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THE POWER OF NO-LAG TECHNICAL INDICATORS IN ALGORITHMIC TRADING

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Abstract. *In the framework of technical analysis for algorithmic trading we introduce an original approach to classical technical indicators. For this, we consider technical indicators as bounded operators: this more abstract, but also more algorithmic view enables us to define in a very simple way the no-lag versions of these tools. Delay in response is indeed a major drawback of many classical technical indicators used in algorithmic trading, which often leads to a wrong information. On the contrary, with the no-lag versions of the indicators that we study here, we get better information that is closer to the instantaneous values of the securities, hence a better expected rate of return of the trading system in which they occur. After having recalled the definitions of weighted and exponential averages as bounded operators, we prove that the lag possesses a fundamental property that is very useful to create no-lag versions of technical indicators. This being done, we apply our results to a basic trading system and test it on the S&P 500 index, in order to compare the classical Elder's impulse system with its no-lag version and the so-called Nyquist-Elder's impulse system: we observe on this example that the no-lag versions of indicators lead to much more profitable systems. More precisely, the Nyquist-Elder's impulse system is much better than the Elder's impulse system without lag, which is itself better than the classical impulse system: the information given by Nyquist-Elder's impulse system is indeed closer to the instantaneous value of the S&P 500 index since it has less delay than the classical impulse system: Nyquist-Elder's impulse system is even the closest to the instantaneous value among the three ones. We eventually compare the profit/loss of four portfolios (a portfolio that replicates S&P 500 index, and one for every of the three impulse systems) in order to better understand the time dynamics of our three Elder's impulse systems. As far as we can see, we also notice a lower draw-down for the portfolio associated to the system using the Nyquist-Elder's impulse system than for the other ones, and this portfolio seems to be more resistant to bearish periods.*

Keywords: *bounded operators; technical indicators without lag; algorithmic trading.*

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СИЛА ТЕХНІЧНИХ ІНДИКАТОРІВ БЕЗ ЗАПІЗНЕНЬ В АЛГОРИТМОВІЙ ТОРГІВЛІ

Бутін, Ф. Сила технічних індикаторів без запізнень в алгоритмовій торгівлі. *Вісник соціально-економічних досліджень* : зб. наук. праць. Одеса : Одеський національний економічний університет. 2021. № 1 (76). С. 90–99.

Анотація. У статті при виконанні технічного аналізу для алгоритмічної торгівлі представлено оригінальний підхід до використання класичних технічних індикаторів. Задля цього технічні індикатори були розглянуті як обмежені оператори: це більш абстрактне, але, водночас, більш алгоритмічне представлення дозволяє дуже простим способом визначати версії цих інструментів без запізнення. З'ясовано, що запізнення з відповіддю дійсно є серйозним недоліком багатьох класичних технічних індикаторів, які використовуються в алгоритмічній торгівлі, що часто призводить до невірної інформації. Протилежна ситуація з версіями індикаторів без запізнення, які розглянуто у статті, ми отримуємо більш точну інформацію, яка ближче до миттєвих значень цінних паперів, отже, очікується найкраща рентабельність торговельної системи, в якій вони зустрічаються. Після того, як наведено визначення зважених і експоненційних середніх як обмежених операторів, доведено, що затримка має фундаментальну властивість, яка дуже корисна для створення версій технічних індикаторів без запізнення. Отримані результати застосовані до базової торговельної системи і протестовані на індексі S&P 500, щоб порівняти класичну імпульсну систему Елдера з її версією без запізнення і так званою імпульсною системою Найквіста-Елдера. Цей приклад ілюструє, що версії індикаторів без запізнення призводять до набагато більш прибуткових систем. Точніше, імпульсна система Найквіста-Елдера набагато краще, ніж імпульсна система без запізнення, а також класична імпульсна система: інформація, що надається імпульсною системою Найквіста-Елдера, дійсно ближче до миттєвого значення S&P 500 індексу, оскільки він має меншу затримку, ніж класична імпульсна система: імпульсна система Найквіста-Елдера навіть найбільш близька із трьох до миттєвого значення. Зрештою, було порівняно прибуток/збиток чотирьох портфель (портфель, який копіює індекс S&P 500, і по одному для кожної із трьох імпульсних систем), щоб краще зрозуміти тимчасову динаміку наших трьох імпульсних систем Елдера. Проведене дослідження дозволяє отримати результат, який ілюструє менше просідання для портфеля, пов'язаного з системою, що використовує імпульсну систему Найквіста-Елдера, ніж для інших, і цей портфель здається більш стійким до ведмежих періодів.

Ключові слова: обмежені оператори; технічні індикатори без затримки; алгоритмічна торгівля.

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СИЛА ТЕХНИЧЕСКИХ ИНДИКАТОРОВ БЕЗ ЗАПАЗДЫВАНИЯ В АЛГОРИТМИЧЕСКОЙ ТОРГОВЛЕ

Бутин, Ф. Сила технических индикаторов без запаздывания в алгоритмической торговле. Вестник социально-экономических исследований : сб. науч. трудов. Одесса : Одесский национальный экономический университет. 2021. № 1 (76). С. 90–99.

Аннотация. В статье при выполнении технического анализа для алгоритмической торговли представлен оригинальный подход к использованию классических технических индикаторов. Для этого технические индикаторы были рассмотрены как ограниченные операторы: это более абстрактное, но, в то же время, более алгоритмическое представление позволяет довольно простым способом определять версии этих инструментов без запаздывания. Определено, что запаздывание с ответом действительно является серьезным недостатком многих классических технических индикаторов, используемых в алгоритмической торговле, что часто приводит к неверной информации.

Противоположная ситуация с версиями индикаторов без запаздывания, которые рассмотрены в статье, мы получаем более точную информацию, которая ближе к мгновенным значениям ценных бумаг, следовательно, ожидается лучшая рентабельность торговой системы, в которой они встречаются. После того, как приведено определение взвешенных и экспоненциальных средних как ограниченных операторов, доказано, что задержка имеет фундаментальное свойство, которое очень полезно для создания версий технических индикаторов без запаздывания. Полученные результаты применены к базовой торговой системе и протестированы на индексе S&P 500, чтобы сравнить классическую импульсную систему Элдера с ее версией без запаздывания и так называемой импульсной системой Найквиста-Элдера. Этот пример иллюстрирует, что версии индикаторов без запаздывания приводят к гораздо более прибыльным системам. Точнее, импульсная система Найквиста-Элдера намного лучше, чем импульсная система без запаздывания, а также классическая импульсная система: информация, предоставляемая импульсной системой Найквиста-Элдера, действительно ближе к мгновенному значению S&P 500 индекса, поскольку он имеет меньшую задержку, чем классическая импульсная система: импульсная система Найквиста-Элдера даже наиболее близка из трех к мгновенному значению. В конечном итоге, было проведено сравнение прибыли/убытка четырех портфелей (портфель, который копирует индекс S&P 500, и по одному для каждой из трех импульсных систем), чтобы лучше понять временную динамику наших трех импульсных систем Элдера. Проведенное исследование позволяет получить результат, который иллюстрирует меньшую просадку для портфеля, связанного с системой, использующей импульсную систему Найквиста-Элдера, чем для других, и этот портфель кажется более устойчивым к медвежьим периодам.

Ключевые слова: ограниченные операторы; технические индикаторы без запаздывания; алгоритмическая торговля.

JEL classification: C600; C630; G110; G150; G170

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1. Introduction and organization of the paper

Delay in response is a major drawback of many classical technical indicators used in algorithmic trading, and this often leads to useless or wrong information. The aim of this paper is to define as operators some of these classical indicators, in order to give and study their no-lag versions: these no-lag versions provide better information that is closer to the instantaneous values of the securities, hence a better rate of return of the trading system in which they occur.

The first section of the article will be devoted to the study of the lag of a weighted moving average and to the presentation of its fundamental property; we will also make use of Nyquist criterium. These results will enable us to give no-lag versions of well-known indicators that we will compare through a very simple example of trading system: this example will show the power of no-lag indicators.

For any security, we denote by $x = (x_0, x_1, x_2, \dots)$ the bounded sequence that consists of its prices at times t_0, t_1, t_2, \dots . For example, we may consider the daily or monthly prices of this security. We also denote by τ the difference between two consecutive time measures, i.e. $\tau = t_n - t_{n-1}$.

2. Lag of a weighted moving average

2.1. Weighted and exponential averages

The *weighted moving average* with p periods and the weights $w = (w_0, w_1, \dots, w_{p-1})$ is the bounded operator M_w defined, for every price series x , by $M_w(x) = y$, where

$$y_n = w_0x_{n-p+1} + w_1x_{n-p+2} + \dots + w_{p-1}x_n.$$

We can also define $M_w^2 = M_w \circ M_w$ and $M_w^k = M_w \circ M_w \circ \dots \circ M_w$ for every positive integer k .

When we want to give more importance to the most recent prices, we usually use *exponential moving averages*, which are bounded operators defined as follows: for α between 0 and 1, we set $E_\alpha(x) = y$, with $y_0 = x_0$ and, for every positive integer n ,

$$y_n = \alpha x_n + (1 - \alpha)y_{n-1}.$$

If α has the form $\frac{2}{p+1}$, we denote the exponential moving average by E_p and we call p the *number of periods* of E_p .

2.2. The fundamental property of the lag

Here, we shall use some results obtained by Patrick Mulloy in the article [8] (see also [2; 4]). The *lag* of the weighted moving average M_w is defined by

$$lag(M_w) = \tau(w_0(p - 1) + w_1(p - 2) + \dots + w_{p-2}).$$

For example, for the simple weighted moving average with p periods defined by

$$w = \left(\frac{2}{p(p + 1)}, \frac{4}{p(p + 1)}, \dots, \frac{2p}{p(p + 1)} \right),$$

its lag equals $\frac{(p-1)\tau}{3}$.

We can show that the lag of a weighted moving average possesses the following fundamental property: the lag of $a_0M_w^0 + a_1M_w + \dots + a_dM_w^d$ is equal to $(a_1 + 2a_2 + \dots + da_d) \times lag(M_w)$. In a more concise way, this means that for every polynomial Q , we have the formula $lag(Q(M_w)) = lag(M_w)Q'(1)$. For math lovers, the proof of this result can be found at the end of the article.

We can now get no-lag versions of weighted moving averages: we can show that $2M_w - M_w^2$ is the only combination of M_w and M_w^2 of the form $aM_w + bM_w^2$ with $a + b = 1$ whose lag is zero. We denote it by $P(M_w)$: this no-lag weighted moving average will be very important in the following. Let also note that $3M_w - 3M_w^2 + M_w^3$ is an interesting no-lag combination of degree 3.

2.3. Using Nyquist criterium

Using Nyquist criterium is another way to reduce the lag of a moving average. Here we use some results obtained by Dürschner in his article [3] and consider two simple

weighted moving averages M_{w_1} and M_{w_2} with respectively p_1 and p_2 periods: their weights are thus defined by

$$w_1 = \left(\frac{2}{p_1(p_1 + 1)}, \frac{4}{p_1(p_1 + 1)}, \dots, \frac{2p_1}{p_1(p_1 + 1)} \right),$$

$$w_2 = \left(\frac{2}{p_2(p_2 + 1)}, \frac{4}{p_2(p_2 + 1)}, \dots, \frac{2p_2}{p_2(p_2 + 1)} \right).$$

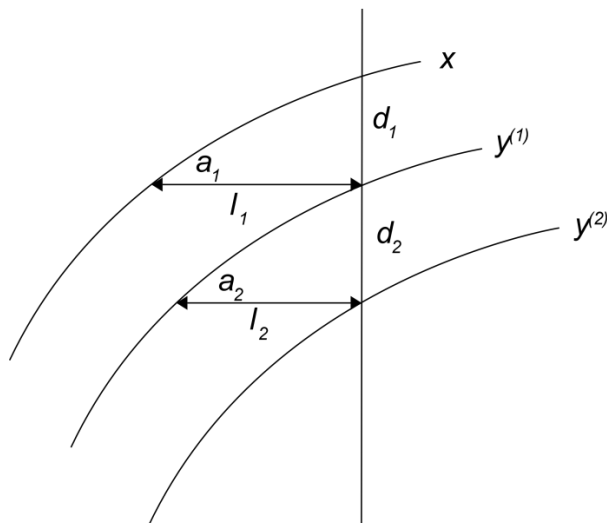


Fig. 1. Nyquist criterium ($\alpha = \frac{p_1 - 1}{p_2 - 1} \simeq \frac{l_1}{l_2}$)
 Source: developed by the author from article [3]

We set $\alpha = \frac{p_1 - 1}{p_2 - 1}$. Then, the Nyquist moving average with periods p_1, p_2 is the bounded operator defined by

$$N_{p_1, p_2} = (1 + \alpha)M_{w_1} - \alpha M_{w_2} \circ M_{w_1}.$$

In order to satisfy stability Nyquist criterium, the numbers p_1 and p_2 must verify the inequality $p_1 \geq 2 p_2$.

3. Technical indicators without lag

Here, we make use of the results of previous section in order to define technical indicators “without lag”. We begin with the exponential moving average and the MACD, and then we use these tools for Elder’s impulse system.

3.1. MACD without lag

Let us recall the definition of the MACD (moving average convergence divergence) introduced by Gerald Appel in 1979 in his financial newsletter “Systems and Forecasts” and presented in his book [1]: first we set $MACD = E_{12} - E_{26}$, then $MACDS = E_9 \circ MACD$ (the average series of MACD) and $MACDH = MACD - MACDS$ (the divergence series).

Now, for any parameter α between 0 and 1, the exponential moving average without lag is defined by $E_{\alpha,wl} = P(E_{\alpha})$. Then, we can define the MACD without lag by replacing every exponential moving average by its no-lag version: first we set $MACD_{wl} = E_{12,wl} - E_{26,wl}$ and $MACDS_{wl} = E_{9,wl} \circ MACD_{wl}$ (the average series of $MACD_{wl}$), and $MACDH_{wl} = MACD_{wl} - MACDS_{wl}$ (the divergence series). These three operators are bounded operators.

In the same way, the Nyquist-MACD can be defined by replacing every exponential moving average by a Nyquist moving average: first we set $MACD_N = N_{12,3} - N_{26,6}$, then $MACDS_N = N_{9,3} \circ MACD_N$ (the average series of $MACD_N$), and finally $MACDH_N = MACD_N - MACDS_N$ (the divergence series). Here again, these three operators are bounded operators.

3.2. Elder’s impulse system without lag

By using the exponential moving average without lag and the MACD without lag, we can give a “no-lag version” of Elder’s impulse system.

Let us first recall the definition of the impulse system introduced by Alexander Elder in his best-seller [5]: this technical indicator can be seen in an algorithmic way as an operator that associates to every time $t_0, t_1, t_2 \dots$, a color that depends on the price series of the security. The chosen colors are red, green, blue, and they are respectively denoted by “R”, “G” and “B”.

We give the algorithmic definition of Elder’s impulse system above, and we define its “no-lag version” and the so-called Nyquist-Elder’s impulse system in the box below.

<p>Let $x = (x_0, x_1, \dots)$ be the price series of a security.</p> <ul style="list-style-type: none"> - Elder’s impulse system is $IS(x) = (y_0, y_1, \dots)$, where $y_0 = B$ and for $n \geq 1$, <ul style="list-style-type: none"> • $y_n = G$ if $E_{12}(x)_n > E_{12}(x)_{n-1}$ and $MACDH(x)_n > MACDH(x)_{n-1}$ • $y_n = R$ if $E_{12}(x)_n < E_{12}(x)_{n-1}$ and $MACDH(x)_n < MACDH(x)_{n-1}$ • $y_n = B$ else - Elder’s impulse system without lag is $IS_{wl}(x) = (y_0, y_1, \dots)$, where $y_0 = B$ and for $n \geq 1$, <ul style="list-style-type: none"> • $y_n = G$ if $E_{12,wl}(x)_n > E_{12,wl}(x)_{n-1}$ and $MACDH_{wl}(x)_n > MACDH_{wl}(x)_{n-1}$ • $y_n = R$ if $E_{12,wl}(x)_n < E_{12,wl}(x)_{n-1}$ and $MACDH_{wl}(x)_n < MACDH_{wl}(x)_{n-1}$ • $y_n = B$ else - Nyquist-Elder’s impulse system is $IS_N(x) = (y_0, y_1, \dots)$, where $y_0 = B$ and for $n \geq 1$, <ul style="list-style-type: none"> • $y_n = G$ if $N_{12,3}(x)_n > N_{12,3}(x)_{n-1}$ and $MACDH_N(x)_n > MACDH_N(x)_{n-1}$ • $y_n = R$ if $N_{12,3}(x)_n < N_{12,3}(x)_{n-1}$ and $MACDH_N(x)_n < MACDH_N(x)_{n-1}$ • $y_n = B$ else
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Table 1. Algorithmic definitions of impulse systems

Source: developed by the author from book [5]

On the graph below we can see the three versions of Elder’s impulse system that we defined. As we can observe it, the beginning of the downtrend (denoted by 1 on the graph) in the index price is detected earlier by Nyquist-Elder’s impulse system than by

the no-lag version of Elder’s impulse system, which itself detects it earlier than its classical version.

In the same way, the beginning of the uptrend (denoted by 2 on the graph) and the beginning of the second downtrend (denoted by 3 on the graph) are first detected by Nyquist-Elder’s impulse system, then by the no-lag version and the classical version of Elder’s impulse system.

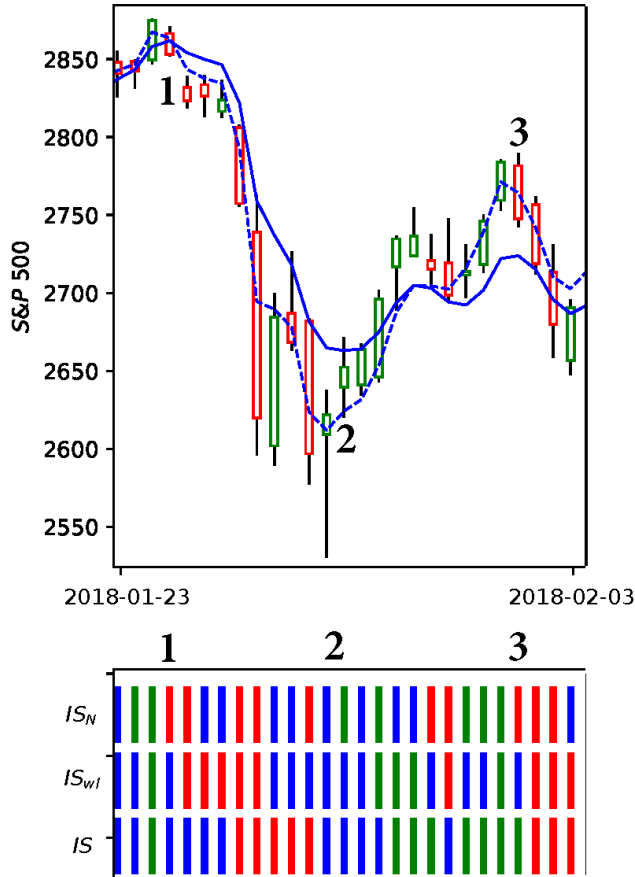


Fig. 2. Comparison of the three impulse systems
 Source: developed by the author

3.3. Comparison of the three versions of Elder’s impulse system

Here we use the very simple trading system given by the following algorithm (box Algorithm 1), in order to compare the three versions of Elder’s impulse system. This trading system simply consists of buying (resp. short selling) 1 E-Mini S&P 500 future when an impulse system given by a function f is green (resp. red) and selling it when this impulse system becomes red (resp. green).

Let us mention that in this trading system, every position is automatically closed one day before the end of the test. We use $x = (x_0, x_1, \dots)$ the daily price series of the S&P 500 index during the time period from 2017-11-01 to 2018-10-31. The reader who wants to learn more interesting information on trading systems can look at the books [7; 9].

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Algorithm
Require:  $x$  (price series),  $f$  (function)
For  $n \in [[0, d]]$ :
  Long entry: if  $f(x)_n = G$ :
    then buy 1 mini contract
  Short entry: if  $f(x)_n = R$ :
    then sell short 1 mini contract
  Long exit: if  $f(x)_n = R$ :
    then sell 1 mini contract
  Short exit: if  $f(x)_n = G$ :
    then buy 1 mini contract
    
```

Algorithm 1. Basic trading system
 Source: developed by the author

The figure below eventually shows the S&P 500 index from 2017-11-01 to 2018-10-31, with the Nyquist moving averages $N_{12,3}$ (dotted line) and $N_{26,3}$ (solid line), the graphs of $MACD_N$ and $MACD_{S_N}$ with the histogram $MACDH_N$, and the three versions of Elder’s impulse system (from bottom to top: IS , IS_{wl} and IS_N).

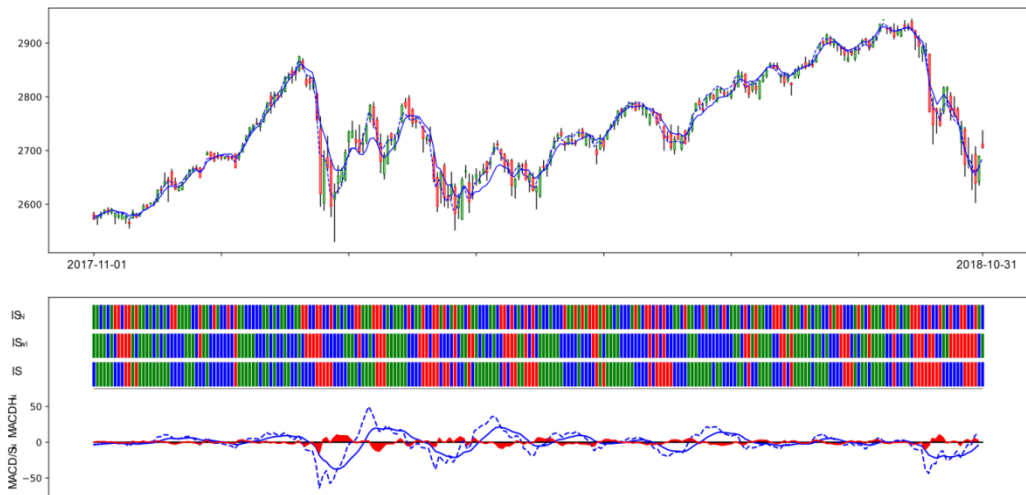


Fig. 3. S&P 500 index from 2017-11-01 to 2018-10-31
 Source: developed by the author

We use the algorithm above with $f = IS$, $f = IS_{wl}$ and $f = IS_N$, and we assume that the transaction cost for every entry/exit is \$3. The results are given by the following table, in which all prices are in USD. Let us note that the value of the S&P 500 index is 2 572.625 (resp. 2 706.125) on 2017-11-01 (resp. 2018-10-31), hence a profit of $TPI = \$6\,675$ for one mini contract during this period.

	$f = IS$	$f = IS_{wl}$	$f = IS_N$
Number of trades	27	35	52
Total net profit	13 549	20 489	31 395
Percentage of winning trades	52%	46%	50%
Average net profit per trade	502	585	604
Total net profit of winning trades (TP)	32 452	41 278	49 835
Average net profit per winning trade (AP)	2 318	2 580	1 917
Total net lost of losing trades (TL)	-18 903	-20 789	-18 440
Average net lost per losing trade (AL)	-1 454	-1 094	-709
Greatest lost between two winning trades	-10 231	-7 793	-3 761
Total net profit of long trades	11 322	10 249	14 930
Average net profit per long trade	755	539	515
Total net profit of short trades	2 227	10 240	16 465
Average net profit per short trade	186	640	716
Profit factor $TP/ TL $	1.72	1.99	2.70
Ratio $AP/ AL $	1.59	2.36	2.70
Ratio TP/TPI	4.86	6.18	7.46

Tab. 2. Comparison of the three trading systems
Source: developed by the author

4. Conclusion and perspectives

Comparing on this example the three versions of Elder's impulse system is very instructive. Here we observe in particular that the Nyquist-Elder's impulse system is much better than the Elder's impulse system without lag, which is itself better than the classical impulse system: the information given by Nyquist-Elder's impulse system is indeed closer to the instantaneous value of the S&P 500 index since it has less delay than the classical impulse system: Nyquist-Elder's impulse system is even the closest to the instantaneous value among the three impulse systems. Therefore, it leads to less wrong trade setups. We can also note that the number of trades as well as the average net profit per trade are increasing when f is respectively equal to IS , IS_{wl} and IS_N . And the repartition of profit among long and short trades is more uniform with IS_{wl} and IS_N than with IS .

In order to better understand the time dynamics of our three Elder's impulse systems, we consider the profit/loss of four portfolios as a function of time:

- portfolio 1 replicates S&P 500 index,
- portfolio 2 consists of profit/loss given by the algorithm above applied to $f = IS$,
- portfolio 3 consists of profit/loss given by the algorithm above applied to $f = IS_{wl}$,
- portfolio 4 consists of profit/loss given by the algorithm above applied to $f = IS_N$.

As we already noticed it, portfolio 4 is much more profitable than portfolio 3, itself better than portfolio 2, and of course much better than portfolio 1. Portfolio 4 also possesses a lower draw-down than the other ones, and it seems to be more resistant to bearish periods.

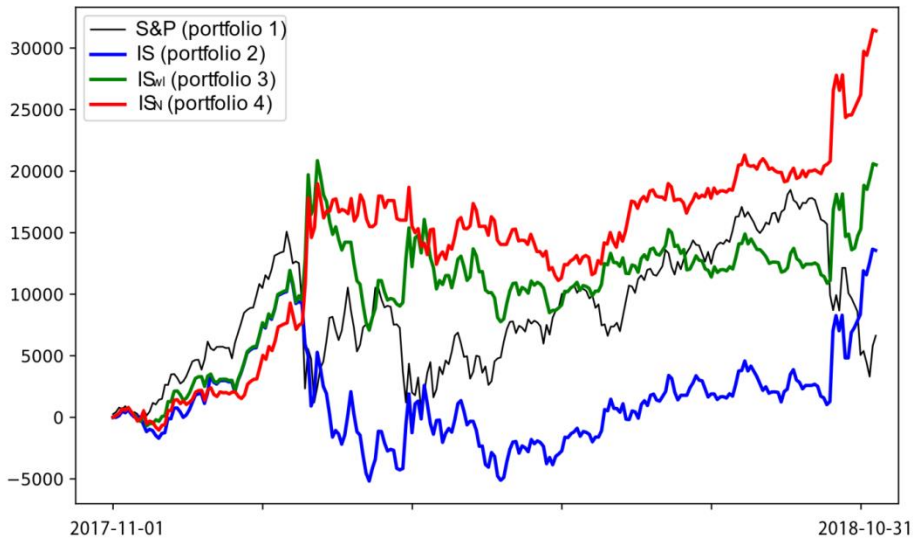


Fig. 4. Comparison of the four portfolios

Source: developed by the author

In view of the results of this example, it could be useful to study how no-lag impulse system and Nyquist-Elder’s impulse system behave in much more elaborate trading systems. Moreover, a statistic study could help us to determine how to adapt the parameters in order to get the fastest response to market motion.

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Appendix. Proof of the fundamental property of the lag

For every positive integer k , we have

$$lag(M_w^k) = \tau \sum_{i_1=0}^{p-1} \sum_{i_2=0}^{p-1} \dots \sum_{i_k=0}^{p-1} \prod_{j=1}^k w_{i_j} (k(p-1) - i_1 - i_2 - \dots - i_k),$$

$$lag(M_w^k) = \sum_{l=1}^k \tau \sum_{i_1=0}^{p-1} \sum_{i_2=0}^{p-1} \dots \sum_{i_k=0}^{p-1} w_{i_1} w_{i_2} \dots w_{i_k} (p-1 - i_l),$$

$$lag(M_w^k) = \sum_{l=1}^k \tau \sum_{i_l=0}^{p-1} w_{i_l} (p-1 - i_l)$$

since $\sum_{j=0}^{p-1} w_j = 1$. Hence $lag(M_w^k) = \sum_{l=1}^k lag(M_w) = k lag(M_w)$.

Finally, $lag(\sum_{k=0}^d a_k M_w^k) = \sum_{k=0}^d a_k lag(M_w^k) = lag(M_w) \sum_{k=0}^d k a_k$.