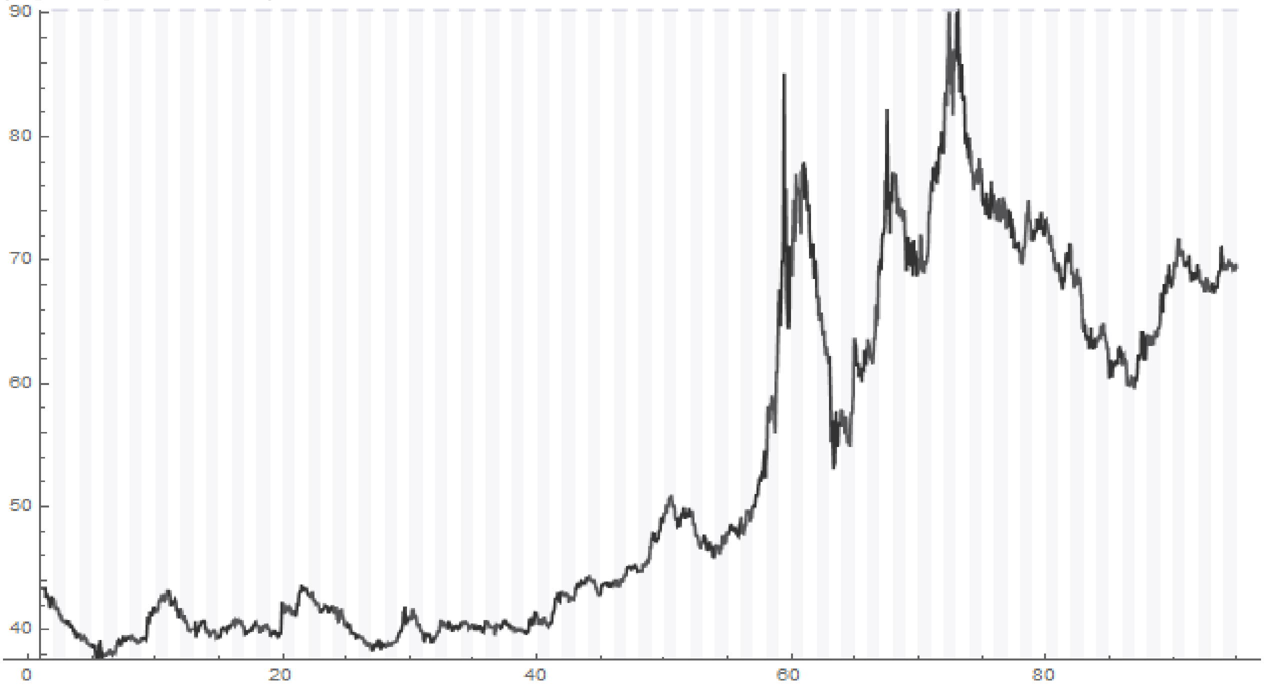

Fig. 1. A flow chart of the euro for the period of 30.12.2009 - 21.12.2017

The data set formed in this way was divided into n parts corresponding to the number of months in the study period, for each of which the value of the fractal dimension was calculated. In our case, they amounted to 94.

It was found that at the time when the course changes were relatively small (for example, in periods 37-39, which corresponds to quarter 1, 2013), the fractal dimension was in the range of 1.00073-1.00493. At the same time, during the crisis, for example, in the periods 59-60 (the end of 2014 - the beginning of 2015), as well as the period 72 (January 2016), the value of the observed parameter increased to values of the order of 1.12 - 1.13.

Using the mechanism of formation of artificial neural networks, built into the Wolfram “Mathematica” package, the clustering function $_1$ was constructed to divide the values of fractal dimensions into 5 clusters.

Figure 2 presents a comparative analysis of flow charts of the euro exchange rate quotations (solid line) and changes in the values of fractal dimensions (dashed line). Since the considered values can only be in the interval from 1 to 2, for clarity, the chart presented their 1000 times increased fractional parts increased ($' = (\quad - 1) \cdot 1000, \quad = 1$).

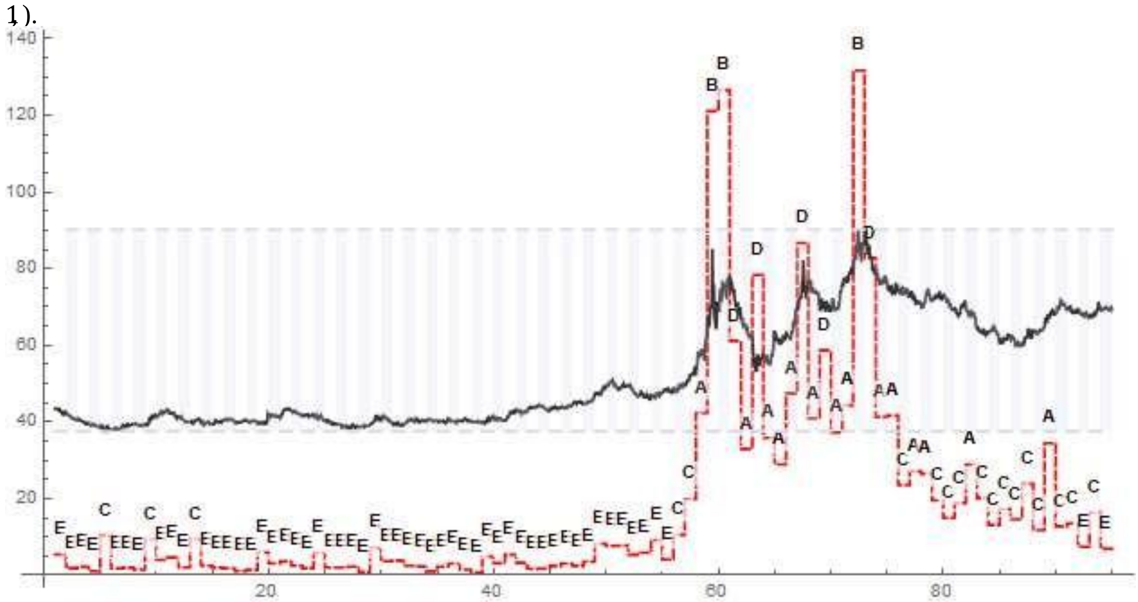

Fig. 2. Comparative analysis of the dynamics of the euro and the values of fractal dimensions for the period of 30.12.2009 - 21.12.2017.

With the help of the comparative analysis, the main 5 clusters were identified that correspond to a certain trend: “A” - pre-crisis situation, “B” - acute phase of the crisis, “C” - exit from the stability zone, “D” - crisis, “E” - stable state.

It is worth noting certain shortcomings of the proposed model. In the given case only separate periods are considered without taking into account their sequence. Therefore, unfortunately, we have no additional information about the aggravation or decline of the crisis situation. To eliminate these shortcomings, we considered not only the values of the fractal dimensions themselves, but also the relationship of the corresponding value in the current month to the previous one. Figure 3 shows the classification made with respect to the dimensions using the generated function $_2$.

When specifying the values of relations of fractal dimensions on the chart, for clarity, the following transformation was used for the relations under study:

$$' = (\quad - 1) \cdot 1000,$$

where

$$= \begin{cases} \frac{}{_{-1}}, & > 1 \\ 1, & = 1 \end{cases}, \quad = 1.$$

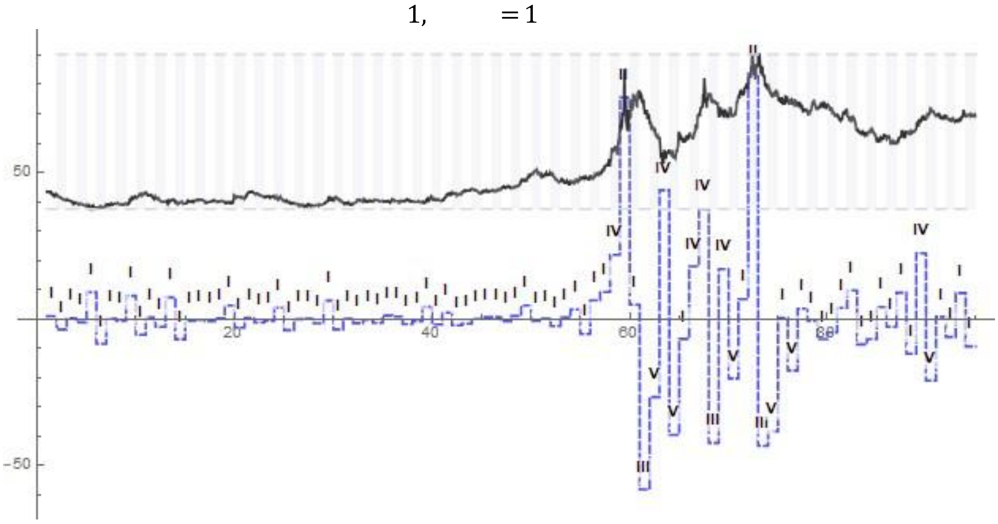

Fig. 3. Comparative analysis of the dynamics of the euro and the values of relations of fractal dimensions for the period of 30.12.2009 - 21.12.2017.

Considering the period of onset of the crisis situation of the end of 2014 - beginning of 2015 (period 59-61), it can be noted that the large positive values here correspond to the acute phase of the crisis (see, for example, period 59, cluster II), while the crisis continues (period 60) the ratio drops and falls into the stability cluster (I), the decline of the acute phase of the crisis in the 61 period leads to the appearance of negative values on the relationship graph (cluster III). Cluster IV can be referred to as “aggravating crisis”, and cluster V - “moderate stabilization”. Thus, having done a comparative analysis of the dynamics of the euro exchange rate quotations and the values of the relations of fractal dimensions, five clusters were identified that characterize the economic situation for the time series under study, namely: I - stability, II - acute phase of crisis, III - decline of acute phase, IV - aggravating crisis, V - moderate stabilization.

In order to distinguish with greater certainty, the phases of a “smooth economic situation” and “stable crisis”, it was decided to use clustering according to two parameters - the fractal dimension and the ratio of fractal dimensions.

As a result, after combining these two methods, the following result was obtained - the clustering function $_3$,, shown in Fig. 4.

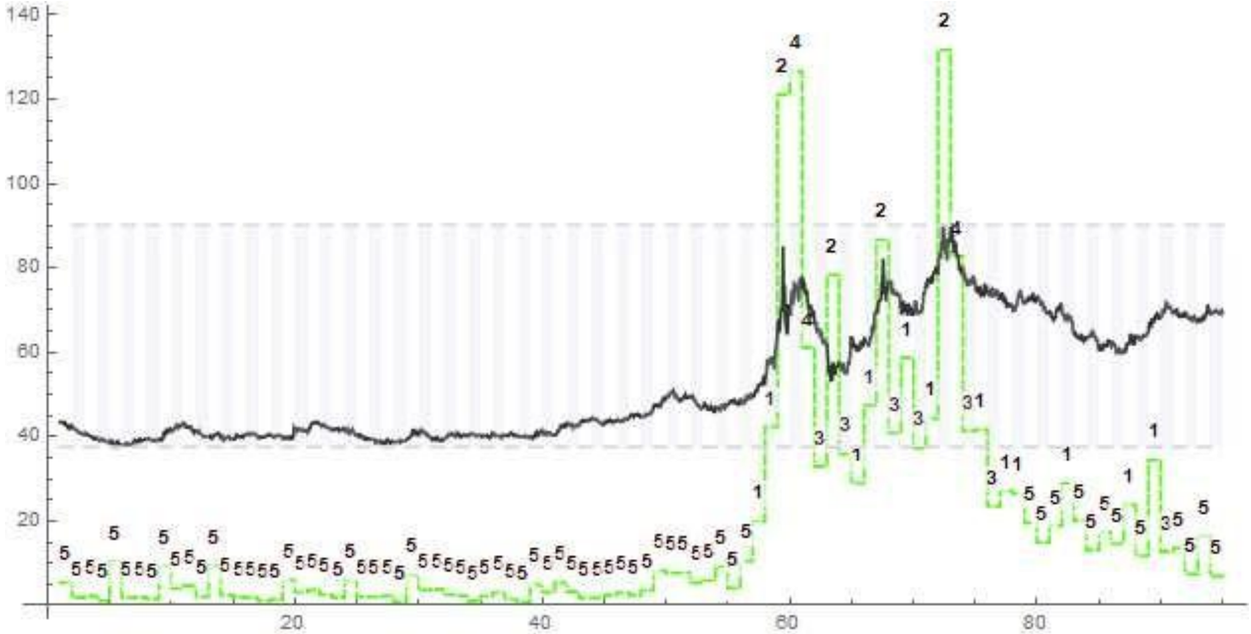

Fig. 4. Comparative analysis of the dynamics of the euro exchange rate quotations and the results of clustering on two indicators for the period of 30.12.2009 - 21.12.2017.

During the analysis of the obtained results, we identified the following clusters: 1 - a dangerous situation; 2 - the beginning of the acute phase of the crisis; 3 - recession of the crisis; 4– continuation of the crisis; 5 - stability.

For verification, a control sample was used - the values of the euro exchange rate quotations for the period 20.12.2017 - 24.08.2018.

Figure 5 presents a comparative analysis of the dynamics of the euro exchange rate quotations and the values of fractal dimensions for the period of 20.12.2017 - 25.08.2018. The classification was carried out using the $_1$ function.

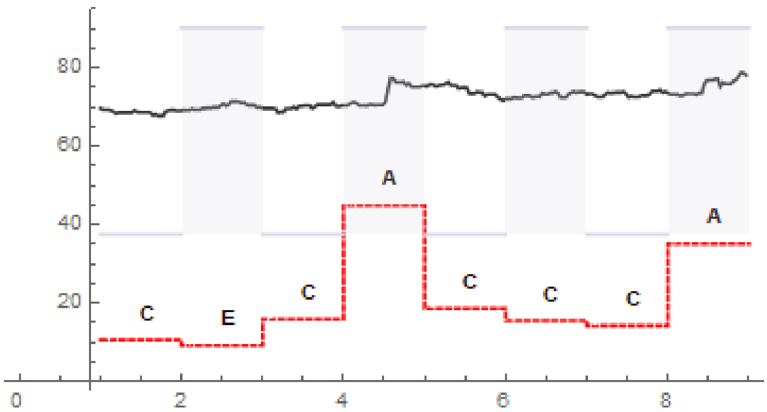

Fig. 5. Comparative analysis of the euro exchange rate quotation and the values of fractal dimensions for the period of 12.20.2017 - 08.24.2018.

It can be seen that periods 4 and 8 were classified by this function as dangerous (class “A”), 1, 3, 5, 6 and 7 - as not quite stable periods (class “C”). Period 2 - as stable (class "E").

The use of the clustering function $_2$ ensured the following results for test data (Fig. 6).

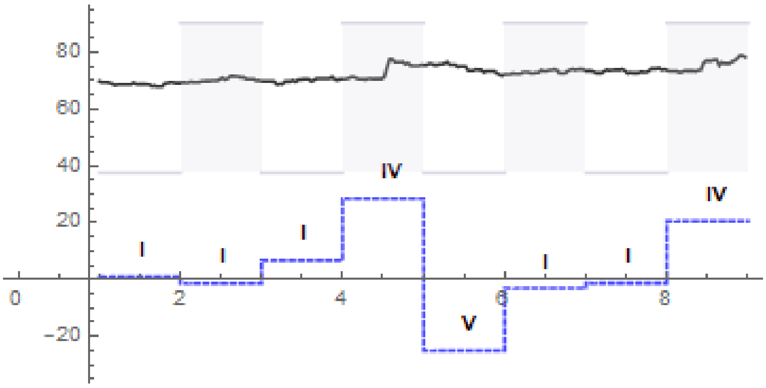

Fig. 6. Comparative analysis of the dynamics of the euro exchange rate quotations and the values of relations of fractal dimensions for the period of 20.12.2017 - 24.08.2018.

In this case, periods 1, 2, 3, 6, and 7 are marked as stable (class I); periods 4 and 8 showed an increase in crisis phenomena, and period 5 – a moderate stabilization.

Fig. 7 shows the conducted classification using the previously constructed function $_3$.

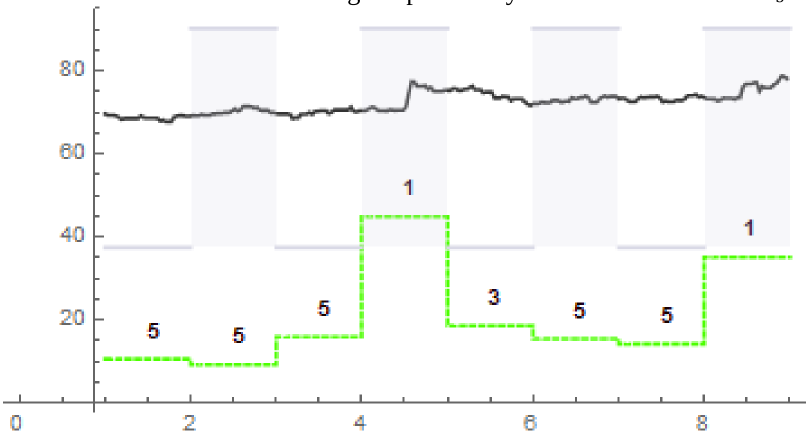

Fig. 7. Comparative analysis of the dynamics of the euro exchange rate quotations and the results of clustering carried out on the basis of a set of indicators for the period of 12.20.2017 - 08.24.2018.

Three states were singled out here. Class 5 (periods 1, 2, 3, 6 and 7) - period of stability, class 1 (periods 4 and 8) - a dangerous situation, class 3 (period 5) - a decrease in tension.

Discussion. The authors conducted the data clustering using three functions: by the magnitude of the fractal dimension ($_1$), by the magnitude of the change in the fractal dimension ($_2$), and by a pair of these two criteria in total ($_3$). After analyzing all three situations and testing all the cases in the control sample, the authors concluded that when clustering took place according to a set of criteria, namely the value of the fractal dimension and the value of the relative change of the fractal dimension, more reliable results were obtained. In this case, the algorithm with a high degree of accuracy began to recognize the difference between a “smooth economic situation” and a “stable crisis”, which is important information for decision-making by investors. The resulting clustering functions can be used in the future to classify new data without additional economic research.

Summary. This methodology, considered on the example of euro exchange rate quotations, is universal and can be applied both for other currencies rates and for securities, as well as financial indicators of commercial enterprises, such as, for example, the revenue of companies. It is based on a mathematical apparatus and does not require calculating any additional economic indicators characterizing the dynamics of the exchange rate or the work of commercial enterprises.

Conclusion. The evolution of science and progress in the modern world can no longer restrict itself to deepening into one specific industry. Economic studies increasingly require the introduction of a complex mathematical apparatus. However, even the simplest statistical studies are not enough for a complete and extensive description of the work of economic systems. In this paper, we reviewed the quotations of securities in terms of fractal analysis. On the basis of such consideration of these economic objects, an algorithm was developed for determining the trend of the time series using fractal and cluster analysis with the help of artificial neural networks. The developed methodology can be considered as a risk management technique, in particular in case of a risk assessment for investors.

Acknowledgements. This work was performed in accordance with the Russian Government Program of Competitive Growth of Kazan Federal University

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